A STOCHASTIC DYNAMIC PROGRAMMING APPROACH FOR OPTIMIZING MIXED-SPECIES FOREST STAND MANAGEMENT POLICIES

by

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The undersigned hereby certify that they have read and recommend to the Faculty of Graduate Studies for acceptance of a thesis entitled "A STOCHASTIC DYNAMIC PROGRAMMING APPROACH FOR OPTIMIZING MIXED-SPECIES FOREST STAND MANAGEMENT POLICIES" by Jules Comeau in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

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Abstract

The main goal is to develop decision policies for individual forest stand management. It addresses three major areas of interest in the optimal management of individual forest stands: incorporating a two-species growth and yield model into a single stand management model, incorporating a comprehensive list of management options into a single stand management model, and incorporating uncertainty into a single stand management model. Dynamic programming (DP) is a natural framework to study forest management with uncertainty. The forest stand management problem, as modelled in this thesis, has a large dimensional state space with a mix of discrete and continuous state variables. The DP model used to study this problem is solved by value iteration with the objective of understanding infinite horizon policies. However, since some of the state variables are continuous, all states can't be examined in an attempt to create the cost-to-go function. Therefore, the cost-to-go function value is calculated at a given stage of the algorithm at a finite set of state points and then the cost-to-go values are approximated on the continuous portion of the state space using a continuous function. All of this is done with random processes impacting state transitions.

With the mixed-species growth model developed in this thesis, a comprehensive list of management options can be incorporated into the DP model and, with the addition of uncertainty from sources such as market prices and natural disasters, near optimal stand management policies are developed. Solving the DP model with the required level of detail lead to the development of insight into function fitting on continuous state spaces and to the development of cost-to-go function approximation bounds. Studying the policies shows that the addition of uncertainty to the model captures the dynamics between market prices and stand definitions, and leads to policies that are better suited to decision making in a stochastic environment, when compared with policies that are developed with a deterministic model. Enough precision is built into the DP model to give answers to typical questions forest managers would ask.

List of Abbreviations and Symbols Used

abv – Commercial thinning from above

age – Stump age of the stand at time t

aht – Average tree height (m)

BA – Total stand basal area (m^2)

$BAFULL^{\theta}$ – Basal area per hectare if the stand was 100% stocked with species θ

$BARem$ – Percentage of total stand basal area to be removed during commercial thinning

$BARem_{split}$ – Percentage of basal area removed that is softwood

blw – Commercial thinning from below

cc – Crown closure fraction

CPU – Central processing unit of a computer

cros – Commercial thinning across the diameter distribution

CT – Commercial Thinning

CTG – Cost-to-go function

d – Stand quadratic mean diameter (cm)

dht – Dominant tree height (m)

DP – Dynamic Programming

DWI – Distance Weighted Interpolation

ECC – Early competition control used to control the growth of specific species of trees on a stand

GNY – Growth and yield

ha – Height age in years

HW – Hardwood

J – Cost function

LC – Land Capability class

MAI – Mean Annuel Increment

$maxtrees$ – Maximum number of trees on one hectare for species θ

MR – Multiple regression

MV – Merchantable Volume (m^3)

NPV – Net Present Value

NS DNR – Nova Scotia Department of Natural Ressources

PCT – Pre-Commercial Thinning

pct^{θ} – Fraction of total basal area of the stand that is species type θ

plt – Plant trees on a stand at a given density per hectare

r – Vector of parameters for cost-to-go function approximation

RBF – Radial Basis Function

Regen – Natural regeneration

ReHar – Regeneration harvest

rmv – Remove trees during a commercial thinning

S^{Eval} – Evaluation subset of state space S

S^{Result} – Result subset of state space S

R_1, R_2, R_3, R_4 – Regeneration states

SI – Site Index : dominant tree height in meters at age 50

SP^{θ} – Average spacing between trees for species θ (m)

splt – Basal area removal proportion that is softwood

SV – Sawtimber Volume (m^3)

SW – Softwood

$TFREQ^{\theta}$ – Number of trees per hectare for species θ

$THINTYP$ – Type of basal area removal which refers to size of tree that will be targeted first when removing a given percentage of BA

tr/ha – Trees per hectare

TRT – Treatment Type

VB – Visual Basic programming language

θ – Species type : S for softwood and H for hardwood

α – Discount rate for dynamic programming algorithm

φ – Basis functions for approximation architectures

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Chapter 1: Introduction

Forests are of vital economic importance throughout Canada. For example, in the province of Nova Scotia, in 2009, the forest industry directly employed 7300 people and exported $0.8 billion worth of forest products (Natural Resources Canada 2010). In order to maximize the benefit to the economy, it is critical that we be able to make proper managerial decisions with respect to the resource.

This work focuses on specific issues related to the optimal management of an individual forest stand. The landowner must decide on what silvicultural treatments to apply and when to apply them to meet current economic objectives and to ensure long term sustainability of the resource. These decisions include whether to do pre-commercial thinning and at what age and spacing, whether to do commercial thinning and at what age and intensity and whether to replant or to let the forests naturally regenerate after regeneration harvesting. The decisions also include whether to pursue even-aged or uneven-aged management. These decisions must be based on knowledge of the current state of the forest but also on the effects of these silvicultural treatments on the future state of the forest.

Because of the length of the growth period of a forest stand and the complexity of management policies, it is necessary to use models in order to predict future effects of the silvicultural treatments on the state of a forest stand. To study these effects, managers need to properly understand the growth dynamics as well as the yield potential of this complex mix of species. Using this knowledge, they need to develop decision models that incorporate market dynamics, growth and yield (GNY) dynamics and potential natural disasters such as insect outbreak or hurricanes. Models will be presented in chapter 2 and the application of those models is discussed in detail in chapter 3. The role of decision models is to compute optimal policies that allow decision makers to manage the resource while facing uncertain outcomes from their decisions.

