State Estimation in Electrical Networks

by

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The undersigned hereby certify that they have read and recommend to the Faculty of Graduate Studies for acceptance a thesis entitled "State Estimation in Electrical Networks" by Hossam Mosbah in partial fulfilment of the requirements for the degree of Master of Applied Science.

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DEDICATION

I dedicate this research to my parents.

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ABSTRACT

The continuous growth in power system electric grid by adding new substations lead to construct many new transmission lines, transformers, control devices, and circuit breakers to connect the capacity (generators) to the demand (loads). These components will have a very heavy influence on the performance of the electric grid. The renewable technical solutions for these issues can be found by robust algorithms which can give us a full picture of the current state of the electrical network by monitoring the behavior of phase and voltage magnitude.

In this thesis, the major idea is to implement several algorithms including weighted least square, extend kalman filter, and interior point method in three different electrical networks including IEEE 14, 30, and 118 to compare the performance of these algorithms which is represented by the behavior of phases and magnitude voltages as well as minimize the residual of the balance load flow real time measurements to distinguish which one is more robust. Also to have a particular understanding of the comparison between unconstraint and constraint algorithms.

LIST OF ABBREVIATIONS AND SYMBOLS USED

LIST OF ABBREVIATIONS

VAr	Random variables.
RTU	Remote Terminal Units.
SCADA	Supervisory Control and Data Acquisitions.
PMUs	Power measurements units.
LF	Load Frequency.
ED	Economic Dispatch.
AGC	Automatic Generation Control.
ECC	Energy Control Center.
WLS	Weighted Least Squares.
KF	Kalman Filter.
EKF	Extended Kalman Filter.
IPMs	Interior Point Methods.
GLS	Generalized Least Squares.
OLS	Ordinary Least Squares.
LAM	Least Absolute Method.
LP	Linear Programming.
SM	Simplex Method.
IPM-PD	Primal-Dual Logarithmic Method.
IPM-PC	Predictor-Corrector Logarithmic Method.
KKT	Kaurush-Kuhn-Tucker.
PSSE	Power System State Estimation.
PAOP	Power Operation and Planning.
UC	Unite Commitment.

LIST OF SYMBOLS

δ_i	Angles of bus voltages.
V_i	Magnitude of Voltages at buses.
P_i	Net Real Power injected into the system.
Q_i	Net Reactive Power injected into the system.
U_{iJ}	Real Power Flow through link i to j.
T_{iJ}	Reactive Power Flow through link i to j.
Z_t	Real Time Available Measurements.
h_t	Non-linear equations of Power System Flow.
v_t	Residuals of Real Time Available Measurements.
σ_t	Standard Deviations.
R_t	Variance of Measurements.
H_t	Jacobian Matrix of non-linear load flow equations.
x_t	State Variables including angles and voltage magnitude.
P_t	Covariance of the expected state variables values.
w_t	Residuals of State variables.
Q_t	Variance of State variables.
K_t	Gain of Kalman Filter.
$f(x_t)$	Objective Function of the residuals.
r_t	Residuals of Measurements in Optimization.
u, v	Slack Variables.
B_μ	Logarithmic Barrier Function.
μ	Barrier Parameter.

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CHAPTER 1 INTRODUCTION

1.1 MOTIVATION

The composition of an electric power system grid is complicated because it contains many internal and external components such as generators, transmission lines, transformers, circuit breakers, and loads. The networks undergo different states such as normal, emergency, and restorative states as well as being subject to missing certain data or having corrupted data which can make it difficult for the control center operator to determine the exact state of the network more complicated. The process of state estimation offers help to obtain the missing or corrupted data by estimating the missing data [1]. Recently, several researchers investigated alternative types of algorithms to enhance their performance to provide an accurate estimator for the network variables [2]. The major purpose of this research is to focus and compare the analytical performance of several state estimation algorithms and implement these algorithms on IEE14, 30, and 118 bus networks to evaluate their performance and to distinguish the best estimate obtained by these algorithms.

1.2 THESIS OBJECTIVES AND CONTRIBUTION

There are many issues in electrical networks that can lead to major and minor or even catastrophic faults. These issues can lead to major system outages. The communications between the system and the operator is necessary to respond to the outages in an early and timely manner. This thesis aims to compare several performances of state estimation algorithms on IEEE14, 30, and 118 bus electrical networks to have a good understanding and determine the best state estimation algorithm. The main contribution of this work is to determine the optimal estimate of state variables including angles and voltages magnitudes. We have contributed this idea using Interior Point Method including logarithmic barrier function which was very successful. And the second objective is to compare the methods by computing the errors as well as comparing the accuracy of the algorithms by computing the residuals for available real time measurements.

1.3 THESIS OUTLINE

This thesis is divided into five main chapters. The first chapter includes the concept of the thesis and the second chapter includes the literature reviews which begins with previous numerical methods in state estimation such as Newton Raphson and Gauss elimination and discuss the history of state estimation and supervisory control and data acquisition. The third chapter discusses modeling of the several state estimation algorithms and how to employ these algorithms in real time process. We include proofs of different algorithms including WLS algorithm, EKF algorithm, and IPM step by step to understand the composition of these algorithms as well as how they processed.

For many years, these algorithms have been employed, developed and used in wide applications in different areas. We use these algorithms to minimize the residual of balanced load flow equation and estimate the dynamic behavior of state variables to check the state of different electrical networks. In chapter four, we gather all the data that we obtain from several electrical networks in order to illustrate and discuss their applications in comparing the error of state variables including phase and magnitudes of voltage as well as the residual of net active power and net reactive power.

CHAPTER 2 LITERATURE REVIEW

2.1 INTRODUCTION

State estimation issues have been among the critical challenges in many different areas and the accuracy of state estimation results is important for realistic operational strategies. One of the important reasons of using state estimation is to determine variables and eliminating errors which may occur because of unreliable measurements of data [3].

State estimation mathematical formulations use the state vector of angles and voltages magnitudes in electrical networks to estimate the real time available measurements including net active and reactive power and power flows. The power flow equations are used by algorithms which have different compositions and strategies. These algorithms rely on the same concept of state estimation which is minimizing the balanced load flow residuals [3, 4].

State estimation is known be the heart of an integrated system which works as a set of sensor sometimes. All parameters and database flow forward from and backward to the state estimator to predict and update new state vector to stabilize the system and then move the information to the control center to analyze the information and take the appropriate decisions based on current state of the system and transmission network[4]

2.2 PREVIOUS NUMERICAL METHODS OF POWER FLOW

It is not straightforward to find the typical solution of the non-linear equations of power system. Several numerical methods have been employed to approximate the non-linear systems and one always hopes that successive systems of linear equations can yield a close solution to the actual non-linear equations. Several methods can approximate the non-linear equations of electric power systems [5].

Newton Raphson method.

Gauss Siedel method.

Optimization methods including convex and non-convex methods. There are two well-known methods described in many power system analysis books.

2.2.1 NEWTON RAPHSON METHOD

Many books cover power flow techniques using Newton Raphson method iterative method as a standard an iterative procedure employed to find the approximate estimates for solving non-linear power flow equations. Newton Raphson algorithm is based on the Taylor series expansion by taking the first order differentiation of Taylor series expansion which based on a Jacobian matrix and ignoring the high order terms of Taylor series expansion [6]. The process of the Newton Raphson method starts by guessing the initial values solution or flat starts of variables which are usually called state vectors including the angles and voltages magnitudes. After some iterations we expect this method to converge to close to the true values if they exist for the loading condition. The process of the iterations can be stopped when it reaches the specified tolerance of our state vectors or the specified error [7].

2.2.2 GAUSS -SIEDEL METHOD

Gauss Seidel method is employed to calculate the solution of the nonlinear equations successively. The basic concept of the Gauss-Seidel method is based on the successive approximation technique which converges if the mapping described by the non-linear equations is one of a contraction where the distance between successive approximations is reduced as iterations progress [9].

2.3 HISTORY OF STATE ESTIMATION

The methods of state estimation are based on the work of many. While Legendre is credited with introducing the least squares idea in his Memoir, some argue that Gauss at least shares in this credit.[8] In 1970 F. C. Schweppe introduced the concept of state estimation to electric power system networks and this concept has been developed since that time [9]. R. E. Larson, Tinney, and Peschon implemented this method in large electrical networks. [10].

R. E. Kalman introduced what has been named after him known as the Kalman filter in 1960 [11], since then, it has been successfully implemented in many applications [12-13.]

A. Debs and R.t E. Larson implemented the equations of Kalman filter in a dynamic power system state estimator [14].

2.4 STATE ESTIMATION IN ELECTRICAL NETWORKS

Usually, all the real time data in electrical networks which are gathered from different sensors on utilities substations (SCADA) are transmitted and aggregated including net active power, net reactive power, real power flow, active power flow, and complex voltages and transferred to the control center where computers-aided tools are used to monitor and analyze the data to have a complete picture of the current state of the network [15]. This data are simulated different time by a state estimator of the electrical network; however, the accuracy of SCADA system has been such as challenge in big electrical networks because of the lack of measurements at all times [16].

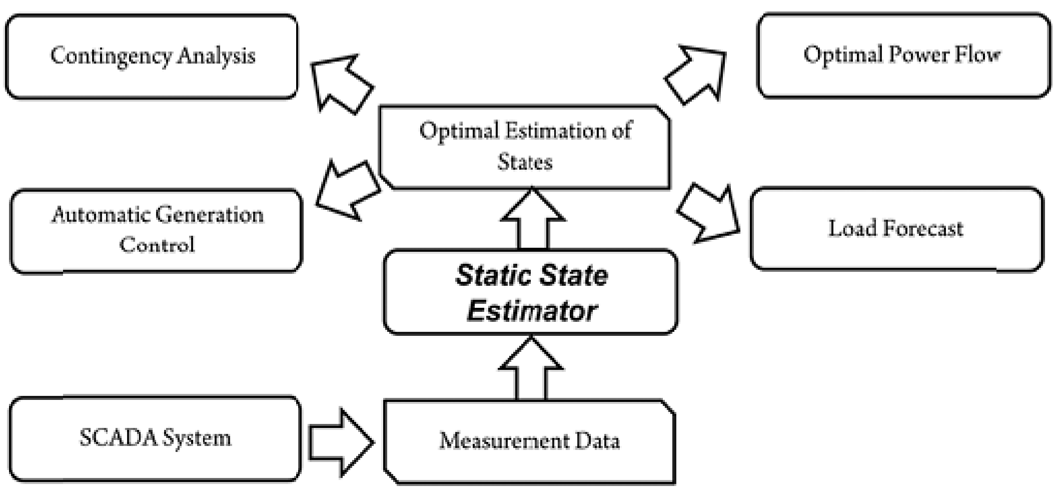

Figure 2-1: State estimation in electrical network[16]

Figure 2 demonstrate that static state estimator is the heart of any electric grid because it detects the past, present, and future state of this grid, so the real time measurements go through the static state estimator by using several algorithms such as WLS, EKF, and optimization (IPM) and find the optimal estimation for this grid, then based on the final results from the different algorithm estimators, we can assess the state of the electric grid.

2.5 SUPERVISORY CONTROL AND DATA ACQUISTION

Supervisory control and data acquisition has been involved in power system grid for well over 30 past years by monitoring and controlling large scale electric grid [17]. These data received from several sensors such as phase measurements unit (PMU) on several locations of the electric grid can be displayed by analogue and digital quantities. These sensors send the data from the electric grid to the remote terminal unit (RTU) by radio communications which transmits these measurements from the different substations to the computers in control room to estimate these data and have an accurate picture of the current state of the electric grid. SCADA systems are unable to avail and collect all the real time measurements (active and reactive power, active and reactive power net, angle, voltage) at all times, this leads to search for new tools to deal with the inadequacy of SCADA system [18].

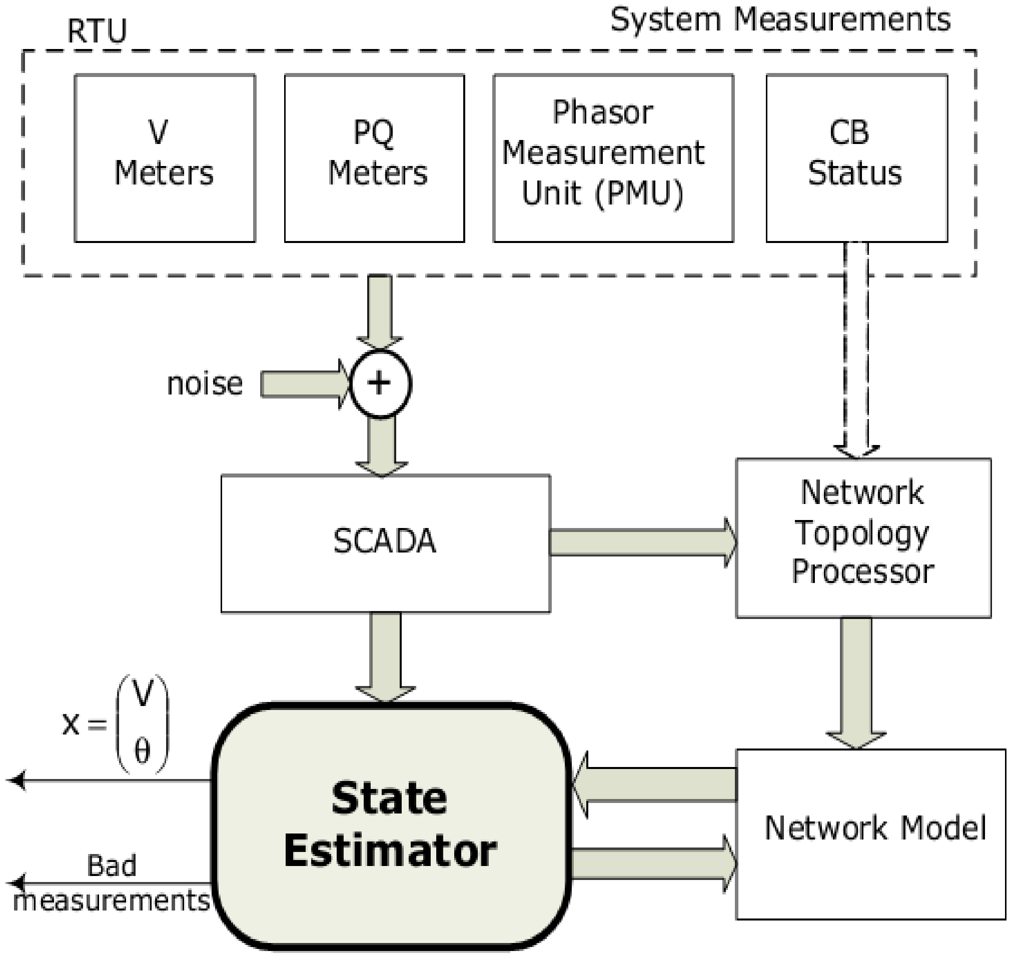

Figure 2-2: Supervisory control and data acquisition [18]

2.6 ENERGY CONTROL CENTER (EGC)

The architecture of control centers has been evolving for the past 20 years because of the revolution of communication and computer technologies in networks.

In the 1950's, collecting of electric networks' data were by analogue computers to monitor the load frequency (LF) and economic dispatch (ED). In the 1960's , digital computers were proposed and used in electric network which lead to transforming the way in which data are collected. By using remote terminal units (RTUs) which have been developed to aggregate the real time measurements including voltage, phase, real and reactive power by digital as well as to check the performance of several devices on the networks such as circuit breakers at substations through transmission lines which is important for the study of automatic generation control (AGC) [19], [20].