In this chapter, we establish the interest of this work which is to develop decision policies for real problems. This leads to a discussion of the challenges in dealing with these real problems and concludes with the contribution of this work.

1.1 Main Areas of Interest

The main goal of this thesis is to develop decision policies for individual forest stand management. More specifically, we deal with the issues faced when developing optimal policies for mixed-species even-aged stands in a stochastic environment. This research addresses three major areas of interest in the optimal management of individual forest stands:

i. Incorporating a two-species growth and yield model into a single stand management model

When studying forest management, growth and yield models are necessary tools. They can be divided into two broad categories: individual tree growth models and whole stand growth models. Both are simple yet useful abstractions of a complex biological system. Individual tree models are not discussed. The reader is directed to these papers for an overview of these models (Andreassen and Tomter (2003), Zhao et al. (2004), Yang et al. (2003), Huang and Titus (1999)). In Nova Scotia, the Department of Natural Resources (NS DNR) has developed separate softwood and hardwood stand level growth and yield models (NSDNR (1993b), O'Keefe and McGrath (2006)). In section 2.7, we propose a method for combining these models to allow modeling of the growth of two species groups together in the same stand. Species groups refer to groups that have distinct growth dynamics and need to be combined together into a single stand. The modeling results are presented in chapter 3.

ii. Incorporating a comprehensive list of management options into a single stand management model

The simplest way to describe an individual forest stand is to use the stand age. A simple stand will have a single species and management can be represented as one decision, the age at which to harvest the stand. This type of decision making, which can be done visually on a graph (Faustmann 1849) is important to understand the basic behaviour of individual forest stands. Chapter 2 will discuss these models in detail.

Growth dynamics and decision making for real forest stands is more complex. Typical stands have more than one species and each species grows differently. Chapter 2 presents the models for combining more than one species on the same stand and chapter 3 presents results of the application of those models. Growth of a stand depends on many factors such as land capability, diameter of the trees, stand basal area, age and previous silvicultural treatments.

If we have complex forest stands that include multiple species that have different continuous growth rates, we need to find a way to make optimal decisions about the silvicultural treatments to apply to these stands. The list of silvicultural options is extensive and complex. They will be presented in chapter 2 and, in chapter 4, we discuss how they are incorporated into one model.

iii. Incorporating uncertainty into a single stand management model

Uncertainty is a fact of life regardless of the planning horizon. Three of these sources of uncertainty are discussed in detail.

1) Market price uncertainty for wood products.
2) Natural disasters such as forest fires, hurricanes and insect outbreaks.
3) The length of the regeneration period of a natural stand.

Dynamic programming (DP) is a natural framework to study forest management with uncertainty. Optimal management regimes (Pelkki 1999), uncertain market price dynamics (Yoshimoto 2002) and changing forest growth conditions (Jacobsen and Thorsen 2003) are some of the areas where DP has been applied. In this thesis, the sources of uncertainty discussed above have been incorporated into a stochastic dynamic programming model aimed at providing optimal decision policies. This model is formulated in chapter 4.

Most individual forest stand management models found in the literature take a simple approach with simplified state variables and decision structures. We attempt to advance the science of individual forest stand modeling by using a more detailed state representation that allows for the use of more realistic growth models and the examination of more complex silvicultural alternatives. In chapter 4, we examine the contrast between the level of detail in previous research and what we propose in this thesis.

1.2 The Challenges for a Dynamic Programming Approach

In this work, we consider a stochastic setting with more treatment types, and management alternatives and a more complex state space than has been found in the literature. Our approach is based on stochastic approximate dynamic programming and we will discuss the insights that can be gained from this approach in the next section.

The complexity of DP models for individual forest stand management optimization can be measured by the amount of detail in 4 areas: forest state descriptors, treatment types, market price levels, and management decisions.

Forest state descriptors are variables that describe the state of the stand at any given point between beginning of growth and regeneration harvesting. In the literature, variables such as volume, residual basal area or number of trees describe stand density (Haight et al. (1985), Brodie and Kao (1979), Amidon and Akin (1968)). In some studies, the use of volume alone was appropriate to discuss rotation ages for pure, single species, even-aged stands. The rotation age is the planned number of years between the formation or regeneration of a crop or forest stand and its final cutting at a specified stage or maturity (Canadian Forest Service 2010). Other studies required the use of residual basal area and number of trees to describe the state of the stand in order to study the impact of partial harvesting on the optimization of stand management. In this work, the number of state variables is sufficient to give an appropriate understanding of the growth and yield dynamics for a stochastic two-species forest stand management model.

Treatment types reflect the history of the past decisions that have been made about the management of the stand. The majority of the reviewed papers optimize models for one treatment type at a time (Arthaud and Klemperer (1988), Peltola and Knapp (2001), Brodie et al. (1978)) or don't differentiate between treatment types. Most studies use variables that can describe the state of the stand regardless of treatment history. Lien et al. (2007) use volume only to describe the stand in a study of the effect of risk aversion on the management of recently harvested forest land in Norway. They study one type of stand. Haight et al. (1985), Brodie and Kao (1979), Amidon and Akin (1968), Arthaud and Klemperer (1988), Brodie et al. (1978) all applied methods that optimize the management of a stand over a set of decisions that keep the stand type as a natural or plantation through the entire optimization horizon without changing from one to the other. Here, we add decisions that can change the treatment type of the stand from plantations to natural stands and vice-versa. Five treatment types are considered.

Market price levels, or market states, are selling prices for wood products removed from a stand. Rollin et al. (2005) use low, medium and high market states to study the management of uneven-aged forests in the French Jura. Haight and Holmes (1991) used 40 price levels to study the relationship between the age of a stand and market prices, and their effect on the cut / no cut decision for loblolly pine plantations in the south-eastern US. In this research, we use a sufficient number of market states to capture the dynamic nature of the decision policies and we'll see that three price levels are not sufficient in some cases.

Management decisions are options available to the forester in order to remove trees from a stand to either encourage better growth of the remaining trees or to create revenue. Most studies limit the number of decisions either for simplicity (Lien et al. 2007) or because of limitations to the size of the model being used (Haight et al. 1985). Here, a forest stand is represented by a dynamic system that is evolving over time. In addition to the dynamics of the stand, market dynamics have an impact on the structure of the DP model and on the decision policies. The decision maker will observe the state of the forest stand and the state of the market, and make decisions about the silvicultural treatments to apply to the stand.

The decision structure in individual forest stand management optimization models is the most important issue. As the state of the stand and the market state evolve, the timing of the observation of those states has an impact on the structure of the DP model.

Whether we observe the state of the stand and of the market before or after making a decision will have an impact on how the stochastics are incorporated into the DP recursion equation. These impacts will be discussed in detail in chapter 4.

Dealing with a high dimensional state space poses challenges for dynamic programming. We use stochastic dynamic programming because it is the only optimization technique that allows us to deal with the stochastic nature of decision making in individual forest stand management. One approach to developing DP in a multi-dimensional state space with several continuous variables is to choose a small number of discrete states and to evaluate the cost-to-go (CTG) function at those chosen states. In our context, we can't analytically determine the exact value of the CTG function for any state. Therefore we develop an approximation function. Regression was the first technique used to approximate the CTG function but the complexity of the state space made it impossible for multiple regression to always give an appropriate approximation without using some sort of bounding scheme to limit its function approximation value. Therefore, we develop an approach, based on approximation based control, for creating bounds on the approximation of the CTG function. A radial basis functions (RBF) gives proper approximations for most values of the cost-to-go function because it interpolates between values. However, its implementation isn't trivial. In addition, in some instances, the shape of the approximating curve between discrete states causes some combinations of state variable values to yield inappropriate approximations. Distance weighted interpolation (DWI) has proven to be accurate in approximating the CTG function as long as the distances used in the weighting scheme are properly scaled. The implementation of these techniques and the challenges they bring are discussed in chapter 4 and 5 and results show that they all lead to policies that are close to optimal.

Stochastics force the structure of the problem to change when compared to a deterministic problem and has been a major challenge for researchers in this field. There are several proper ways of dealing with the stochastics but the deciding factor is the timing of the silvicultural decision with the timing of when you learn the value of the uncertain variables such as market prices. Many studies give policies that don't allow the decision maker to observe the state of the forest stand and market prices before the making a decision (Kao and Brodie (1979), Pelkki and Arthaud (1997), Pelkki (1997), Arthaud and Pelkki (1996)). These studies do not take advantage of DP to develop proper decision policies. Studies which properly account for the stochastic aspect of forest management include Haight et al. (1985), Haight (1993), Haight (1991) and Hool (1966). We want to add more detail to make the resulting management policies more useful to forest managers. We can develop policies that account for the stochastic aspect of forestry by using DP as described in chapter 4 while structuring the problem so that the variable nature of the prices of the wood products market and the unpredictable nature of natural disasters such as insect outbreaks, forest fires and hurricanes are modeled at a level of detail that allows us to study their effect on policy.

1.3 Contributions of this Research

The model developed in this research solves an infinite horizon discounted stochastic dynamic programming problem for a mixed-species forest stand. As discussed briefly in section 1.1, mixed-species in our context refers to two species groups. Value iteration is used to solve the DP algorithm and cost-to-go function approximations, for the continuous portions of the state space, are done using three different approximation architectures, the results of which are discussed in chapter 5. This section gives a brief overview of the contributions of this thesis which are mostly in the areas of the impact of uncertainty and mixed-species modelling on decision making in individual forest stand management.

There were no mixed-species whole stand models readily available to us so we develop a method, based on the concept of crown closure, for combining single-species models together in a mixed-species growth model. Essentially, spacing between trees drives diameter growth and spacing is a function of crown closure. We take advantage of this relationship and develop a methodology for combining and growing two or more species types together in one stand. This has helped us understand the relationship between two species at a stand level and is an important contribution. Simulation and function fitting were used to develop knowledge of how stands react to having different types and percentages of basal area removal. The final model includes user adjustable variables such as forest stand characteristics along with economic parameters such as silvicultural costs, market prices and user definable discount rates. The proposed approach gives us a great deal of flexibility in studying the interaction between species in the same stand.

Silviculture is a complicated process and the approach developed in this thesis allows us to examine pre-commercial thinning, commercial thinning, plantations and regeneration in one model and leads to the development of complex transitory policies. These complex policies then allow us to discuss the trade-off between taking all the wood on a stand with a regeneration harvest versus taking a proportion of it with a commercial thinning and leaving some behind for a future harvest. The impact of this added decision flexibility is discussed and is a contribution of this work.