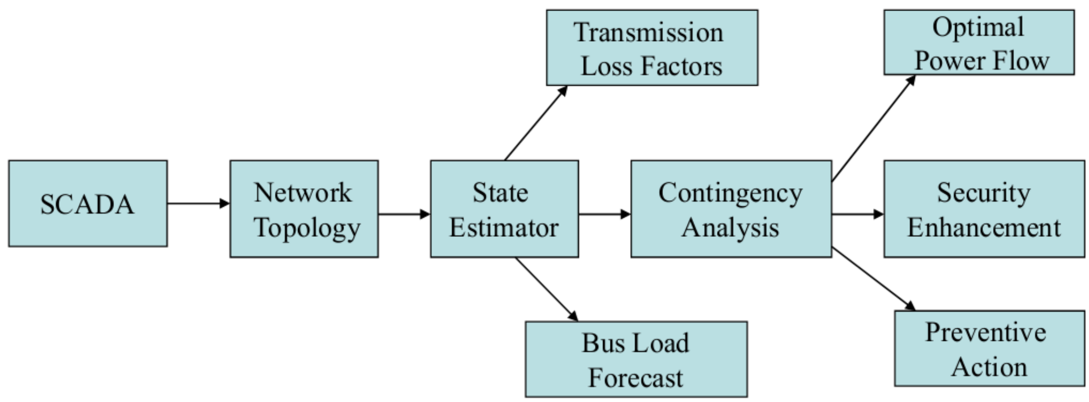

Figure 2-3: Diagram of energy control center[19]

The security control system re-directs the real-time measurements which come from SCADA to state estimator to estimate these measurements and eliminate the gross errors in measurements. The results of the state estimator flows through the contingency analysis which can be expressed as several types of disturbances such as transmission lines/generators outages. Contingency analysis can take a decision based on the output of the state estimator to respond the current state of the system [21].

To sum up, the Energy control center contains all the forecasting net of the electric grid. This center includes many computer- aided tools for the operators (engineers) to monitor and manage the electric grid [22].

CHAPTER 3 MODELING OF STATE ESTIMATION ALGORITHMS

3.1 INTRODUCTION

Several algorithms have been implemented in different electrical networks to evaluate the performance of these algorithms which have different levels of accuracies. Therefore by employing these algorithms in several electrical networks, we can clearly distinguish the best state estimation algorithm which can be based on the comparisons in errors and residuals of these algorithms. This comparison can assist the operators in energy control center (ECC) which we mentioned in chapter 2 to have an accurate understanding of the present and future of the electrical network state [23]. Implementing these algorithms in several electrical networks is not easy. However, all these algorithms have the same concept of state estimation which is minimizing the sum of the residuals; each algorithm has a specific composition and process sequence.

In this chapter, we discuss three state estimation algorithms such as WLS, EKF, and IPM as well as we will prove these algorithms to understand their details and the way they progress.

3.2 WEIGHTED LEAST SQUARES (WLS)

The earliest publication related to the least squares method was in 1805 by the French Mathematician Legendre' [24]. In 1909, Gauss, a German mathematician, emphasized that he used this method in late seventeen century [25]. In the eighteenth century, Francis Galton employed this method statistically for human measurements such as weights and heights data, as well as in cross-culture data which is now called auto-correlation or Galton's problem [26].

There are many ways to define weighted or generalized least squared (GLS.) It is the most efficient method that can accurately be used for small data sets. WLS is useful to estimate some variables among several methods to compute numerical values of parameters by minimizing the sum of squared deviations between observed responses and functional portions of certain models [27].

Generally, WLS is based on assumptions including linear and non-linear least square regression [28]. The major justification making WLS more appropriate for estimation is the heavy reliance of ordinary least squares (OLS) on homoscedasticity which means that all variances around the regression line have the same approximate residual that occurs when some variables are skewed and the other are not, so this can lead to inferential imprecise statements [29-30].

Recently, WLS was employed to solve some contingency load flow problems by transferring on-line data telemetered periodically to the Energy Control Center (ECC) which were discussed in chapter 1 and subsequently calculate all power flow parameters such as angles, voltages, net active power, and net reactive power. As a matter of fact, these measurements have always been associated with errors because of on-line system topology errors, infrequent malfunctioning of measuring instruments, as well as some measurement redundancy.

In this chapter we discuss the application of the Weighted Least Squares to evaluate (h) different power flow parameters such as the power angles and voltage magnitudes to find solutions for these errors which we have discussed earlier and how to use least estimation techniques to minimize the errors.

3.2.1 BASIC SOLUTION METHOD OF WLS

The basic formula of power flow equations is augmented by measurement errors caused by redundant measurements or noisy frequency measurements to allow good precision estimator. In power system analysis, a number of quantities need to be evaluated such as unknown voltages (V_{ij}), net active power (P_i), net reactive power (Q_i), real power flow (U_{ij}) and finally, reactive power flow (T_{ij}).

$$z_t = h\left(x_t^k\right) + v_t \tag{3.1}$$

$$\begin{bmatrix} z1 \\ z2 \\ zn \end{bmatrix} = \begin{bmatrix} h_1\left(x_1^k, x_2^k \ \dots\dots\dots\dots\dots\dots\dots\dots.. x_n^k\right) \\ h_2\left(x_1^k, x_2^k \ \dots\dots\dots\dots\dots\dots\dots\dots.. x_n^k\right) \\ h_n^k\left(x_1^k, x_2^k \ \dots\dots\dots\dots\dots\dots\dots\dots.. x_n^k\right) \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ v_n \end{bmatrix} \tag{3.2}$$

Where

z_t : m vector of the measurements.

$h(x_t^k)$: mxn matrix of non-linear equations of power system flow.

v_t : m vector of the measurements errors or residuals.

The major idea is to minimize the residuals to obtain the best estimates of parameters (x_t^k).

$$R_t = E(v_t^T * v_t) = E(v_t^2) = \sigma_t \tag{3.3}$$

Because we have independent parameters, equation (3.2) represents the variance of Gaussian-distributed measurements which is the square of the residual equals to the square of the standard deviation, thus R_t is a diagonal matrix.

$$\text{Standrad deviation}(\sigma_t) = \sqrt{}\text{var}(v_t^2) \tag{3.4}$$

By taking the square of both sides of equation (3.3,) we obtain,

$$\sigma_t^2 = \text{var}(v_t^2) = R_t \tag{3.5}$$

$$R_t = [\sigma_1^2, \sigma_2^2, \sigma_3^2, \dots \dots \dots, \sigma_n^2] \tag{3.6}$$

The standard deviation can offer an impression about the expected accuracy of an estimator.

The measurements can just be represented as a Z (true) which are actual or true vector of measured quantities; in other applications, they are called scheduled values, Z (estimator) is the vector of the measured values by updating different parameters such as angles per iteration and v_t or e is the measured errors or variance/residuals as in estimation theory which illustrates the difference (distance) between the estimated model and the standard model that is controlled by the mean and standard deviations.

We can use the linearized set of non-linear power flow equation by converting $h(x_t^k)$ to Hx_t^k :-

$$z_t = Hx_t^k + v_t \tag{3.7}$$

$$J\left(x_t^k\right) = \frac{1}{\sigma^2} * [z_t - Hx_t^k]^T [z_t - Hx_t^k] \tag{3.8}$$

$$\frac{dJ}{dx} = \left[z_t - Hx_t^k\right] * R_t^{-1} * H^T = 0 \tag{3.9}$$

$$z_t * R_t^{-1} * H^T = H * x_t^* * H^T \tag{3.10}$$

$$x_t^* = (H * R_t^{-1} * H^T) * R_t^{-1} * H^T * z_t \tag{3.11}$$

We can also work with the original non-linear equation of power system flow:

$$v_t = z_t - h\left(x_t^k\right) \tag{3.12}$$

$$J\left(x_t^k\right) = \frac{1}{\sigma^2} * (v_t^T * v_t) \tag{3.13}$$

$$J\left(x_t^k\right) = \frac{1}{\sigma^2} * [z_t - h(x_t^k)]^T [z_t - h(x_t^k)] \tag{3.14}$$

$$\frac{dJ}{dx} = 0 \tag{3.15}$$

$$0 = \frac{1}{\sigma^2} * [z_t - h(x_t^k)] * -H^T \tag{3.16}$$

$$0 = -\left[z_t - h\left(x_t^k\right)\right] * R_t^{-1} * H^T \tag{3.17}$$

$$0 = H^T * R_t^{-1} * [z_t - h(x_t^k)] \tag{3.18}$$

This equation represents the best estimator of x_t^k by expanding the non-linear equation using Taylor series expansion to approximate the non-linear equation of power systems flow.

$$h\left(x_t^k\right) \simeq h\left(x_t^k\right) + H * \Delta x_t^{k+1} = 0 \tag{3.19}$$

$$0 = H^T * R_t^{-1} * [z_t - h\left(x_t^k\right) - H * \Delta x_t^{k+1}] \tag{3.20}$$

$$H^T * R_t^{-1} z_t - H^T * R_t^{-1} h\left(x_t^k\right) - H^T * R_t^{-1} H * \Delta x_t^{k+1} = 0 \tag{3.21}$$

$$H^T * R_t^{-1} H * \Delta x_t^{k+1} = H^T * R_t^{-1} z_t - H^T * R_t^{-1} h\left(x_t^k\right) \tag{3.22}$$

$$\Delta x_t^{k+1} = (H^T * R_t^{-1} * H)^{-1} * H^T * R_t^{-1} * [z_t - h(x_t^k)] \tag{3.23}$$

$$x_t^{k+1} - x_t^k = (H^T * R_t^{-1} * H)^{-1} * H^T * R_t^{-1} * [z_t - h(x_t^k)] \tag{3.24}$$

$$x_t^{k+1} = x_t^k + (H^T * R_t^{-1} * H)^{-1} * H^T * R_t^{-1} * [z_t - h(x_t^k)] \tag{3.25}$$

x_t^k : n state vector matrix which presents the voltage and angles of power system flow.

H : $m \times n$ matrix which presents the Jacobian matrix of non-linear power system flow equations.

Earlier we solved these equations using methods such as Newton-Raphson method, and the fast decoupled method. It turns out that there is always some remarkable difference in values between the measured observations and the true observations or measurements. Thus it is hoped that by using weighted least squares algorithms to minimize the residual values [31], [32], [33], [34].

3.3 THE KALMAN FILTER ALGORITHM

The Kalman filter approach was introduced in 1960 by R. E. Kalman to provide a solution to discrete data linear filter issues. Since that time, the Kalman filter approach has been employed in a wide range of applications. Profoundly, Kalman filter approach is a set of mathematical equations which are used to estimate states of different processes by taking into account both incoming measurements and predictors to obtain an optimal estimation of a certain system state [35]. In 1969, Friedland provided two important stages of Kalman filter approaches which have been successfully used in many linear applications; specifically, in linear dynamic processes [36].

Because we deal with the non-linear equations of power system flow, so we choose to focus our attention on the discrete time EKF.

3.3.1 DISCRETE TIME EXTENDED KALMAN FILTER

The basic or original equations of the Kalman filter have failed in some non-linear applications. The success or failure of the extended KF based on the incorrect use of the Jacobian lead to inaccurate model process. If the model equations are linear, the equations of the extended KF will reduce to the original or basic equations of the Kalman filter. In particular EKF has been successfully used in several non-linear applications.

3.3.2 THE EQUATIONS OF EXTENDED KALMAN FILTER

We choose to divide the derivation of EKF to two parts:

3.3.3 EXTENDED KALMAN FILTER PREDICTION FORMULA

$$x_t^{+} = f(x_t^{-}, u_t) + \omega_t \tag{3.26}$$

We take the Taylor series expansion in the equation (3.26) to linearize it.

$$x_t^{+} = f(x_t^{-}, u_t) + \Delta f_x (x_t - x_t^{-}) + \omega_t \tag{3.27}$$

$$x_t^{+} = f(x_t^{-}, u_t) + \Delta f_x x_t - \Delta f_x x_t^{-} + \omega_t \tag{3.28}$$

x_t^{-} : A prior state estimator.

x_t^{+} : A posterior state estimator.

ω_t : m vector which represents the parameter white noise.

Δf_x : mxn vector which represents the Jacobian matrix.

We just consider the first and second order derivative; high order derivative will be ignored.

We re-express the previous equation to make it simpler by

$$F = \Delta f_x \; ; \qquad U_t = f(x_t^{-}, u_t) - \Delta f_x x_t^{-} \tag{3.29}$$

$$x_t^{+} = F x_t^{-} + U_t + \omega_t \tag{3.30}$$

So now we apply the linear Kalman filter prediction formula using
the linear process model:

$$x_t^{+} = F x_t^{-} + U_t + \omega_t = \Delta f_x x_t^{-} + f(x_t^{-}, u_t) - \Delta f_x x_t^{-} + \omega_t \tag{3.31}$$

$$x_t^{+} = f(x_t^{-}, u_t) + \omega_t \tag{3.32}$$

$$x_t^{+} = F\, x_t^{-} + U_t + \omega_t \tag{3.33}$$

Before we start deriving the equations of the Kalman filter, we assume that the input is equal to zero, U_t=0, so we reduce the linear dynamic system equation as well as we have a Gaussian distribution in our measurements. So we normalize the measurements to have zero mean and variance including predictors' and measurements' variance:-

$$x_t^{+} = Fx_t^{-} + \omega_t \tag{3.34}$$

$$\mathrm{Var}\,(\omega_t) = E(\omega_t * \omega_t^T) = Q_t \tag{3.35}$$

$$\omega_t \sim N(0, Q_t) \tag{3.36}$$

So from the distinguish categories, we can see that the Kalman filter has a clear dependence on the initial value to estimate the next one.

We assume that all the measurements and estimators have a normal distribution, so the distribution is denoted by N (x_t^k, p_t^k).

We first consider the a-priori covariance of the weighted least squares.

$$x_t^{+} = F * x_t^{-} + \omega_t \tag{3.37}$$

$$(x_t^{+} - x_t) = F(x_t^{-} - x_t) + \omega_t \tag{3.38}$$

$$E\left[(x_t^{+} - x_t^{*})(x_t^{+} - x_t^{*})^T\right] = E[[F * (x_t^{-} - \overline{x_t})\omega_t]\left[F * (x_t^{-} - \overline{x_t})\right) + \omega_t]^T \tag{3.39}$$

$$P_t^{+} = F * F^T * E\left[(x_t^{-} - \overline{x_t})\right) * (x_t^{-} - \overline{x_t})^T] + E(\omega_t * \omega_t^T) \tag{3.40}$$

$$P_t^{+} = F * P_t^{-} * F^T + Q_t \tag{3.41}$$

P_t^{+} : Posteriori covariance of the expected error of the predictors.

P_t^{-} : Priori covariance of the expected error of the predictors.

Q_t : A diagonal matrix mxn which represents the variance of the parameters or estimators.

3.3.4 LINERAZATION OF OBSERVATION MODEL

$$z_t = h(x_t{}^+) + v_t \tag{3.42}$$

By also using the Taylor series to linearize the observation equation:-

$$z_t = h((x_t{}^+) + \Delta h_x(x_t - x_t{}^+) + v_t \tag{3.43}$$

Δh_x : A Jacobian matrix of function h with respect to $x_t{}^+$.