In chapter 4, we establish the framework for incorporating stochastics into the DP model and we establish that the timing of the decision in reference to the observations made by the decision maker of the state of the market and the forest stand has an important impact on the structure of the DP algorithm. Essentially, the foresters need to understand uncertainty and the impact of not incorporating it into their models. The key is to focus on the decision problem, to exploit uncertainty and to not make decisions based on average prices. In the same chapter, the importance of incorporating this information into the DP model is discussed. The value iteration approach developed for solving the DP algorithm with the level of detail required to make decisions while considering a high dimensional state space, a comprehensive list of management options and stochastic elements is an important contribution. To the best of our knowledge, this level of detail has not previously been combined into a single stand-level management model.

Trying out different function fitting methodologies for the CTG function has led to some insight into the relationship between the methodology and the definition of the state space it is fitting on. As we demonstrate in chapter 5, proper scaling of the values of the variables in these models can be the difference between a good fit and a very bad fit. This work will show that some methodologies are good at approximating a function value for points that are close to evaluation states in state space $\{S\}$ but aren't very good at approximating a function value for the continuous portion of the state space between evaluation states. We'll discuss why this is so. The complexity of the state space and the challenge it poses has led to the implementation of CTG function approximation bounds, derived from control theory. These bounds guarantee consistency as defined by Bertsekas (2000) and, although it is mostly a technical issue, their application in this context is a contribution of this research.

1.4 Overview of Thesis

Figure 1-1 shows a flow chart which gives an overview of this thesis and guides the reader in understanding the links between topics. The three main areas of interest are described in section 1.1 and are identified at the top of figure 1-1. The basics of forest management and the details of combining two species together are presented in chapters 2 and 3. Chapter 4 focuses on the development of approximate dynamic programming approaches to the individual forest stand management optimization problem and critical issues related to these approaches. More specifically, it's the amount of detail required to study the main areas of interest that has created challenges which are discussed in detail in chapter 4. Chapter 5 presents results in four sections, each of which is related directly to the three main areas of interest. Chapter 6 discusses how the goals set out in the thesis have been met, reflects on the contribution of this work, and presents some opportunities for future research.