We also consider the first and second order derivative, so high order derivative will also be ignored.

$$z_t = h(x_t{}^+) + \Delta h_x x_t - \Delta h_x x_t{}^+ + v_t \tag{3.44}$$

We assume that

$$z_t{}^* = \Delta h_x x_t + v_t \tag{3.45}$$

$$z_t{}^* = z_t - h(x_t{}^+) + \Delta h_x x_t{}^+ \tag{3.46}$$

We apply the linear Kalman filter formula using linear observation model.

$$x_t^{k+1} = x_t{}^k + K_t(z_t{}^* - \Delta h_x * (x_t{}^k)) \tag{3.47}$$

We compensate $z_t{}^*$ in previous equation:-

$$x_t^k = x_t{}^+ \tag{3.48}$$

$$x_t^{k+1} = x_t^k + K_t(z_t - h(x_t{}^k) + \Delta h_x x_t{}^k - \Delta h_x * x_t^k) \tag{3.49}$$

$$x_t^{k+1} = x_t{}^k + K_t(z_t - h_x(x_t{}^k)) \tag{3.50}$$

$$E\left[(x_t^{k+1} - \overline{x_t})(x_t^{k+1} - x_t^{*})^T\right] = E[[\left(x_t^{k} - \overline{x_t}\right)\left(x_t^{k} - \overline{x_t}\right)^T] + [(K_t v_t)(K_t v_t)^T] \tag{3.51}$$

$$P_t^+ = P_t^- - K_t H K_t^T \tag{3.52}$$

From the linear Kalman filter formula, we can take the proof of the innovation covariance and the Kalman gain equation.

$$S = R_t +* H^T * P_t^- * H \tag{3.53}$$

$$K_t = H^T * P_t^- * S^{-1} \tag{3.54}$$

Finally, we collect all different equations from part one and part two:-

Hence, our measurements are independent, so we always have a Gaussian distribution, so we normalize our measurements by zero-mean and variance instead of using zero-mean and unit standard deviation in standard normal distribution.

$$\text{Var}\ (v_t) = E(v_t * v_t^T) = R_t \tag{3.55}$$

$$v_t \sim N(0, R_t) \tag{3.56}$$

$$\text{Cov}\ (v_t^T * \omega_t) = E(v_t^T * \omega_t) = 0 \tag{3.57}$$

$$\text{Cov}\ (\omega_t^T * v_t) = E(\omega_t^T * v_t) = 0 \tag{3.58}$$

R_t : A diagonal matrix mxn which represents the variance of the measurements.

Q_t : A diagonal matrix mxn which represents the variance of the parameters or estimators.

$$P_t^+ = F * P_t^- * F^T + Q_t \tag{3.59}$$

$$S = R_t + * H^T * P_t^- * H \tag{3.60}$$

$$K_t = H^T * P_t^- * S^{-1} \tag{3.61}$$

$$x_t^{k+1} = x_t{}^k + K_t(z_t - h_x(x_t{}^k)) \tag{3.62}$$

$$P_t^+ = P_t^- - K_t R K_t{}^T \tag{3.63}$$

The major process of EKF is to estimate unmeasured states. The equations of EKF can be categorized to different parts. First part is time update equations (predictions) which update the state of the process by forwarding the time and error covariance to obtain the prior estimate for the next time. Second part is measurements update (corrections) which work by correcting the state estimate each iteration. The process is iterating until the innovation equation $K_t(z_t - h_x(x_t{}^k))$ close to zero to be no more iterations or change in the states.

In this work, we assume the a-priori covariance at any value with low predictors' noise and we calculate the variance of the measurements from the updated prior covariance, then we determine the value of Kalman gain and we add it to the innovation equation. Finally we update the corrected covariance to converge the predictors from both the prior covariance and the Kalman gain.

In these equations, we cannot directly use generic non-linear equations, so we should derivative them and add them to the error covariance equations.

The EKF can result in very accurate estimates because it can linearize the system at each point during the trajectory of the states [37], [38] [39] [40] [41].

3.4 LEAST ABSOLUTE METHOD (LAM)

Least absolute estimator has been proposed and suggested by many mathematicians such as Bosivich and Laplace since 1793.

The least absolute estimator was second ranked in solving regression problems because of the development and uniqueness of least squares estimator [42]. The computations and formulations of the least absolute value estimator require more computer time.

Least absolute value method is an alternative to least squares because it overcomes the drawbacks of least squares method such as lower sensitivity to gross outliers which can give us a better starting point than the least squares method. Many applications have proved that statistically using least absolute value is more efficient than least square method [43].

Recently, several algorithms have been used to solve the least absolute value estimator problem such as the simplex method and interior point method.

In this work we are interested in using interior point method in non-linear equations of power system specifically logarithmic barrier method to minimize the sum of the measurements residuals (L1 norm).

$$L_1 = \min \sum W_t^T |z - h(x_t)| \tag{3.64}$$

3.4.1 INTERIOR POINT METHOD (IPM)

IPMs have been involved to solve many large linear programming problems because the simplicity of using IPMs and being much easier to understand. Frisch introduced the logarithmic barrier method since 1955 to replace the linear inequality constraints to linear equality constraints by adding non-negative slack variables. In 1984 Narendre Karmarkar pioneered the idea of IPMs. His research has shown the advantages of using IPMs instead of using simplex method which was first used method in linear programming (LP). The advantages of IPMs are being faster than the simplex method and also have a better starting point. Fiacco and McCormick converted the inequality constraints to the sequence of unconstraint. IPMs contain several different methods such as primal-dual logarithmic method (IPM-PD) which was introduced by Megiddo, and predictor-corrector method (IPM-PC) which has been proposed since 1992 by Mehrota. In the past twenty years IPMs have been widely used in solving power system problems such as power system state estimation (PSSE) problems and power operation and planning (POAP) especially in unit commitment (UC) and economic dispatch (ED) [44] [45].

We denote z as the difference between the capacity (generation) and demand (load) on electric nodes and h represents the non-linear power system flow equations, so we minimize the distance between z and h as well as we estimate the state variables (phase and voltage magnitude). Abnormally, instead of taking the second the estimated values of state variables as optimal values, we take them as the initial values for the second linear programming and we iterate them over and over until we obtain less error in state variables.

Objective function

$$\mathrm{Min}\ W^{T}|r| \tag{3.65}$$

subject to

$$r = z - h(x) \tag{3.66}$$

$$-r \leq z - h(x) \tag{3.67}$$

$$-r \leq -z + h(x) \tag{3.68}$$

$$-r - z + h(x) \leq 0 \tag{3.69}$$

$$-r + z - h(x) \leq 0 \tag{3.70}$$

$$r = z - h(x) = 0 \tag{3.71}$$

$$g(x) = 0 \qquad (3.72)$$

Where $W = [1,1, \dots \dots \dots ,1]^T$

We add the non-negative slack variables (u_i, v_i) to certain inequality constraints to turn the in-equality constraints to equality constraints where the linear combination of these variables is less than or equal to a given constraints; based on that, slack variables are always positive.

$$-\mathrm{r} - \mathrm{z} + \mathrm{h}(\mathrm{x}) + \mathrm{u} = 0 \qquad (3.73)$$

$$-\mathrm{r} + \mathrm{z} - \mathrm{h}(\mathrm{x}) + \mathrm{v} = 0 \qquad (3.74)$$

Now we organize our equations

$$\mathrm{Min}\ \mathrm{W}^{\mathrm{T}}|\mathrm{r}| \qquad (3.75)$$

Subject to

$$-\mathrm{r} - \mathrm{z} + \mathrm{h}(\mathrm{x}) \leq 0 \qquad (3.76)$$

$$-\mathrm{r} + \mathrm{z} - \mathrm{h}(\mathrm{x}) \leq 0 \qquad (3.77)$$

$$\mathrm{r} = \mathrm{z} - \mathrm{h}(\mathrm{x}) = 0 \qquad (3.78)$$

$$\mathrm{g}(\mathrm{x}) = 0 \qquad (3.79)$$

$$\mathrm{u}, \mathrm{v}, \mathrm{e} \geq 0$$

3.4.2 LOGARITHMIC BARRIER FUNCTION

We subject the residual function with barrier function and we differentiate it to find the optimal solution for state variables.

$$B(w, r, x, \mu, u, v) = W^T r - \mu \sum_{i=1}^{n} (\ln u_i + \ln v_i) \tag{3.80}$$

$$\text{Min } W^T r - \mu \sum_{i=1}^{n} (\ln u_i + \ln v_i) \tag{3.81}$$

Subject to

$$z - h(x) - r + u = 0 \tag{3.82}$$

$$-z + h(x) - r + v = 0 \tag{3.83}$$

$$g(x) = 0 \tag{3.84}$$

$$u, v, e \geq 0$$

By using the Karush -Kuhn-Tucker (KKT) optimality conditions of the lagrangian multipliers function and then using Newton's method to iterate the equations by reducing barrier parameter to be close to zero.

$$W^T r - \mu \sum_{i=1}^{n} (\ln u_i + \ln v_i) + \lambda^T (z - h(x) - r + u) + \pi^T (-z + h(x) - r + v) + \quad \rho^T (g(x)) \tag{3.85}$$

$$\frac{\partial L}{\partial u} = -\mu U^{-1} e + \lambda = 0 \tag{3.86}$$

$$\frac{\partial L}{\partial v} = -\mu V^{-1} e + \pi = 0 \tag{3.87}$$

$$\frac{\partial L}{\partial \lambda} = z - h(x) - r + u = 0$$

$$\frac{\partial L}{\partial \pi} = -z + h(x) - r + v = 0 \tag{3.88}$$

$$\frac{\partial L}{\partial \rho} = g(x) = 0 \tag{3.89}$$

$$\frac{\partial L}{\partial x} = -WH^T\Delta x - H^T\lambda + H^T\pi + G^T\rho = 0 \tag{3.90}$$

We apply Taylor expansion series to approximate the previous equations.

$$-H\Delta x + \Delta u = -z + h\left(x^k\right) + r - u^k \tag{3.91}$$

$$H\Delta x + \Delta v = z - h\left(x^k\right) + r - v^k \tag{3.92}$$

$$G\Delta x = -g(x^k) \tag{3.93}$$

$$\mu\left(U^k\right)^{-2} - \lambda = -\mu\left(U^k\right)^{-1}e \tag{3.94}$$

$$\mu\left(V^k\right)^{-2} - \pi = -\mu\left(V^k\right)^{-1}e \tag{3.95}$$

$$H^T\lambda - H^T\pi - G^T\rho = -WH^T\Delta x \tag{3.96}$$

$$\begin{bmatrix} 0 & 0 & 0 & I_n & 0 & -H \\ 0 & 0 & 0 & 0 & I_n & H \\ 0 & 0 & 0 & 0 & 0 & G \\ -I_n & 0 & 0 & \mu\left(U^k\right)^{-2} & 0 & 0 \\ 0 & -I_n & 0 & 0 & \mu\left(V^k\right)^{-2} & 0 \\ H^T & -H^T & -G^T & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda \\ \pi \\ \rho \\ \Delta u \\ \Delta v \\ \Delta x \end{bmatrix} = - \begin{bmatrix} z - h\left(x^k\right) - r + u^k \\ -z + h\left(x^k\right) - r + v^k \\ g(x^k) \\ \mu\left(U^k\right)^{-1}e_n \\ \mu\left(V^k\right)^{-1}e_n \\ WH^T\Delta x \end{bmatrix} \tag{3.97}$$

$$\lambda^{k+1} = \lambda^k + \alpha\Delta\lambda \tag{3.98}$$

$$\rho^{k+1} = \rho^k + \alpha\Delta\rho \tag{3.99}$$

$$x^{k+1} = x^k + \alpha\Delta x \tag{3.100}$$

$$u^{k+1} = u^k + \alpha\Delta u \tag{3.101}$$

$$v^{k+1} = v^k + \alpha\Delta v \tag{3.102}$$

$$\pi = \pi^k + \alpha\Delta\pi \tag{3.103}$$

The superscript k is used to denote the estimate of the previous parameters at each iteration. The step length (α) is selected to lead the primal solution to be feasible and inside the constraints region; moreover, the step length is used to consider the violation of the constraints.

$$\alpha = 0.9995\tilde{\alpha}$$

$$\tilde{\alpha} = \min(u_i + \alpha_i\Delta u \geq 0, v_i + \alpha_i\Delta v \geq 0)$$

The stopping iterations is going to be based on the duality gap which is the difference between the primal and dual cost, so when the cost of the duality gap is smaller than the step size or length, the iterations must stop[44] [46] [47] [48] [49] .

The barrier parameter μ is defaulted in Matlab to 0.1 and is subsequently reduced to be close to zero to determine the new search direction and update the new estimate of previous parameters until these parameters converged which can be the optimal solutions for both primal and duality problems, so if the duality gap is positive that means the points in feasible region, but it does not mean we compute the optimal solution, so we obtain the optimal solution when the duality gap turns to zero. The determination of duality gap can give us a picture of the nearness of our optimal solutions.

3.5 SUMMARY

This chapter has discussed the modeling of state estimation algorithms which is the core subject of this thesis, so by knowing the compositions and the techniques of these three algorithms, it will help us to understand how they work step by step and how they help to solve state estimation issues. In next chapter, we will employ these algorithms in different IEEE electrical networks and we will observe the behaviour of these algorithms by calculating errors for state variables and residuals for available real time measurements.

CHAPTER 4 RESULTS

4.1 INTRODUCTION

This chapter presents the computational results for the three different test cases namely the IEEE 14, 30, and 118 buses systems. The object is to compare the performance of state estimation algorithms by taking the optimal solution of both phase and magnitude voltage as the initial value until the iterations converge , then, we calculate the residuals for available real time measurements including real power net and reactive power net and compare the accuracy of these algorithms.

4.2 APPROACH

Knowing the voltage magnitudes and angles at buses in the network is important for steady state in PSSE. By minimizing the residual of load flow node equation which can give the operators (engineers) in energy control center (ECC) a clearer picture about the state of the electrical network [50].

$$P_G - P_D - \sum |V_i||V_j|\left[G_{ij}\cos(\theta_i - \theta_j)\right) + \left[B_{ij}\sin(\theta_i - \theta_j)\right) = 0 \qquad (4.1)$$

$$Q_G - Q_D - \sum |V_i||V_j|\left[G_{ij}\sin(\theta_i - \theta_j)\right) - \left[B_{ij}\cos(\theta_i - \theta_j)\right) = 0 \qquad (4.2)$$

The data obtained from the official website of University of Washington [51] and estimate these data by three different algorithms in matlab.
Software programs have coded in matlab in three different algorithms WLS, EKF, and IPM linear programming based on least absolute value estimation criterion [51].

4.3 TEST CASES

The test cases include two major estimation problems including state vector estimation such as phase and magnitude voltage as well as measurements such as net active power into the system and net reactive power into the system. Also we calculate the percentage errors for state vectors and residuals for available real time measurements.

Measured Values= the values provided from balanced load flow equations.

Estimated Values = the optimal solution or the outcome of estimation for both state vectors and real time measurements under tolerance conditions [51].

$$\text{Error} = \left|\frac{\text{Measured Value} - \text{Estimated Value}}{\text{Measured Value}}\right| \times 100 \tag{4.3}$$

$$\text{Residuals} = |\text{Measured Value} - \text{Estimated Value}| \tag{4.4}$$

4.4 IEEE 14 BUS TEST CASE

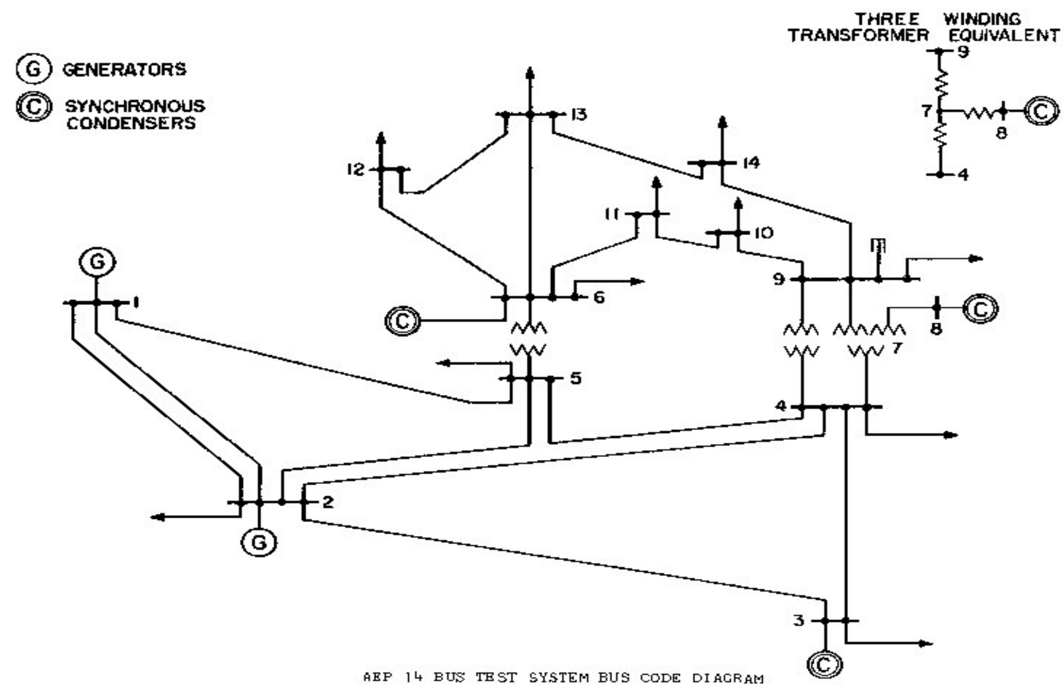

Figure 4-1: Single line diagram of 14 bus system [52]

Name of Network Components	Number of Network Components
Generators	11
Transformers	3
Loads	5
Transmission Lines	20

4.4.1 WEIGHTED LEAST SQUARES METHOD

The weights in WLS vary from value to value to obtain the best converge for both state variables and measurements. The weights for the angles equal 1E-6, for the voltages are 1E-5, for the net active power are 1E-4, and for the net reactive power 3E-3. We also observe the input number of state variables in WLS is 2 times. Finally we checked all the state vectors and measurements in WLS for IEEE 14 bus system and all the values have successfully converged.