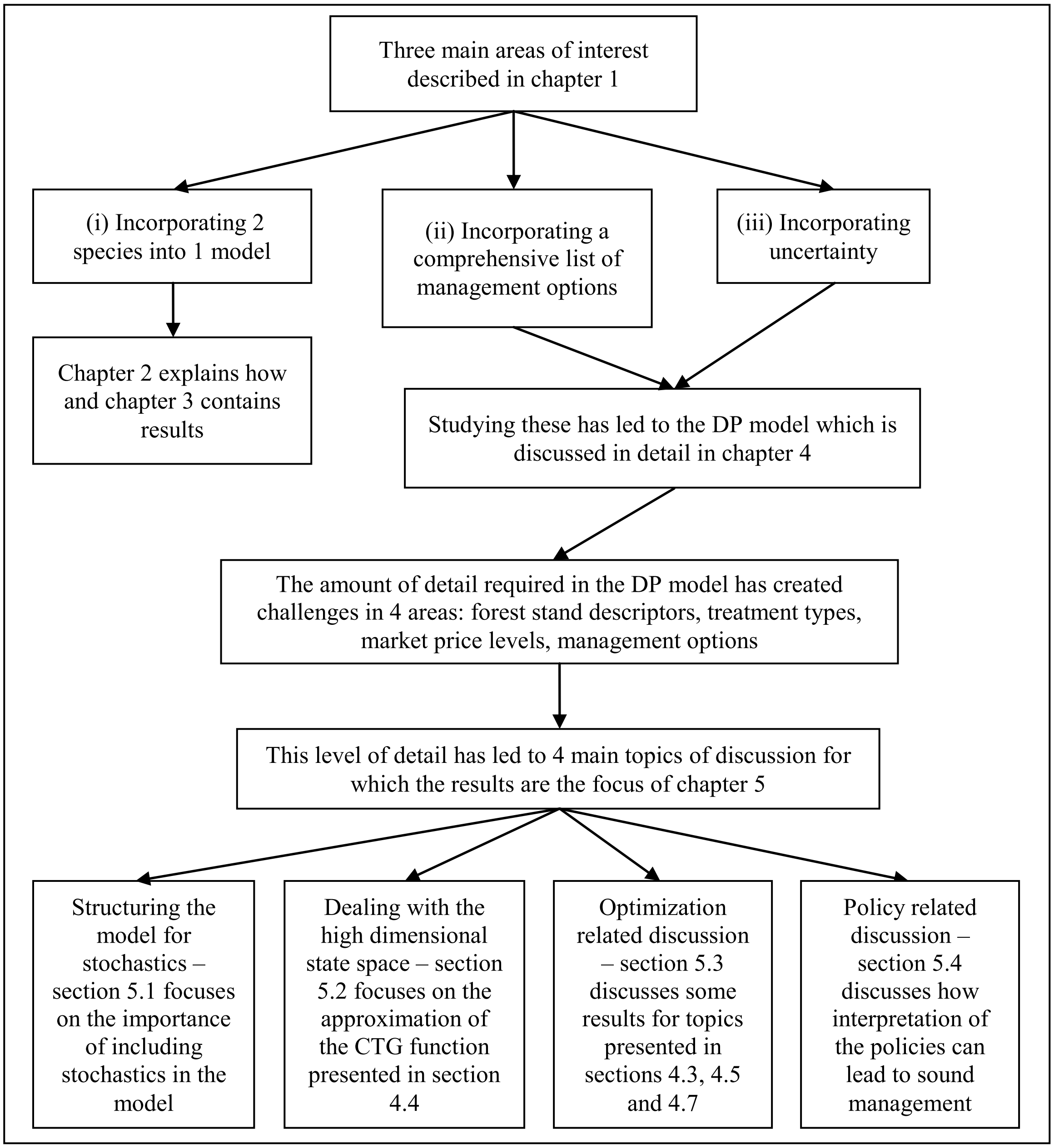


Figure 1-1 – Flow chart of thesis structure

Chapter 2: Issues in Forest Management and Forest Stand Growth Models

Some of the forest management terminology and approaches used in this document are specific to Nova Scotia. The single-species growth models have been developed by Nova Scotia Department of Natural Resources based on data collected in Nova Scotia which renders them specific to this jurisdiction. Applying this approach to another jurisdiction would require replacing the growth models in the DP model with jurisdiction specific growth models. Forest stand management options used as actions in the DP model are applied as suggested in the Forestry Field Handbook published by NSDNR. This chapter presents forest management as it applies to Nova Scotia and as implemented in this thesis. The context of the models in this work is established in the introduction. Sections 2.2 to 2.4 present detailed forest stand definitions and section 2.5 discusses the regeneration period that precedes the establishment of growing stock in a recently harvested stand. Section 2.6 presents methods for growing a single-species forest stand while section 2.7 discusses a proposed approach to combining two species into a mixed-species stand along with arguments which support its use. Section 2.8 discusses yield calculations for single-species stands and how this extends to a mixed-species stand. Section 2.9 presents approaches to modeling natural disasters. The focus of this chapter is on establishing single-species individual stand definitions, on describing procedures for combining those two species into a single stand and growing it, and on discussing the effects of natural disasters on mixed-species stands.

2.1 Introduction

Three levels of forest management are recognized: individual tree level, stand level and forest level (Davis et al. 2005). Each of these levels of management requires the use of specific modeling and optimizing techniques which are briefly presented here.

Individual tree management models require knowledge about the characteristics of every tree in a stand and thus can’t be developed without having a very well structured long term data collection program. Many studies have focused on using individual tree models as the basis for better understanding the growth and yield dynamics of a forest stand (Andreassen and Tomter (2003), Hynynen and Ojansuu (2003), and Zhao et al. (2004)). These models look to understand the interactions between trees in a stand by considering descriptive information such as tree species, tree age and tree diameter, and spatial information such as the spacing between trees. This information is used to develop detailed knowledge about individual tree growth and mortality so stand models can be constructed in which growth and mortality for all individual trees are combined to get average stand growth. Multiple linear regression and artificial neural networks (ANN) have proven to be useful tools in the development of these growth and mortality models. ANN`s are particularly well suited for the binary response of tree mortality (Guan and Gertner (1995), Hasenauer et al. (2001)). The ANN can be programmed to give a binary output based on any given number of inputs. This type of analysis can lead to very detailed knowledge of how stands grow but the level of detail and data, descriptive and spatial, required in order to obtain that knowledge is difficult to find.

"A forest stand is a community of trees possessing sufficient uniformity in composition, age, arrangement, or condition to be distinguishable from the forest or other growth on adjoining areas, thus forming a silvicultural or management entity" (Canadian Council of Forest Ministers 2010). Stand level models can vary from simple single state representations of volume over time to more complex, multi state models which attempt to more accurately represent the factors that determine tree growth and mortality. These more complex stands are discussed later in this chapter.

Forest level management refers to the large scale landscape level management where, typically, the goal is to maximize long term objectives such as maximum sustained yield and/or maximum connectedness of unharvested areas to protect fauna, among others. In forest management, the term forest refers to an area managed for the production of timber and other forest produce, or maintained under woody vegetation for such indirect benefits as the protection of watersheds, the provision of recreation areas, or the preservation of natural habitat (Canadian Council of Forest Ministers 2010). Individual stands that make up a forest will typically have different characteristics. A forest can contain many thousands of stands thus requiring each stand to have a simple definition so the model doesn't become too cumbersome to manage. Optimizing the management of a forest requires the optimization of a forest level objective while making individual stand harvesting decisions. Optimal management policies for this type of application are often developed using a number of operations research (OR) techniques including, but not limited to, linear programming, integer programming, simulation and goal programming.

Many textbooks have been written on the subject of using OR techniques to develop forest management policies (Buongiorno and Gilless (2003), Davis et al. (2005)). As discussed in the first chapter, the objective of this thesis is to develop optimal management policies for mixed-species even-aged individual forest stands. In this chapter, we discuss stand level management especially as it applies here. We start by discussing simple single-species even-aged stand level growth and yield (GNY) along with a basic set of stand management options or decisions to be made. A host of management objectives are briefly discussed and put into context. Some of them aren't applicable to our problem and the reasons are discussed. The addition of a second species to a stand requires additional stand variables and the expansion of the list of stand management options. We will discuss why this is important and how it affects this work. The last section discusses the impact of natural disasters on forest stand management and presents our modeling approach.

2.2 Basic Forest Stand Definitions

In general, the literature distinguishes between 4 types of forest stands (Smith et al. 1997). Pure even-aged stands are stands in which at least 80% of the trees in the main crown canopy are of a single species (Canadian Council of Forest Ministers 2010) and are typically stands that, after the last regeneration harvest or removal of all trees on the stand, were planted or were treated to remain single-species stands by removing most of the competing vegetation. Slightly more complicated are the stands that grow from advanced regeneration, small trees growing under the canopy, present before a regeneration harvest. These stands aren't studied directly here but the concept of a stand containing advanced regeneration is important and will be discussed in section 2.5. Uneven-aged stands have trees or groups of trees at different stages of development and have more complex growth patterns. Uneven-aged stands are not discussed in this thesis.

Mixed-stands have more than one species and can be even-aged or uneven-aged which makes them even more complicated to manage. This discussion starts with pure even-aged stands and moves towards the more complex stands.

The simplest way to describe a pure even aged stand is to use one stand variable such as age where the age refers to the age of the stand since its last regeneration harvest and is the first basic variable used in calculating the volume of a stand. Volume refers to the total volume of wood products such as boards, veneer, fibre and other commercial products that can be extracted from the trees on a stand and is typically measured in m^3. There are charts available to calculate these volumes for any given stand based on characteristics that can be observed on the stand such as age (Keys and McGrath 2002).

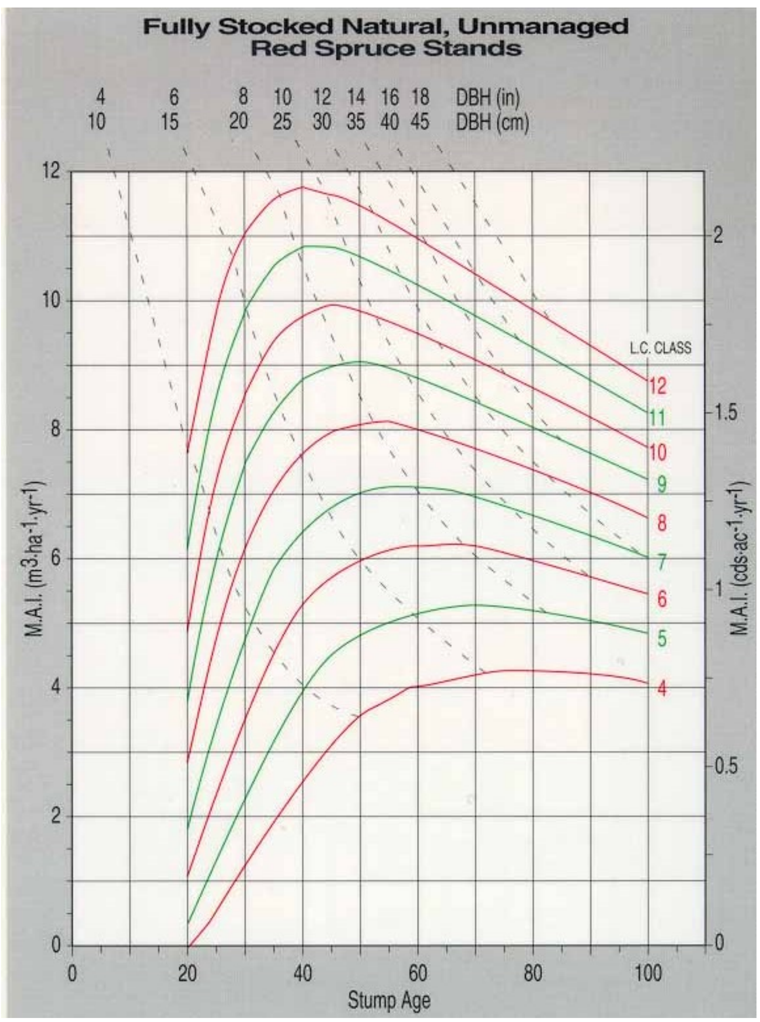


Figure 2-1 – MAI curves for a fully stocked natural unmanaged red spruce stand (NSDNR 1993a)

Figure 2-1 shows the rate of growth of fully stocked natural unmanaged red spruce stands in Nova Scotia (NSDNR 1993a). The concept of stocking is discussed in the next section. The graph has 4 different scales: the bottom axis is the age in years since the last regeneration harvest, the left axis is mean annual increment (MAI) in metric units which is defined below, the top axis is the quadratic mean stand diameter which we will discuss in detail in section 2.3 and the right axis is MAI in imperial units. MAI is the average annual volume growth of the stand per year since the last regeneration harvest and is measured in m^3 per hectare per year ($m^3 \cdot ha^{-1} \cdot yr^{-1}$) in metric units and cords of stocked wood per acre per year ($cds \cdot ac^{-1} \cdot yr^{-1}$) in imperial units. Metric units are used in this thesis. MAI is calculated by dividing the total volume of wood on a stand by the age. For example, a stand with 212 m^3 of wood at age 40 would have a mean annual volume increment of 5.3 $m^3 \cdot ha^{-1} \cdot yr^{-1}$ at age 40.

LC class in this graph refers to land capability class of the stand. LC is an indicator of the rate of growth of the volume of wood on a stand. A higher LC indicates a stand that has capability for a higher rate of volume growth. The red and green curves on the graph plot the average volume growth of the stand per year since the last regeneration harvest for stands of different LC classes. Total volume on any stand continues to grow until the stand dies. But the rate of growth, MAI, changes throughout the entire life of the stand. The steep part of the curve at low stump ages reflects the rapid growth of wood volume in young stands. MAI rises to a maximum value and then drops as the rate of growth of the trees on an old stand slows down. The example of MAI given in the previous paragraph falls on the curve for LC 6.

Ideally, from the point of view of volume growth, the stand would be cut at peak MAI. Based on the curves in figure 2-1, the stand would be cut at different ages depending on the LC of the stand. If the objective is to maximize total volume over time, the rotation age can be chosen to be the age at peak MAI. The rotation age is a recurring age at which a regeneration harvest is done.

A stand with LC 12 has a peak MAI at age 40 and figure 2-1 indicates a sharp fall off before and after that age therefore there is a very narrow range of ages at which MAI is at or near its maximum. However, most stands in Nova Scotia are measured as LC 4 to 6 and very few stands are above LC 6. Although the MAI curve for LC 5 in figure 2-1 shows peak MAI is reached around age 70, MAI stays relatively constant between ages 55 and 90 ranging between 5.0 and 5.3 $m^3 \cdot ha^{-1} \cdot yr^{-1}$. So, choosing a rotation age that maximizes MAI, in this case, may be mathematically simple but, in practice, other management objectives may need to be considered and there may be times where cutting before or after peak MAI is a better decision.