4.4.1.1. Measured and estimated angles with error using WLS

Table 4-1: Measured and estimated angles with error using WLS

Bus no	Measured Angle Values	Estimated Angle Values	%Error
4	-0.02322	-0.02322	2.99E-4
5	-0.027	-0.027	2.59E-4
6	-0.1197	-0.1197	7.1E-5
7	-0.084	-0.084	6.37E-05
8	-0.08462	-0.08462	2.09E-05
11	-0.12345	-0.12345	2.97E-05
13	-0.13461	-0.13461	3.01E-05

4.1.1.2. Measured and estimated voltages magnitudes with error using WLS

Table 4-2: Measured and estimated voltages magnitudes with error using WLS

Bus no	Measured voltage Values	Estimated voltage Values	% Error
4	1.01665	1.01665	4.8E-06
5	1.019748	1.019748	6.2E-06
6	1.054439	1.054439	8.1E-06
7	1.041635	1.041636	6.6E-06
8	1.07083	1.07083	2.6E-06
11	1.035528	1.035528	2E-07
13	1.03208	1.03208	4.8E-06

4.1.1.3. Measured and estimated net active power into the system with residuals using WLS.

Table 4-3: Measured and estimated net active power with residuals using WLS

Bus no	Measured Values of net active power into the system	Estimated Values of net active power into the system	Residuals
4	-0.478	-0.4819	3.89E-3
5	-0.076	-0.0799	3.89E-3
6	-0.112	-0.1159	3.88E-3
7	0.000	-0.0039	3.89E-3
8	0.000	-0.0039	3.89E-3
11	-0.035	-0.0389	3.89E-3
13	-0.135	-0.1389	3.91E-3

4.1.1.4. Measured and estimated net reactive power into the system with residuals using WLS

Table 4-4: Measured and estimated net reactive power with residuals using WLS

Bus no	Measured Values of net reactive power into the system	Estimated Values of net reactive power into the system	Residuals
4	-0.359	-0.3553	3.7E-3
5	0.039	0.0426	3.6E-3
6	-0.016	-0.0124	3.6E-3
7	0.047	0.0508	3.8E-3
8	0.000	0.0037	3.7E-3
11	-0.058	-0.0542	3.8E-3
13	-0.016	-0.0122	3.8E-3

4.4.2 EXTENDED KALMAN FILTER

We suggest the initial covariance (P) in EKF to be equals 100 and the noise variance (Q) of state vectors is 0.001 because we work on the tolerance $\mathrm{tol} < 1\mathrm{e} - 5$ in IEEE 14 bus system, so we do not observe the number of the iteration; however, the time to obtain the optimal solution in EKF is more faster both WLS and IPM. Also the weights have chosen to give us accurate results for all errors and residuals which are 1E-4 for angles, 1E-6 for voltages, and 1E-3 for both net active and reactive power into the system. We also observe the input number of state variables in EKF is 2 times. Finally we checked all the state vectors and measurements in EKF for IEEE 14 bus system and all the values have successfully converged.

4.4.2.1. Measured and estimated angles with error using EKF

Table 4-5: Measured and estimated angles with error using EKF

Bus no	Measured Angle Values	Estimated Angle Values	%Error
4	-0.0225291	-0.0225291	1.25E-4
5	-0.0264024	-0.0264024	1.61E-4
6	-0.1202456	-0.1202456	3.7E-06
7	-0.0840094	-0.0840094	3.46E-05
8	-0.0846317	-0.0846317	1.5E-5
11	-0.1238493	-0.1238493	2.27E-05
13	-0.1351328	-0.1351328	2.41E-05

4.4.2.2. Measured and estimated angles with error using EKF

Table 4-6: Measured and estimated voltages with error using EKF

Bus no	Measured voltage Values	Estimated voltage Values	% Error
4	1.01334	1.01334	3.1E-06
5	1.01670	1.01670	4.2E-06
6	1.04736	1.04736	9E-07
7	1.03543	1.03543	6E-6
8	1.06424	1.06424	1.1E-06
11	1.02785	1.02785	1.1E-06
13	1.02419	1.02419	5.3E-06

4.4.2.3. Measured and estimated net active into the system with residuals using EKF

Table 4-7: Measured and estimated net active power with residuals using EKF

Bus no	Measured Values of net active power into the system	Estimated Values of net active power into the system	Residuals
4	-0.478	-0.4824	4.4E-3
5	-0.076	-0.0803	4.3E-3
6	-0.112	-0.1157	3.7E-3
7	0	-0.0039	3.9E-3
8	0	-0.0039	3.9E-3
11	-0.035	-0.0386	3.6E-3
13	-0.135	-0.1386	3.6E-3

4.4.2.3. Measured and estimated net active into the system with residuals using EKF

Table 4-8: Measured and estimated net reactive power with residuals using EKF

Bus no	Measured Values of net reactive power into the system	Estimated Values of net reactive power into the system	Residuals
4	0.039	0.0374	1.6E-3
5	-0.016	-0.0200	4E-3
6	0.047	0.0455	1.5E-3
7	0.000	-0.0020	2E-3
8	0.174	0.1709	3.1E-3
11	-0.018	-0.0196	1.6E-3
13	-0.058	-0.0598	1.8E-3

4.4.3 INTERIOR POINT METHOD

The weights in IPM have chosen to be 1.0 to obtain one solution for both state variables and measurements. We also observe the input number of state variables in IPM is 1 times. Finally we checked all the state vectors and measurements in IPM for IEEE 14 bus system and all the values have successfully converged.

4.4.3.1. Measured and estimated angles with error using IPM

Table 4-9: Measured and estimated angles with error using IPM

Bus no	Measured Angle Values	Estimated Angle Values	%Error
4	-0.02574791	-0.02574792	2.74E-05
5	-0.03045719	-0.03045719	9.06E-06
6	-0.13137174	-0.13137174	2.86E-06
7	-0.09030380	-0.09030380	3.53E-06
8	-0.09041721	-0.09041721	2.69E-06
11	-0.13339247	-0.13339247	2.53E-06
13	-0.14793937	-0.14793937	3.12E-06

4.4.3.2. Measured and estimated voltage magnitudes with error using IPM.

Table 4-10: Measured and estimated voltage magnitudes with error using IPM

Bus no	Measured voltage Values	Estimated voltage Values	% Error
4	0.96400	0.96400	4E-07
5	0.96831	0.96831	6E-07
6	1.01350	1.01350	3E-07
7	0.98880	0.98880	1E-07
8	1.01836	1.01836	6E-07
11	0.98912	0.98912	3E-07
13	0.99413	0.99413	9E-07

4.4.3.3. Measured and estimated net active power into the system with residuals using IPM.

Table 4-11: Measured and estimated net active power with residuals using IPM

Bus no	Measured Values of net active power into the system	Estimated Values of net active power into the system	Residuals
4	-0.478	-0.4856	7.6E-3
5	-0.076	-0.0834	7.4E-3
6	-0.112	-0.1135	1.5E-3
7	0.000	-0.0027	2.7E-3
8	0.000	-0.0006	6E-4
11	-0.035	-0.0350	0
13	-0.135	-0.1353	3E-4

4.4.3.4. Measured and estimated net reactive power into the system with residuals using IPM.

Table 4-12: Measured and estimated net reactive power with residuals using IPM

Bus no	Measured Values of net reactive power into the system	Estimated Values of net reactive power into the system	Residuals
4	0.039	0.0390	5E-5
5	-0.016	-0.0159	6E-5
6	0.047	0.0471	9E-5
7	0.000	0.0000	3E-5
8	0.174	0.1740	3E-5
11	-0.018	-0.0180	4E-5
13	-0.058	-0.0580	2E-5

We summarize all the results that we have in tables in IEEE 14 buses test case in four figures to easily compare the performance of each algorithm.

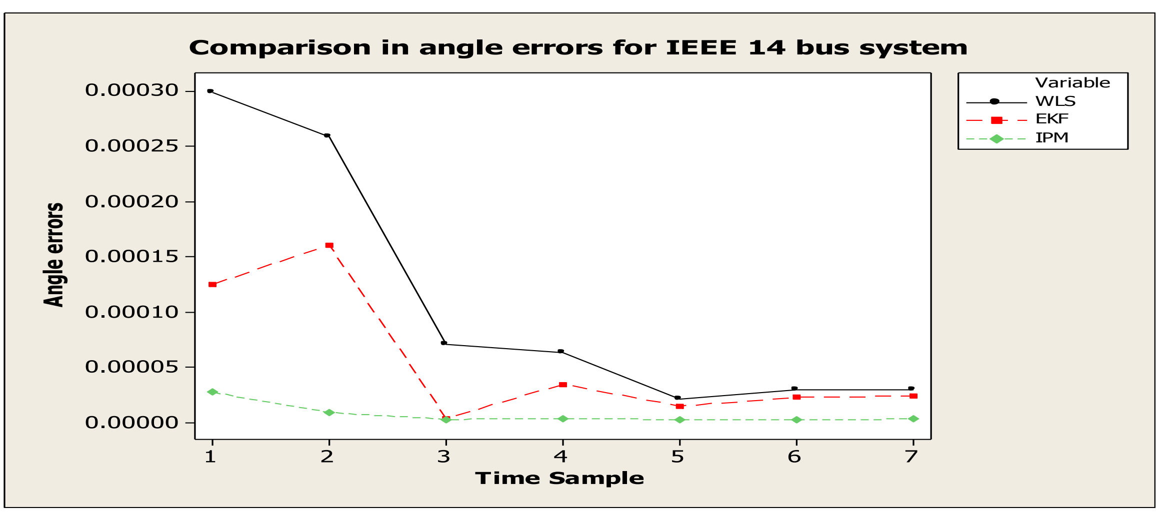

Figure 4-2: Comparison in angles error for IEEE 14 bus system

Figure 4.5 compares angle errors for IEEE 14 buses which are represented in Tables 4.1, 4.5, and 4.9. The weighted least square has more variation in the first four angle errors, so we suggest that it is lesser all weights the same accurate. The weights (R) in WLS for angle errors are 1E-6 and in EKF are 1E-4.

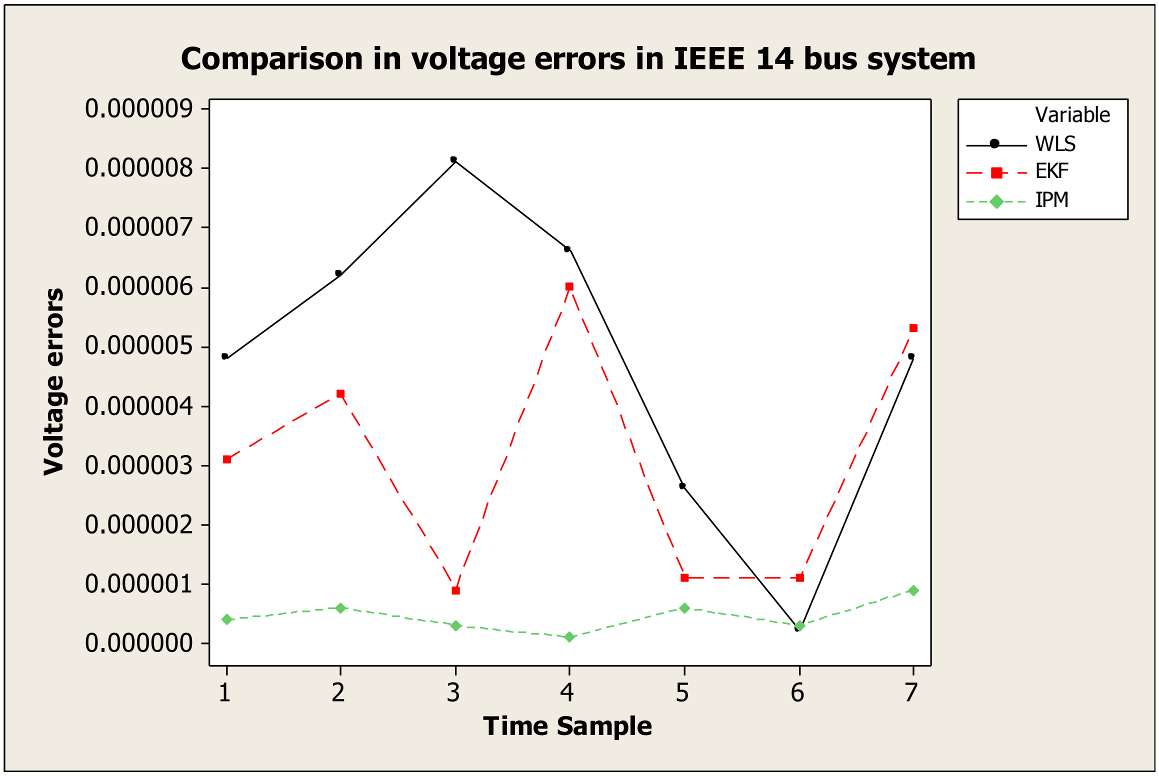

Figure 4-3: Comparison in voltages error for IEEE 14 bus system

Figure 4.6 shows the comparison of voltage errors for the IEEE 14 buses system represented in Table 4.2, 4.6, and 4.10 for the three algorithms. Again the interior point method (green line) gives better accuracy because all voltage errors are close to zero. EKF results are closer to the WLS than in the figure 4.5 which was close the interior point method. The weights in WLS (R) of net active power are 1E-5 and in EKF are 1E-6.

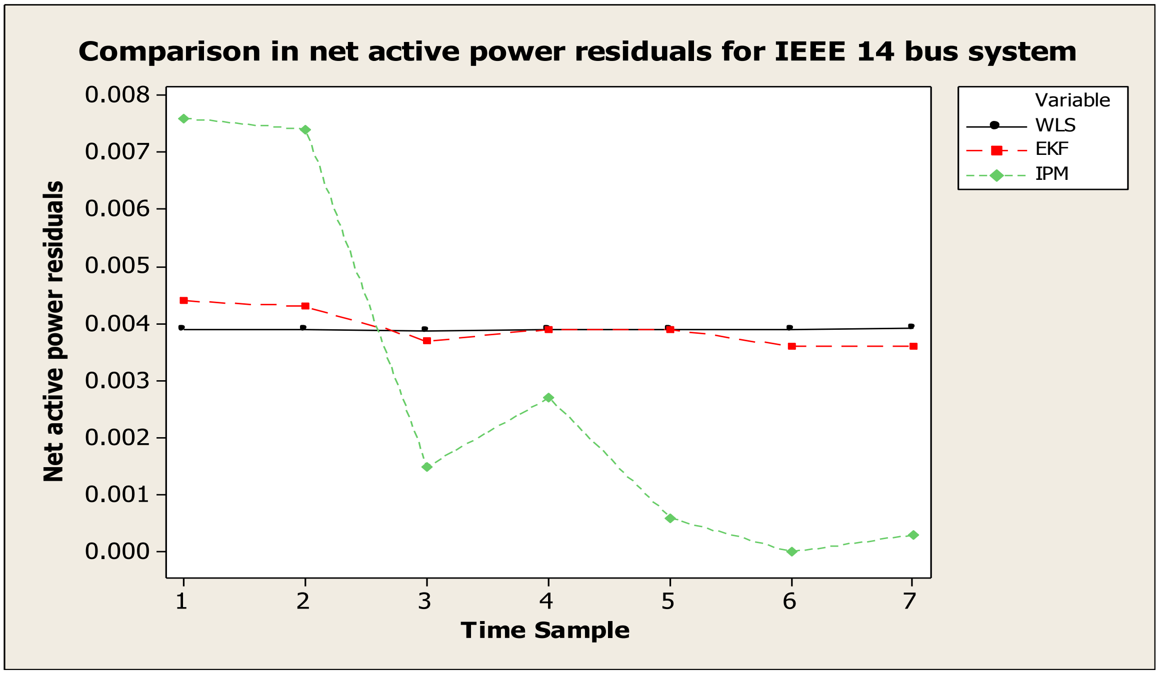

Figure 4-4: Comparison in net active power residuals for IEEE 14 bus system

Figure 4.7 illustrates the comparison of net active power residuals for IEEE 14 bus system represented in Table 4.3, 4.7, and 4.11 IPM is more precise because most of the error values are lesser than both WLS and EKF and also from the above figure seems there is no different in accuracy between EKF and WLS except in time simulation. EKF takes lesser iteration and CPU time than WLS. The weights in WLS (R) of net active power are 1E-4 and in EKF are 1E-3.

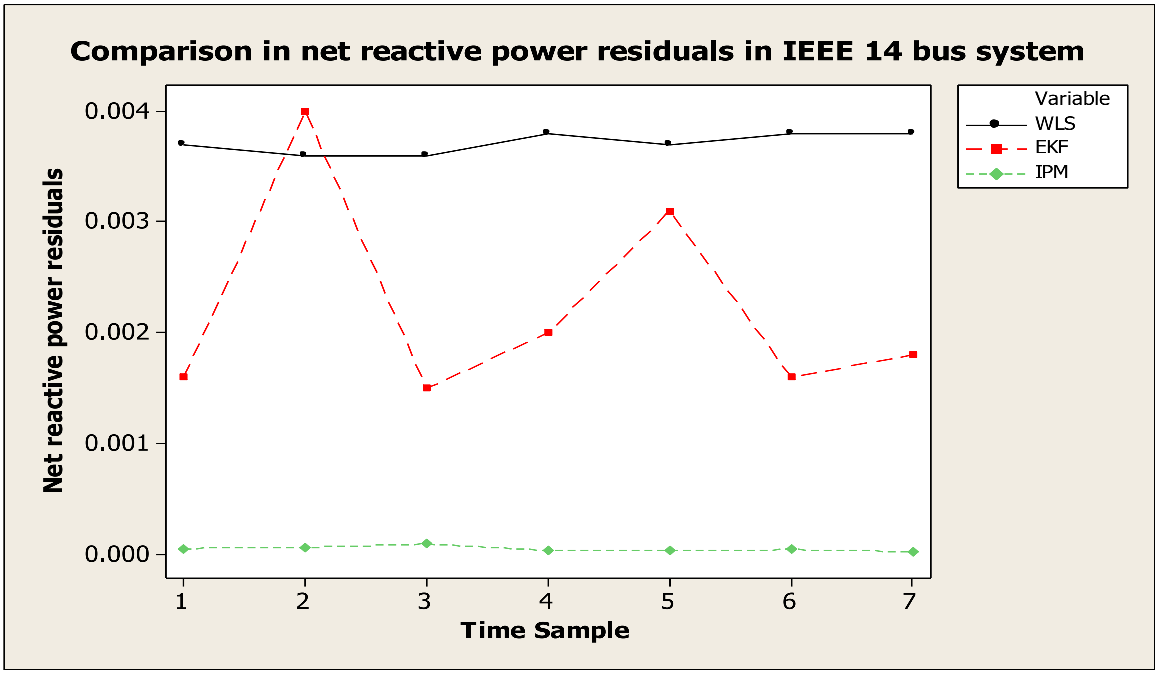

Figure 4-5: Comparison in net reactive power residuals for IEEE 14 bus system

Figure 4.8 demonstrates the comparison of net reactive power residuals for IEEE 14 bus system showed in Table 4.4, 4.8, and 4.12.The residuals in IPM is almost zero and EKF is lesser than WLS with more variation in EKF. The weights in WLS are 3E-3 and in EKF are 1E-3.

4.5 TEST CASE IEEE 30 BUSES

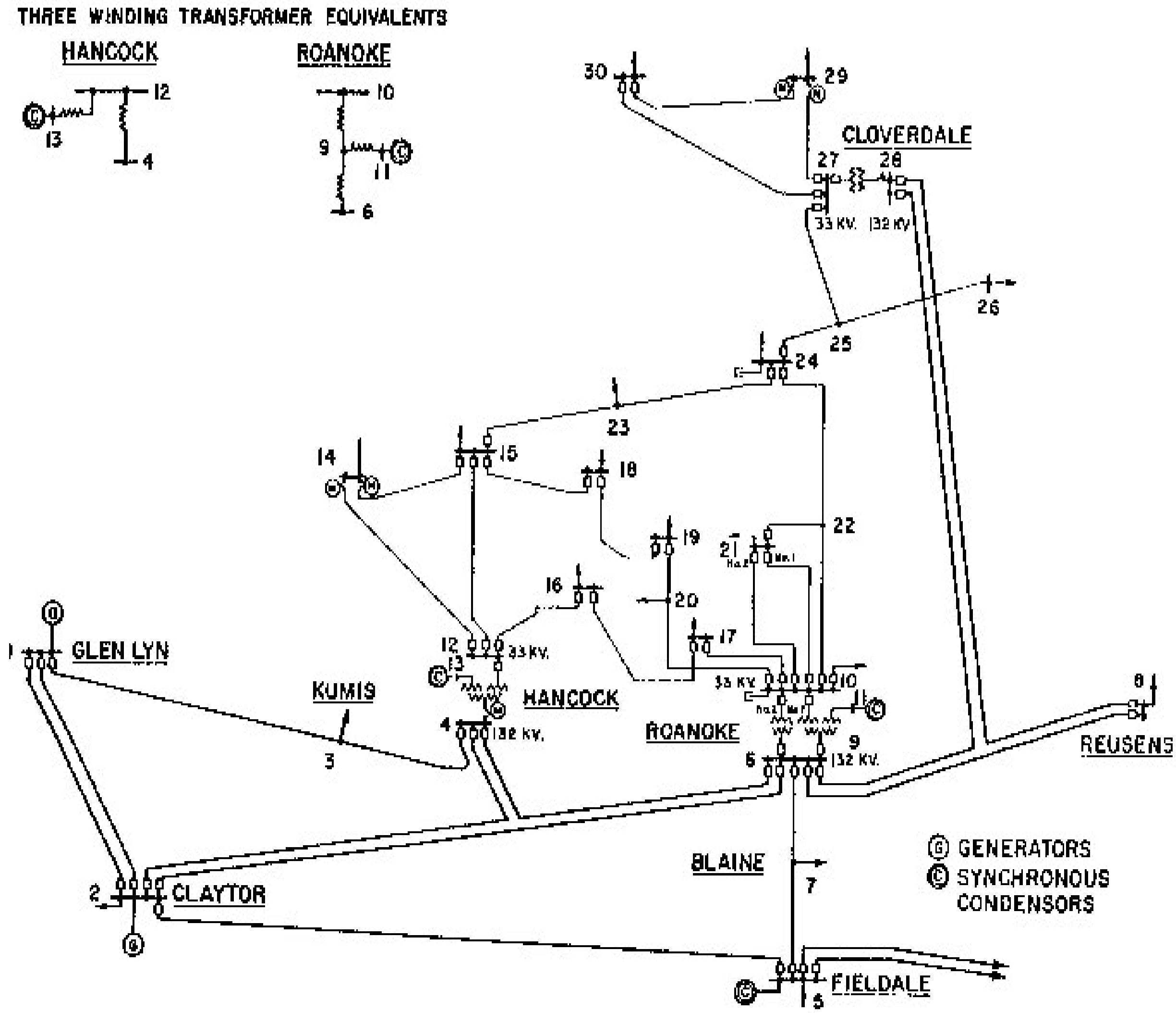

Figure 4-6: Single line diagram of 30 bus system [53]

Name of Network Components	Number of Network Components
Generators	21
Transformers	5
Loads	6
Transmission Lines	42

4.5.1 WEIGHTED LEAST SQUARES

The weights in WLS vary from value to value to obtain the best converge for both state variables and measurements. Because we work on the tolerance $\text{tol} < 1\text{e} - 5$ in IEEE 30 bus system, so we do not observe the number of the iteration in WLS. The weights for the angles equal 6E-5, for the voltages are 9E-4, for the net active power are 1E-4, and for the net reactive power 1E-3. We also observe the input number of state variables in WLS is 3 times. Finally we checked all the state vectors and measurements in WLS for IEEE 30 bus system and all the values have successfully converged.

4.5.1.1. Measured and estimated angles with error using WLS.

Table 4-13: Measured and estimated angles with error using WLS

Bus no	Measured Angle Values	Estimated Angle Values	%Error
8	-0.0609816	-0.0609816	6.41E-05
9	-0.0910379	-0.0910379	4.71E-05
12	-0.1007560	-0.1007560	2.75E-05
14	-0.1162253	-0.1162253	3.27E-05
15	-0.1172526	-0.1172526	3.67E-05
16	-0.1111657	-0.1111657	3.15E-05
18	-0.1278145	-0.1278145	3.23E-05
21	-0.1247513	-0.1247513	4E-5
22	-0.1200416	-0.1200416	4.14E-05
27	-0.1177957	-0.1177957	3.82E-05
30	-0.1542477	-0.1542477	2.41E-05

4.5.1.2. Measured and estimated voltages with error using WLS

Table 4-14: Measured and estimated voltage magnitudes with error using WLS

Bus no	Measured voltage Values	Estimated voltage Values	% Error
8	0.96417	0.96417	2.5E-06
9	0.97682	0.97682	1.5E-06
12	0.98642	0.98642	2.1E-06
14	0.96737	0.96737	3.2E-06
15	0.95973	0.95973	1E-6
16	0.96545	0.96545	3.5E-06
18	0.94467	0.94467	3.3E-06
21	0.94171	0.94171	4.4E-06
22	0.94588	0.94588	1.3E-06
27	0.95242	0.95242	4E-07
30	0.91800	0.91800	4.8E-06

4.5.1.3. Measured and estimated net real power into the system with residuals using WLS

Table 4-15: Measured and estimated net real power with residuals using WLS

Bus no	Measured Values of net active power into the system	Estimated Values of net active power into the system	Residuals
8	-0.3000	-0.2920	7.97E-3
9	0.0000	0.0080	7.98E-3
12	-0.1120	-0.1040	8.01E-3
14	-0.0620	-0.0540	7.97E-3
15	-0.0820	-0.0740	7.98E-3
16	-0.0350	-0.0270	7.98E-3
18	-0.0320	-0.0240	7.95E-3
21	-0.1750	-0.1670	7.96E-3
22	0.0000	0.0080	7.96E-3
27	0.0000	0.0080	7.96E-3
30	-0.1060	-0.0981	7.90E-3

4.5.1.4Measured and estimated net reactive power into the system with residuals using WLS

Table 4-16: Measured and estimated net reactive power with residuals using WLS

Bus no	Measured Values of net reactive power into the system	Estimated Values of net reactive power into the system	Residuals
8	0.073	0.0674	5.57E-3
9	0.000	-0.0058	5.77E-3
12	-0.075	-0.0810	6.02E-3
14	-0.016	-0.0221	6.10E-3
15	-0.025	-0.0311	6.09E-3
16	-0.018	-0.0241	6.07E-3
18	-0.009	-0.0151	6.13E-3
21	-0.112	-0.1181	6.13E-3
22	0.000	-0.0061	6.12E-3
27	0.000	-0.0061	6.12E-3
30	-0.019	-0.0252	6.23E-3

4.5.2 EXTENDED KALMAN FILTER

We suggest the initial covariance (P) in EKF to be equals 10 and the noise variance (Q) of state vectors is 0.01 because we work on the tolerance $\text{tol} < 1\text{e} - 5$ in IEEE 30 bus system, so we do not observe the number of the iteration; however, the time to obtain the optimal solution in EKF is more faster both WLS and IPM. Also the weights have chosen to give us accurate results for all errors and residuals which are 1E-5 for angles, 1E-5 for voltages, and 9E-3 for net active power and 4E-3 for reactive power into the system. We also observe the input number of state variables in EKF is 3 times. Finally we checked all the state vectors and measurements in EKF for IEEE 30 bus system and all the values have successfully converged. The time to obtain the optimal solution in EKF is faster than both IPM and WLS.

4.5.2.1. Measured and estimated angles with error using EKF

Table 4-17: Measured and estimated angles with error using EKF

Bus no	Measured Angles Values	Estimated Angles Values	%Error
8	-0.0592009	-0.0592009	1.78E-05
9	-0.0882774	-0.0882774	5E-6
12	-0.0977978	-0.0977978	3E-6
14	-0.1126929	-0.1126929	6E-6
15	-0.1136344	-0.1136344	6.2E-06
16	-0.1077675	-0.1077675	7E-6
18	-0.1237298	-0.1237298	1.5E-06
21	-0.1208737	-0.1208737	1.21E-05
22	-0.1162480	-0.1162480	2.3E-06
27	-0.1141459	-0.1141459	1.27E-05
30	-0.1492807	-0.1492807	6.8E-06

4.5.2.2. Measured and estimated voltages with error using EKF

Table 4-18: Measured and estimated voltages with error using EKF

Bus no	Measured voltage Values	Estimated voltage Values	% Error
8	0.97738	0.97738	3.6E-06
9	0.98981	0.98981	0
12	0.99962	0.99962	2.8E-06
14	0.98054	0.98054	9E-07
15	0.97290	0.97290	3E-07
16	0.97858	0.97858	1.5E-06
18	0.95774	0.95774	1.1E-06
21	0.95486	0.95486	1.9E-06
22	0.95885	0.95885	6E-07
27	0.96516	0.96516	5E-07
30	0.93077	0.93077	2E-07

4.5.2.3. Measured and estimated net active power into the system with residuals using EKF

Table 4-19: Measured and estimated net active power with residuals using EKF

Bus no	Measured Values of net active power in the system	Estimated Values of net active power in the system	Residuals
8	-0.300	-0.2915	8.5E-3
9	0.000	0.0081	8.1E-3
12	-0.112	-0.1042	7.8E-3
14	-0.062	-0.0542	7.8E-3
15	-0.082	-0.0742	7.8E-3
16	-0.035	-0.0272	7.8E-3
18	-0.032	-0.0242	7.8E-3
21	-0.175	-0.1672	7.8E-3
22	0.000	0.0078	7.8E-3
27	0.000	0.0079	7.9E-3
30	-0.106	-0.0982	7.8E-3

4.5.2.4. Measured and estimated net reactive power into the system with residuals using EKF

Table 4-20: Measured and estimated net reactive power with residuals using EKF

Bus no	Measured Values of net reactive power into the system	Estimated Values of net reactive power into the system	Residuals
8	-0.109	-0.1138	4.8E-3
9	0.073	0.0682	4.8E-3
12	0.162	0.1573	4.7E-3
14	0.106	0.1009	5.1E-3
15	-0.016	-0.0213	5.3E-3
16	-0.025	-0.0303	5.3E-3
18	-0.058	-0.0633	5.3E-3
21	-0.007	-0.0123	5.3E-3
22	-0.112	-0.1173	5.3E-3
27	-0.023	-0.0283	5.3E-3
30	-0.009	-0.0143	5.3E-3

4.5.3 INTERIOR POINT METHOD

The weights in IPM have chosen to be 1.0 to obtain one solution for both state variables and measurements. We also observe the input number of state variables in IPM is 1 times. Finally we checked all the state vectors and measurements in IPM for IEEE 30 bus system and all the values have successfully converged.