Table 2-1 shows the total volume and MAI for a stand with LC 6. MAI is the total volume divided by the age of the stand. Based on the table and using an objective of maximizing MAI for a stand with LC 6, we should cut the stand between the ages of 60 and 70 years even if the volume of wood on the stand continues to grow up to 100 years. It only takes 60 years to get to 372 m^3 of wood. If we wait to age 100 before harvesting the stand, the volume will grow to 550 m^3 which is a gain of 178 m^3 in 40 years. If the stand is cut at age 60 and begins growing again, it will grow to 212 m^3 by the time it reaches age 40 which is 34 m^3 more than letting the stand grow to age 100 without harvesting at age 60. A 60 year cycle followed by a 40 year cycle yields 584 m^3 while a 100 year cycle yields only 550 m^3. Strictly on a volume basis, the first option is the better one. We observe that two 50 year cycles would yield an even higher total volume of 600 m^3. MAI stays relatively unchanged between the ages of 50 and 90 years which will lead to a variety of optimal policies when the management period is short enough to include only a few growth and harvest cycles.

Table 2-1 – MAI and corresponding volume for a fully stocked natural unmanaged red spruce stand (LC 6)

Age (years)	20	30	40	50	60	70	80	90	100
Total volume (m^3)	24	105	212	300	372	434	480	522	550
MAI ($m^3 \cdot ha^{-1} \cdot yr^{-1}$)	1.2	3.5	5.3	6	6.2	6.2	6	5.8	5.5

The data shown in figure 2-1 and table 2-1 are for a fully stocked natural unmanaged red spruce stand in Nova Scotia and the curves would be different for other types of stands. In the case described above, if the only objective is the maximization of total volume, the management policy is simple: determine LC for the stand, check the chart and cut the stand at the age indicated by the highest point of the MAI curve that corresponds to the land capability of the stand. Repeat the cycle.

Wood products created from cutting trees on a stand can be divided into many types such as wood fibre for pulp, firewood, construction material, and veneer. For the purpose of our discussion and for the rest of this document, those products will be divided into two categories: merchantable volume (MV) and sawtimber volume (SV). Merchantable refers to a tree or stand that has attained sufficient size, quality, and/or volume to make it suitable for harvesting (Canadian Council of Forest Ministers 2010). In Nova Scotia, merchantable volume is the inside bark volume, per unit area, of trees with minimum diameter of 9 cm at diameter breast height. The merchantable bole excludes the stump (15 cm height) and top portion of the bole < 7.6 cm diameter inside bark. Sawtimber refers to trees that will yield logs suitable in size and quality for the production of lumber (Canadian Council of Forest Ministers 2010). Here, sawtimber volume is the inside bark bole volume, per unit of area, of trees greater than 14 cm at diameter breast height. The sawtimber bole excludes the stump (15 cm height) and top portion of the bole < 10 cm diameter inside bark. When a stand begins growing, it doesn't contain any MV and when the trees reach a certain minimum diameter, which depends on the tree species, it contains MV but no SV. As the stand ages, the total volume grows but the proportion of SV gradually shifts towards 100% SV. Therefore, depending on the type of product the land owner wants to produce on his/her land, strictly managing to maximize MAI might not be the best objective.

Maximizing the value of the wood products instead of only the volume on the stand will most certainly change the optimal policy. The unit used for calculating the value of wood on the stand is $\$/m^3$ and is higher for SV than the rest of MV. As the volume of wood products on the stand continues to grow throughout the life of a stand, so does the total value of the stand. Similarly to MAI, we can calculate mean annual value increment of the stand over its entire life since the last regeneration harvest. Because of the length of planning periods in forestry, net present value (NPV) criterion will be used in the rest of this work and the objective will be to maximize NPV. The discount value used to calculate those NPVs is a much debated topic and it will be discussed in detail in chapter 4.

Thus far, volume, which was derived from age and transformed to dollar value, has been used to describe a stand. Generally speaking, larger trees bring a premium in price because of the value of the large timber products, and market values fluctuate and vary depending on demand for different wood products, so there may be advantages to having one size of tree over another. In addition, the same tree may be transformed differently depending on the value of the different wood products on the market. Simply put, volume shouldn't be the only measure by which the stand is evaluated. Therefore, the next section describes an unmanaged natural stand using more than one variable which leads to the discussion of more complex stands.

2.3 Expanded Definition of an Unmanaged Natural Stand

In the previous section, a stand was defined using ages and volume which can be converted to dollars. An unmanaged stand can be defined using only age. All other stand characteristics such as quadratic mean stand diameter, average height of the trees and the average spacing between the trees depend on the age for an unmanaged natural stand and those characteristics are all used to calculate wood volume. Therefore, it is important to properly define the relationship between age and quadratic mean stand diameter, height and spacing before continuing. NS DNR has developed equations to represent these relationships and they have been published in a series of research reports that are available on the forestry section of the NS DNR website (http://www.gov.ns.ca/natr/forestry/).

Quadratic mean stand diameter can't be properly defined without first discussing basal area. The basal area of a tree is the round surface area of the stem of a tree when the tree is cut at 1.37m from the ground which is known as breast height (Canadian Council of Forest Ministers 2010). So a tree with a diameter at breast height of 10 cm would have a tree basal area (TBA) of 0.007854 m^2 ($\pi \cdot r^2$). The total basal area of a stand is the sum of the TBA of all the trees on a stand. The quadratic mean stand diameter of the trees on a stand is a basal area weighted average. More precisely, it is the diameter of a tree of average basal area (Gove 2003). For example, if a stand has 150 m^2 of basal area and 2800 trees, the stand would have a quadratic mean stand diameter of 26.12 cm ($2 \times \sqrt{(150/2800)/\pi}$). Here, quadratic mean stand diameter is referred to as diameter (d) and the total stand basal area is referred to as basal area (BA).