4.5.3.1. Measured and estimated angles with error using IPM

Table 4-21: Measured and estimated angles with error using IPM

Bus no	Measured Angles Values	Estimated Angles Values	%Error
8	-0.06415701	-0.06415701	2.87E-06
9	-0.10165675	-0.10165675	1.93E-06
12	-0.11013451	-0.11013451	7.1E-07
14	-0.12754103	-0.12754103	3.11E-06
15	-0.12924841	-0.12924841	2.33E-06
16	-0.12285486	-0.12285486	1.48E-06
18	-0.14222134	-0.14222134	1.07E-06
21	-0.13779067	-0.13779067	3.28E-06
22	-0.13484394	-0.13484394	9.9E-07
27	-0.13407190	-0.13407190	2.77E-06
30	-0.17349562	-0.17349562	1.5E-06

4.5.3.2. Measured and estimated voltages with error using IPM

Table 4-22: Measured and estimated voltages with error using IPM

Bus no	Measured voltage Values	Estimated voltage Values	% Error
8	0.99699	0.99699	2E-07
9	1.01344	1.01344	5E-07
12	1.02575	1.02575	3E-07
14	1.00737	1.00737	2E-07
15	0.99969	0.99969	3E-07
16	1.00522	1.00522	4E-07
18	0.98502	0.98502	1E-07
21	0.98214	0.98214	0
22	0.98596	0.98596	3E-07
27	0.99262	0.99262	1E-07
30	0.96000	0.96000	3E-07

4.5.3.3. Measured and estimated net active power into the system with residuals using IPM

Table 4-23: Measured and estimated net active power with residuals using IPM

Bus no	Measured Values of net active power into the system	Estimated Values of net active power into the system	Residuals
8	-0.3000	-0.2992	8E-4
9	0.0000	-0.0016	1.6E-3
12	-0.1120	-0.1123	3E-4
14	-0.0620	-0.0620	0
15	-0.0820	-0.0820	0
16	-0.0350	-0.0356	6E-4
18	-0.0320	-0.0327	7E-4
21	-0.1750	-0.1753	3E-4
22	0.0000	-0.0021	2.1E-4
27	0.0000	-0.0023	2.3E-4
30	-0.1060	-0.1060	0

4.5.3.4. Measured and estimated net reactive power into the system with residuals using IPM

Table 4-24: Measured and estimated net reactive power with residuals using IPM

Bus no	Measured Values of net reactive power into the system	Estimated Values of net reactive power into the system	Residuals
8	0.0730	0.0682	4.8E-3
9	0.0000	-0.0061	6.1E-3
12	-0.0750	-0.0834	8.4E-3
14	-0.0160	-0.0177	1.7E-3
15	-0.0250	-0.0267	1.7E-3
16	-0.0180	-0.0197	1.7E-3
18	-0.0090	-0.0107	1.7E-3
21	-0.1120	-0.1131	1.1E-3
22	0.0000	-0.0023	2.3E-3
27	0.0000	-0.0053	5.3E-3
30	-0.0190	-0.0185	5E-4

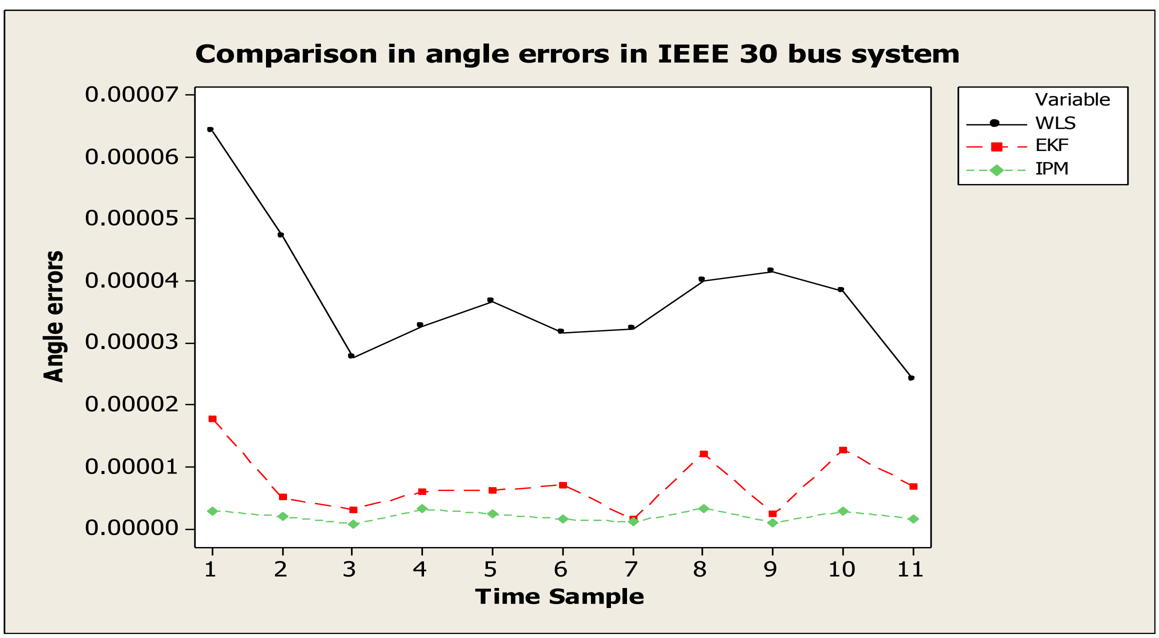

Figure 4-7: Comparison in angles error for IEEE 30 bus system

Figure 4.10 compares angle errors for IEEE 30 buses which are showed in Table 4.13, 4.17, and 4.21. The Weighted least square has more variation, so we compile it as the bad estimation algorithm even-though the values of angle errors are more reasonable. The weights (R) in angles error in WLS are 6E-5 and in EKF are 1E-5.

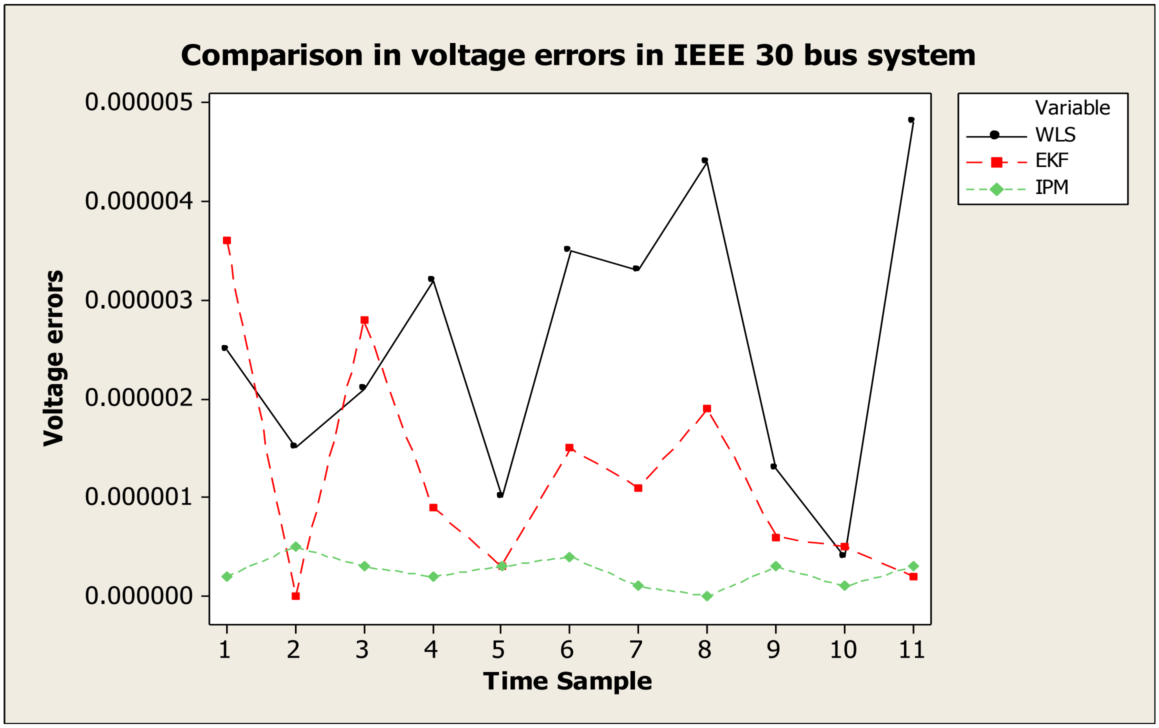

Figure 4-8: Comparison in voltages error for IEEE 30 bus system

Figure 4.11 shows the comparison of voltage errors for IEEE 30 bus system tabled in Table 4.14, 4.18, and 4.22 for the three algorithms including WLS, EKF, and IPM. Again the IPM gives a better accuracy because most of voltage errors closer to zero than EKF and WLS. EKF is the second best estimation in most voltage error values. The weights (R) in voltages error in WLS are 9E-4 and in EKF are 1E-5.

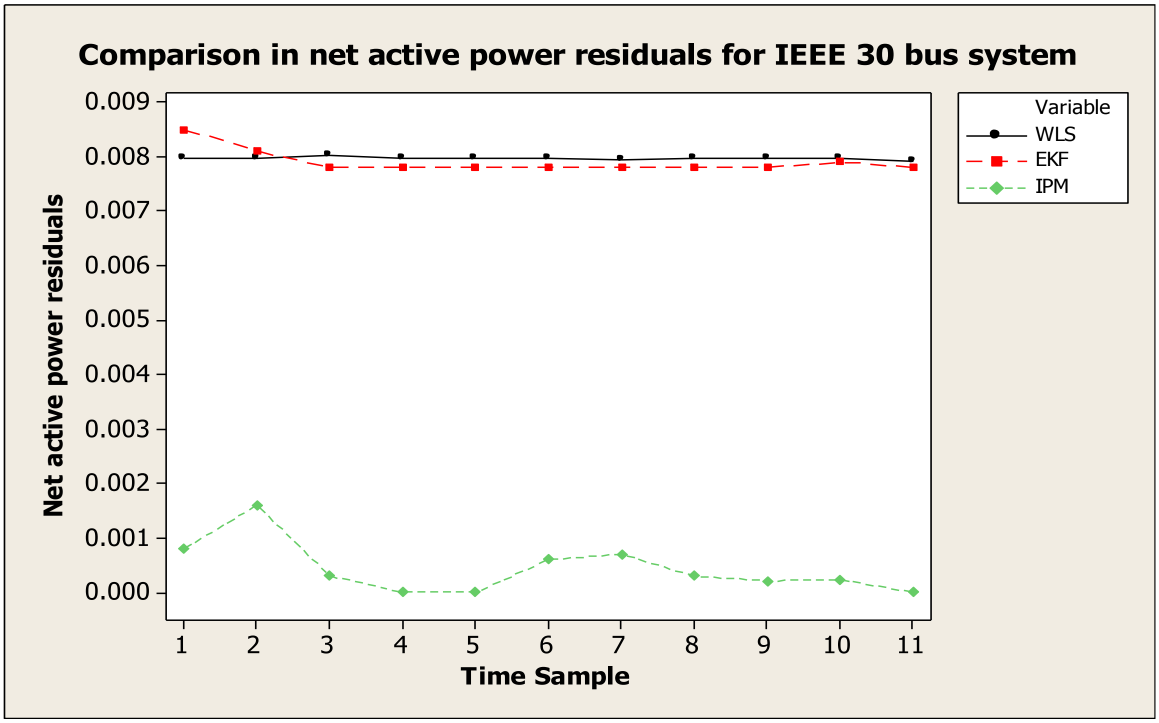

Figure 4-9: Comparison in net active power residuals for IEEE 30 bus system

Figure 4.12 demonstrates the comparison of net active power residuals for IEEE 30 bus system presented in Table 4.15, 4.19, and 4. Again IPM is more accurate because all the values are lesser than both WLS and EKF. Also from the above figure seems there is no different in accuracy between EKF and WLS except that EKF takes lesser iteration and CPU time than WLS. The weights in WLS (R) of net active power are 1E-4 and in EKF are 9E-3.

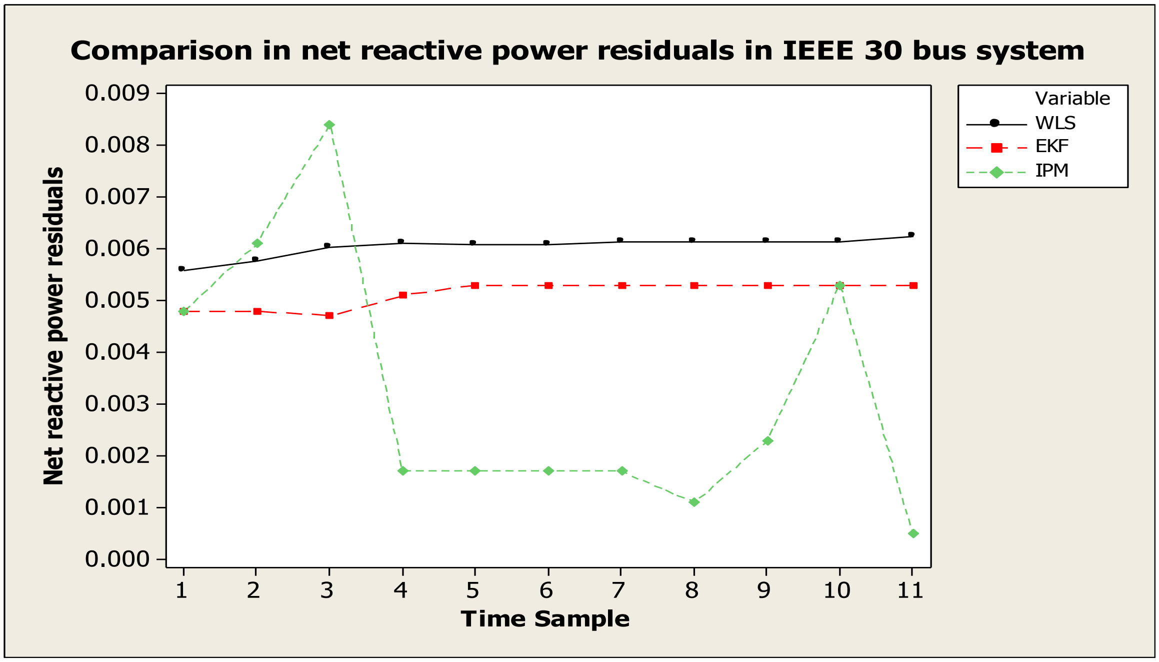

Figure 4-10: Comparison in net reactive power residuals for IEEE 30 bus system

Figure 4.13 compares the net reactive power residuals for IEEE 30 bus system showed in Table 4.16, 4.20, and 4.24. IPM is closer to zero than WLS and EKF. Also residuals of EKF are lesser than WLS. The weights in WLS (R) of net active power are 1E-3 and in EKF are 4E-3.

4.6 TEST CASE IEEE 118 BUSES

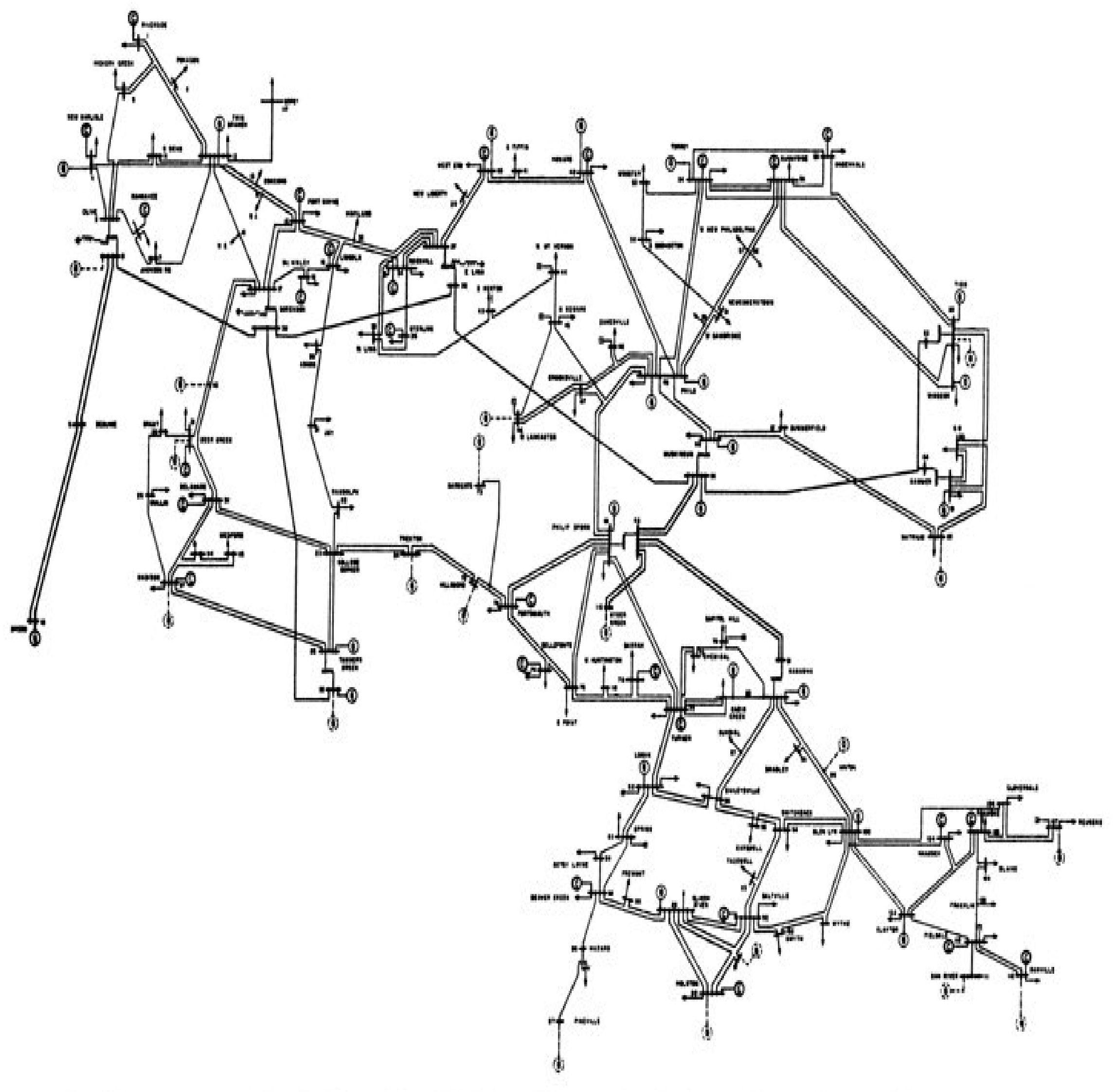

Figure 4-11: Single line diagram of 118 bus system

Name of Network Components	Number of Network Components
Generators	112
Transformers	9
Loads	90
Transmission Lines	186

4.6.1 WEIGHTED LEAST SQUARES METHOD

The new thing in IEEE 118 bus system that we could not use the tolerance condition in WLS Matlab program because some of values diverge. The weights for the angles equal 5E-6, for the voltages are 5E-5, for the net active power are 1E-6, and for the net reactive power 5E-4. We also observe the input number of state variables in WLS is 5 times. Finally we checked all the state vectors and measurements in WLS for IEEE 118 bus system and some values have not successfully converged.

4.6.1.1. Measured and estimated angles with error using WLS

Table 4-25: Measured and estimated angles with error using WLS

Bus no	Measured Angles Values	Estimated Angles Values	%Error
13	-0.0446196	-0.0446196	3.2E-5
34	-0.0426906	-0.0426906	7.57E-05
40	-0.0921856	-0.0921856	2.09E-05
43	-0.0314880	-0.0314880	4.15E-05
59	0.1007783	0.1007783	4.11E-05
67	0.1672282	0.1672282	2.02E-05
73	0.1483344	0.1483344	9.8E-06
95	0.2131232	0.2131232	1.61E-05
102	0.2714584	0.2714584	1.5E-5
117	-0.0440711	-0.0440711	1.35E-05
118	0.1371820	0.1371820	1.22E-05

4.6.1.2. Measured and estimated voltages with error using WLS

Table 4-26: Measured and estimated voltages with error using WLS

Bus no	Measured Voltages Values	Estimated Voltages Values	% Error
13	0.91717	0.91717	1.4E-06
34	0.98695	0.98695	2.3E-06
40	1.00106	1.00106	1E-6
43	1.03549	1.03549	2E-6
59	1.19586	1.19586	5E-07
67	1.22432	1.22432	1.7E-06
73	1.02489	1.02489	6E-07
95	1.15950	1.15950	7E-07
102	1.17753	1.17753	1E-6
117	0.94439	0.94439	1.4E-06
118	1.10251	1.10251	3.3E-06

4.6.1.3. Measured and estimated net active power into the system with residuals using WLS

Table 4-27: Measured and estimated net active power with residuals using WLS

Bus no	Measured Values of net active power into the system	Estimated Values of net active power into the system	Residuals
13	-0.3400	-0.2929	4.71E-2
34	-0.5900	-0.2249	3.651E-1
40	-0.6600	-0.3881	2.719E-1
43	-0.1800	-0.0989	8.11E-2
59	-1.2200	-0.6853	5.347E-1
67	-0.2800	-0.1905	8.95E-2
73	-0.0600	0.0005	6.05E-2
95	-0.4200	-0.1706	2.494E-1
102	-0.0500	0.0078	5.78E-2
117	-0.2000	-0.1687	3.13E-2
118	-0.3300	0.0411	3.711E-1

4.6.1.4. Measured and estimated net reactive power into the system with residuals using WLS

Table 4-28: Measured and estimated net reactive power with residuals using WLS

Bus no	Measured Values of net reactive power into the system	Estimated Values of net reactive power into the system	Residuals
13	-0.16	-0.2826	1.2263E-1
34	-0.26	-0.4052	1.4523E-1
43	-0.07	-0.1556	8.561E-2
59	-1.13	-1.1578	2.775E-2
67	-0.07	-0.2032	1.3317E-1
73	0.00	-0.0773	7.731E-2
95	-0.31	-0.2953	1.474E-2
102	-0.03	-0.0036	2.636E-2
117	-0.08	-0.1079	2.793E-2
118	-0.15	-0.1524	2.35E-3

4.6.2 EXTENDED KALMAN FILTER

We suggest the initial covariance (P) in EKF to be equals 0.01 and the noise variance (Q) of state vectors is 0.01 because we work on the tolerance $tol < 1e - 1$ in IEEE 118 bus system, so we do not observe the number of the iteration; however, the time to obtain the optimal solution in EKF is faster than both WLS and IPM. Also the weights have chosen to give us accurate results for all errors and residuals which are 6E3 for angles, 6E3 for voltages, and 6E6 for net active power and 6E4 for reactive power into the system. We also observe the input number of state variables in EKF is 3 times. Finally we checked all the state vectors and measurements in EKF for IEEE 118 bus system and all the values have successfully converged. The time to obtain the optimal solution in EKF is faster than both IPM and WLS.

4.6.2.1. Measured and estimated angles with error using EKF

Table 4-29: Measured and estimated angles with error using EKF

Bus no	Measured Angles Values	Estimated Angles Values	% Error
13	-0.0403495	-0.0403495	7.3E-06
34	-0.0313728	-0.0313728	1.49E-05
40	-0.0529834	-0.0529834	7.6E-06
43	-0.0245089	-0.0245089	4.4E-06
59	-0.0243231	-0.0243231	1.56E-05
67	0.0393583	0.0393583	7.9E-06
73	-0.0165501	-0.0165501	6.4E-06
95	-0.0172618	-0.0172618	1.69E-05
102	0.0158082	0.0158082	1.2E-06
117	-0.0280828	-0.0280828	5.5E-06
118	-0.0386295	-0.0386295	1.04E-05

4.6.2.2. Measured and estimated voltages with error using EKF

Table 4-30: Measured and estimated voltages with error using EKF

Bus no	Measured Voltages Values	Estimated Voltages Values	%Error
13	0.94577	0.94577	4E-07
34	0.99956	0.99956	3E-07
40	0.96917	0.96917	2E-07
43	0.98554	0.98554	1E-07
59	0.98841	0.98841	3E-07
67	1.01937	1.01937	2E-07
73	0.99324	0.99324	1E-07
95	0.98433	0.98433	2E-07
102	1.00450	1.00450	1E-07
117	0.94190	0.94190	3E-07
118	0.96778	0.96778	1E-07

4.6.2.3. Measured and estimated net active power with residuals using EKF

Table 4-31: Measured and estimated net active power with residuals using EKF

Bus no	Measured Values of net active power into the system	Estimated Values of net active power into the system	Residuals
13	-0.340	-0.2094	1.3058E-1
34	-0.590	-0.4361	1.5385E-1
40	-0.660	-0.3363	3.2372E-1
43	-0.180	0.0073	1.8729E-1
59	-1.220	-1.2387	1.874E-2
67	-0.280	-0.5064	2.2637E-1
73	-0.060	-0.0575	2.54E-3
95	-0.420	-0.4797	5.972E-2
102	-0.050	-0.2320	1.8196E-1
117	-0.200	-0.0485	1.5151E-1
118	-0.330	-0.2547	7.526E-2

4.6.2.3. Measured and estimated net reactive power with residuals using EKF

Table 4-32: Measured and estimated net reactive power with residuals using EKF

Bus no	Measured Values of net reactive power into the system	Estimated Values of net reactive power into the system	Residuals
13	-0.1600	-0.1497	1.03E-2
34	-0.2600	-0.2307	2.93E-2
40	-0.2300	-0.1611	6.89E-2
43	-0.0700	-0.0981	2.81E-2
59	-1.1300	-1.1690	3.90E-2
67	-0.0700	-0.0616	8.4E-3
73	0.0000	-0.0112	1.12E-2
95	-0.3100	-0.0800	2.30E-1
102	-0.0300	-0.0248	5.2E-3
117	-0.0800	-0.0579	2.21E-2
118	-0.1500	-0.0619	8.81E-2

4.6.3 INTERIOR POINT METHOD

The weights in IPM have chosen to be 1.0 to obtain one solution for both state variables and measurements. We also observe the input number of state variables in IPM is 2 times. Finally we checked all the state vectors and measurements in IPM for IEEE 118 bus system and all the values have successfully converged.

4.6.3.1. Measured and estimated angles with error using IPM

Table 4-33: Measured and estimated angles with error using IPM

Bus no	Measured Angles Values	Estimated Angles Values	%Error
13	-0.123598290000	-0.123598288951	8.49E-07
34	-0.089992160000	-0.089992161155	1.28E-06
40	-0.094141030000	-0.094141033840	4.08E-06
43	-0.084234960000	-0.084234957978	2.4E-06
59	-0.052515130000	-0.052515133780	7.2E-06
67	-0.032948630000	-0.032948627784	6.73E-06
73	-0.052170650000	-0.052170653465	6.64E-06
95	-0.034757540000	-0.034757543387	9.74E-06
102	-0.017238210000	-0.017238209919	4.71E-07
117	-0.122747730000	-0.122747732250	1.83E-06
118	-0.038877140000	-0.038877142725	7.01E-06

4.6.3.2. Measured and estimated voltages with error using IPM

Table 4-34: Measured and estimated voltages with error using IPM

Bus no	Measured Voltages Values	Estimated Voltages Values	%Error
13	0.960003676000	0.960003675700	3.13E-08
34	0.970598894000	0.970598894137	1.41E-08
40	0.966540942000	0.966540942454	4.69E-08
43	0.973337862000	0.973337862304	3.12E-08
59	0.981782375000	0.981782375472	4.8E-08
67	0.992572351000	0.992572350786	2.15E-08
73	0.980129862000	0.980129861586	4.22E-08
95	0.996036685000	0.996036684907	9.3E-09
102	1.010844766000	1.010844765924	7.5E-09
117	0.960009763000	0.960009763490	5.11E-08
118	0.976276903000	0.976276903033	3.3E-09

4.6.3.3. Measured and estimated net active power into the system with residuals using IPM

Table 4-35: Measured and estimated net active power with residuals using IPM

Bus no	Measured Values of net active power into the system	Estimated Values of net active power into the system	Residuals
13	-0.340	-0.3360	4.0E-3
34	-0.590	-0.5891	9E-4
40	-0.660	-0.6613	1.3E-3
43	-0.180	-0.1826	2.6E-3
59	-1.220	-1.2290	9.0E-3
67	-0.280	-0.2897	9.7E-3
73	-0.060	-0.0715	1.15E-2
95	-0.420	-0.4324	1.24E-2
102	-0.050	-0.0611	1.11E-2
117	-0.200	-0.1978	2.2E-3
118	-0.330	-0.3421	1.21E-2

4.6.3.4. Measured and estimated net active power into the system with residuals using IPM

Table 4-36: Measured and estimated net reactive power with residuals using IPM

Bus no	Measured Values of net reactive power into the system	Estimated Values of net reactive power into the system	Residuals
13	-0.10	-0.0235	7.65E-2
34	-0.09	-0.0662	2.38E-2
40	-0.11	-0.0962	1.38E-2
43	-0.23	-0.2179	1.21E-2
59	-0.03	-0.0279	2.1E-3
67	-0.18	-0.1784	1.6E-3
73	0.00	-0.0007	7E-4
95	-0.16	-0.1753	1.53E-2
102	-0.15	-0.1680	1.80E-2
117	0.00	-0.0019	1.9E-3
118	-0.08	-0.0498	3.02E-2

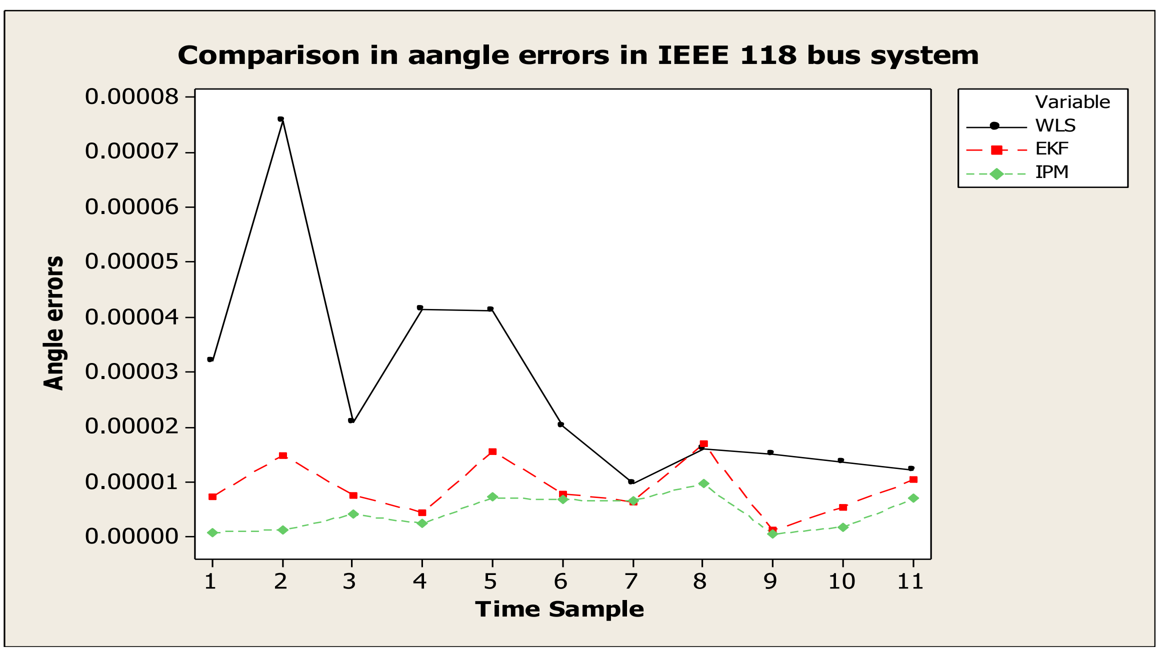

Figure 4-12: Comparison in angles error for IEEE 118 bus system

Figure 4.15 displays the comparison of angle errors for IEEE 118 bus system illustrated in Table 4.25, 4.29, and 4.33. Both of IPM (green line) and EKF (red line) are more accurate because all angle errors are closer to zero than WLS (black line) which has more variation. The weight (R) in angles error in WLS is 5E-6 and in EKF are 6E3.

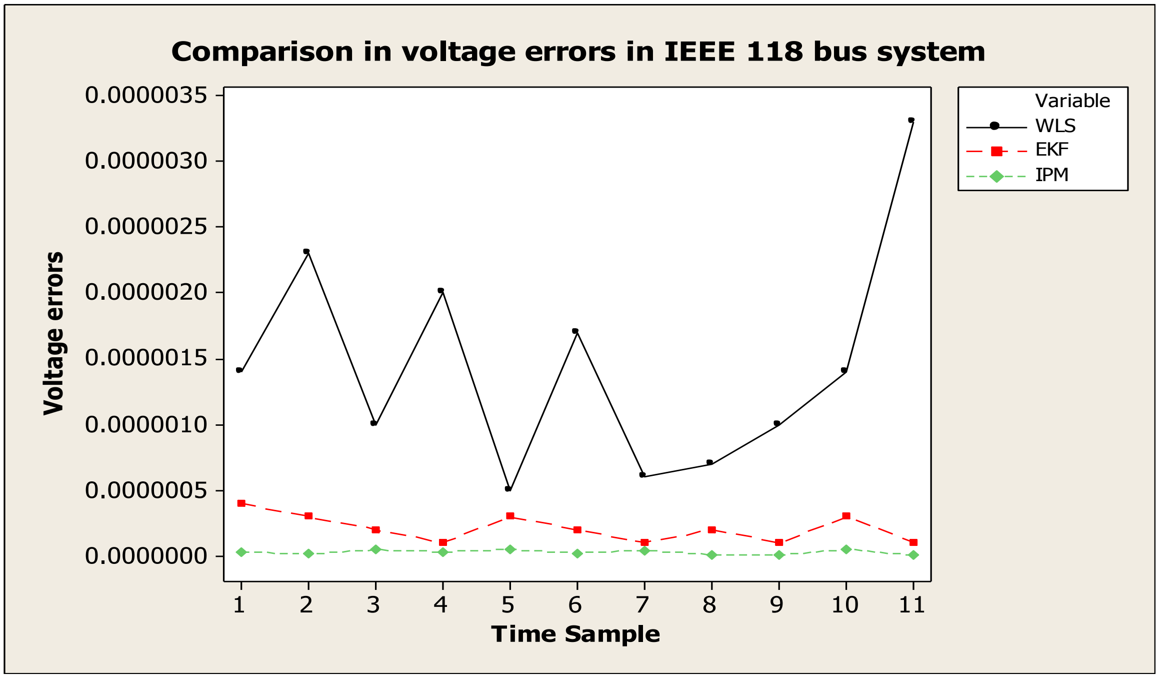

Figure 4-13: Comparison in voltages error for IEEE 118 bus system

Figure 4.16 shows the comparison of voltage errors for IEEE 118 bus system showed in Table 4.26, 4.30, and 4.34. Again in large scale electrical network proved that both of IPM and EKF are more accurate than WLS at all the voltage error points. The weight (R) in voltages error in WLS is 5E-5 and in EKF are 6E3.

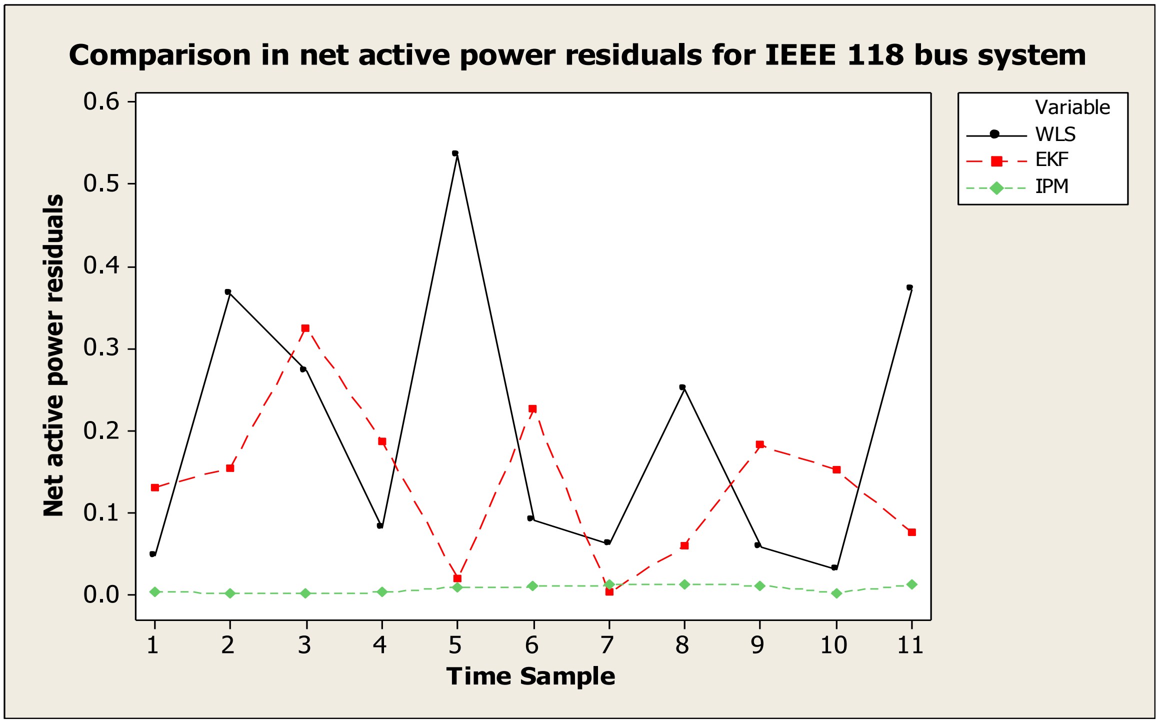

Figure 4-14: Comparison in net active power residuals for IEEE 118 bus system

Figure 4.17 shows the comparison of net active power residuals for IEEE 118bus system represented in Table 4.27, 4.31, and 4.35. As always IPM is more accurate because all the residual values are almost zero and both of WLS and EKF are both variations, so it is hard to distinguish the accuracy. The weights of net active power in WLS are 1E-6 and in EKF are 6E6.

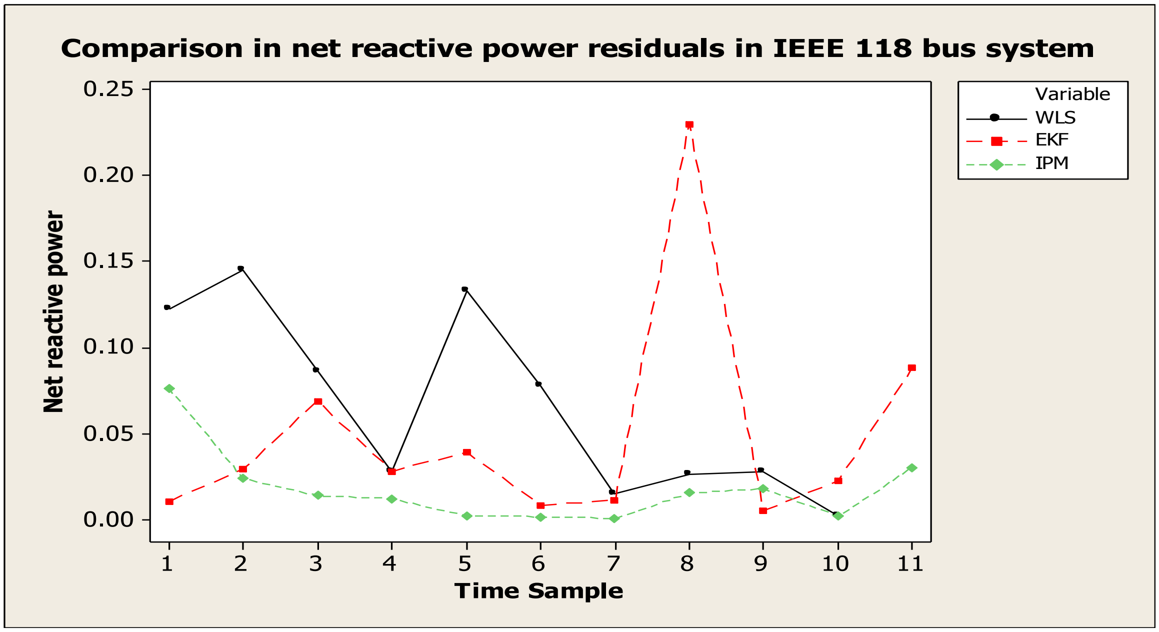

Figure 4-15: Comparison in net reactive power residuals for IEEE 118 bus system

Figure 4.18 compares the net reactive power residuals for IEEE 118 bus system showed in Table 4.28, 4.32, and 4.36. The IPM net active residuals is lesser and closer to zero than both WLS and EKF. The weights of net reactive power in WLS are 5E-4 and in EKF are 6E4.

Table 4-37: Comparison in state estimation algorithms

	COMPARISION IN STATE ESTIMATION ALGORITHMS					
	INPUT ITERATIONS			CONVERGENCE		
ALGORITHMS	IEEE 14	IEEE 30	IEEE 118	IEEE 14	IEEE 30	IEEE 118
WLS	2 times	3 times	5 times	converged	converged	Non-converged
EKF	2 times	3 times	3 times	converged	converged	converged
IPM	1 time	1 time	2 times	converged	converged	converged

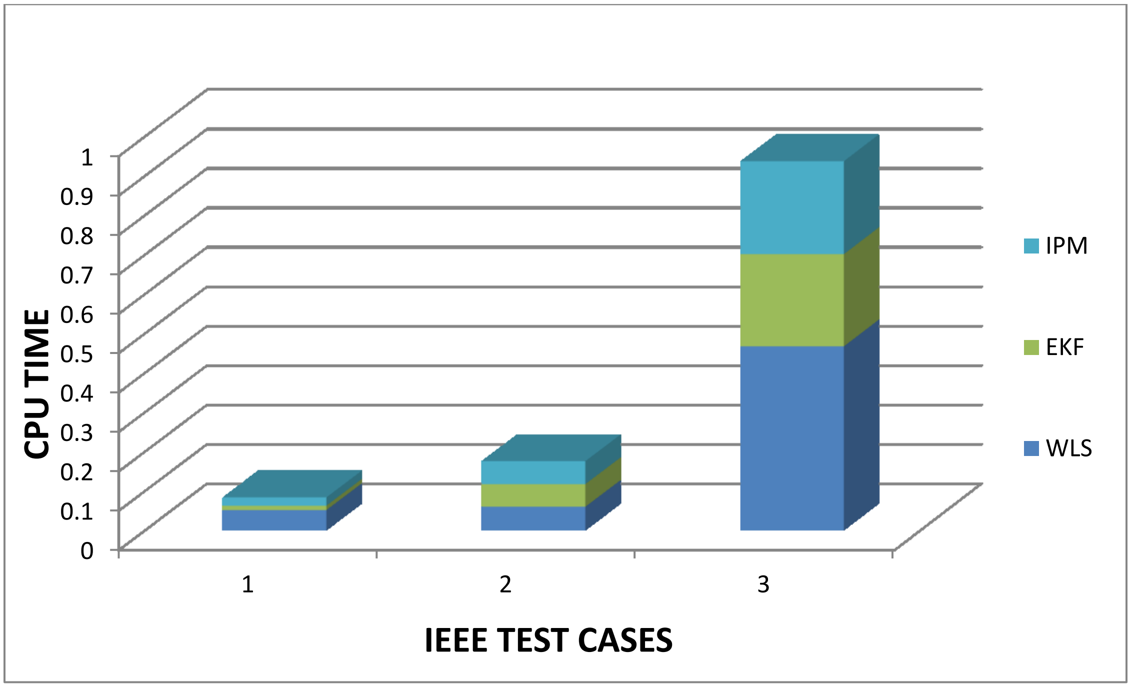

Figure 4-16: Comparison in CPU time in IEEE test cases

4.7 DISCUSSION

From all the tables and graphics that we have demonstrated in this thesis, we sum up the comparison in state estimation algorithms that weighted least square clearly is very sensitive to the influence of outliers which can make a gap between the statistical theory and practical applications. This case can be obvious in IEEE 118 buses because some of state variables did not converge or the optimal values resulted from the estimation is not related to the actual nominal values of state variables; moreover, several references have mentioned that there is no different between weighted least square and extend kalman filter in accuracy except the number of iterations to obtain the optimal points; however, in this thesis, some of results have shown that extend kalman filter can even challenge interior point method algorithm in case of accuracy which can be clear in figure 4.5,4.10,4.15,4.16,and 4.18.

Even-though weighted least square is not successful in large scale electrical networks, it has intervened in several interpretations to provide variety of statistical intervals for many parameters such as predictions, estimations, calibrations, and finally optimizations.

An interior point method specifically logarithmic barrier functions has been used and succeed in this thesis, but CPU time is not faster as EKF is, so in case of accuracy, interior point method is the best estimation and in case of less CPU time, EKF is the best; however, WLS is a widely use in practical life because everyone knows the composition of this algorithm and is more trust than other algorithms.

CHAPTER 5 CONCLUSION AND FUTURE WORK

5.1 CONCLUSION

State estimation techniques in area of power system operation and control analysis have been constantly developing topics to find more precise solution for many electrical networks issues. Many algorithms with different techniques have been extensively implemented to accomplish the best performance which can give us the best estimation of electrical networks.

In this thesis, three different algorithms including WLS, EKF, and IPM are proved, discussed, and employed in order to achieve the most optimum solution. Specifically in IPM, we have concentrated on the minimization sum of real time available measurements by considering the maximization and the dual-algorithms in explanation as well.

We have done three different test cases including IEEE 14, 30, and 118 buses to compare the state estimation algorithms based on the errors and residuals which can help us have an indicate of the best algorithm which can provide us an accurate or optimum view of the state of our electrical networks .

From this comparison, it has found that weighted least absolute value (WLAV) in interior point method (IPM) is the best estimation at most of test cases, extend kalman filter (EKF) is the second best estimation, but it is clear at some of test cases, and so obvious that weighted least square is the worst estimation because of reasons that we have explained in discussion in chapter 4.

5.2 FUTURE WORK

The area of this thesis can be further extent in wide research areas such as:-

The problems discussed in this thesis can be further extent to employ more electrical network components such as IEEE 9, 57, and 300 buses by considering the additional constraints in WLAV interior point method.

Contingency analysis is one of a challenge part in electrical networks, so we can use the state estimation to evaluate the type of emergency state N, N-1, or even N-2 which is outage in two or more generations or transmission lines. Also state estimation can be used to deal with load change challenge and lack of generation production.

Smart grid electrical network can be an interesting topic to employ these algorithms to minimize the sum of real time available measurements residuals.

This thesis can be further extent to explore new optimization algorithms including deterministic and heuristic algorithms as well as implementing these algorithms in several IEEE electrical networks.

Renewable energy such as wind, solar, and tidal energy can be considered in state estimation.

This thesis can be further extent in using phasor measurements units (PMUs) in these electrical networks and compare the performance of these algorithms with and without using phasor measurements units (PMUs).

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