We consider two measures of height for a stand: dominant height (dht) and average height (aht). Dominant trees are trees with crowns extending above the general level of the main canopy of even-aged groups of trees and receiving full light from above and partial light from the sides (Canadian Council of Forest Ministers 2010). Dominant height refers to the average height of the dominant trees on a stand and the number of trees used to calculate this average is not fixed but is limited to a few trees.

The average height refers to the average of the dominant and codominant trees on the stand. Codominant trees have crowns which form the general level of the main canopy in even-aged groups of trees and receive full light from above and comparatively little from the sides. The dominant trees in a stand are easier to measure and it has been shown that there exists a relationship between the dominant height and the average height of a stand (Staebler 1948). This concept has been implemented by NS DNR and the equations for the relationship between dominant height and average height used in this thesis are taken from their work (NSDNR (1993b), O'Keefe and McGrath (2006)). The height growth of the dominant and codominant trees on a stand is only dependent on the growth-supporting factors of a piece of land (Smith et al. 1997, p.52).

Site index (SI) is an expression of forest site quality based on the height, at a specified age, usually 50 years in Nova Scotia, of dominant and codominant trees in a stand (Canadian Council of Forest Ministers 2010). Figure 2-2 shows the relationship between LC and site index for Nova Scotia softwoods.

A stand with LC 5 would have an approximate site index of 15.5m at age 50. NS DNR has established equations for the relationship between age and site index for softwood and hardwood in Nova Scotia (NSDNR (1993b), O'Keefe and McGrath (2006)) and has incorporated it into all of its growth and yield models. In the rest of this document, SI will be used in the formulas but LC will be used when referring to the figures taken from the Forestry Field Handbook (NSDNR 1993a).

In an unmanaged natural stand, stand quadratic mean diameter (d) and average height (aht) only depend on age and, once determined, the volume of wood on the stand can be calculated using formulas that incorporate tree shape parameters for different species (NSDNR (1993b), O'Keefe and McGrath (2006)). That volume is then used to create decision tools such as those in figure 2-1 and table 2-1. The equations for calculating the yield are presented in section 2.8.

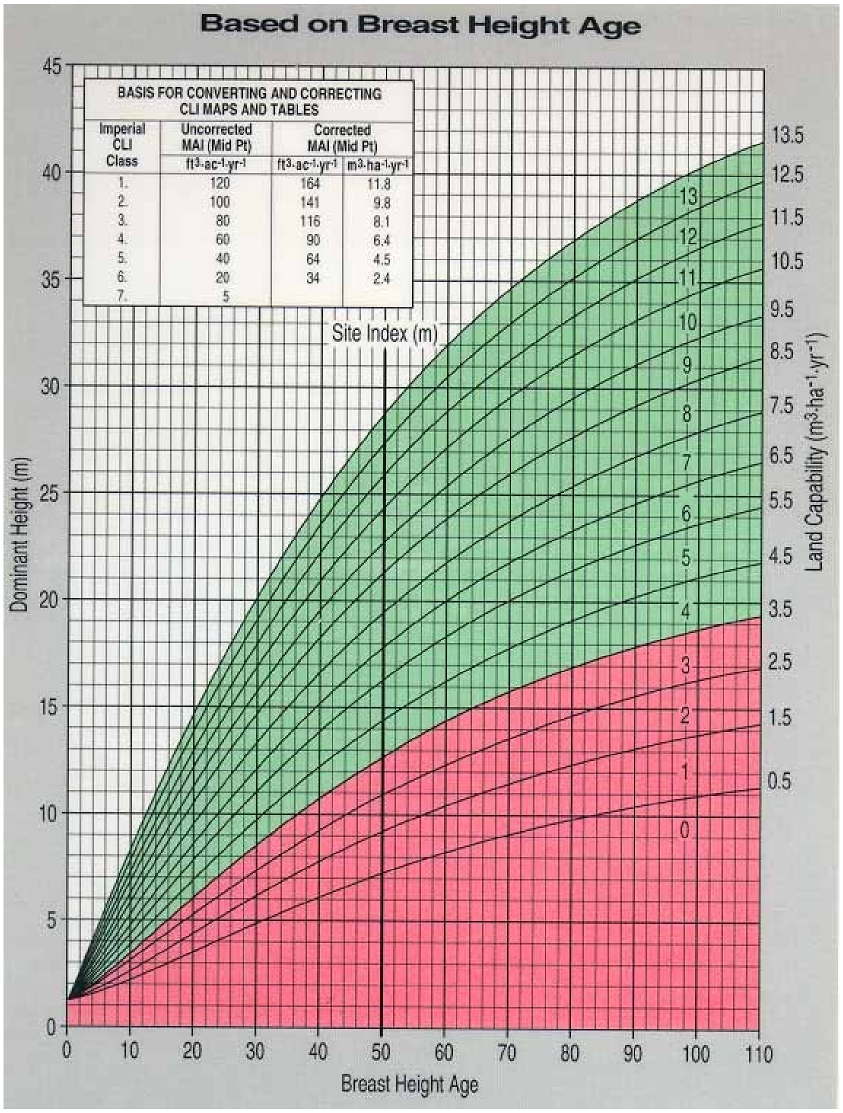
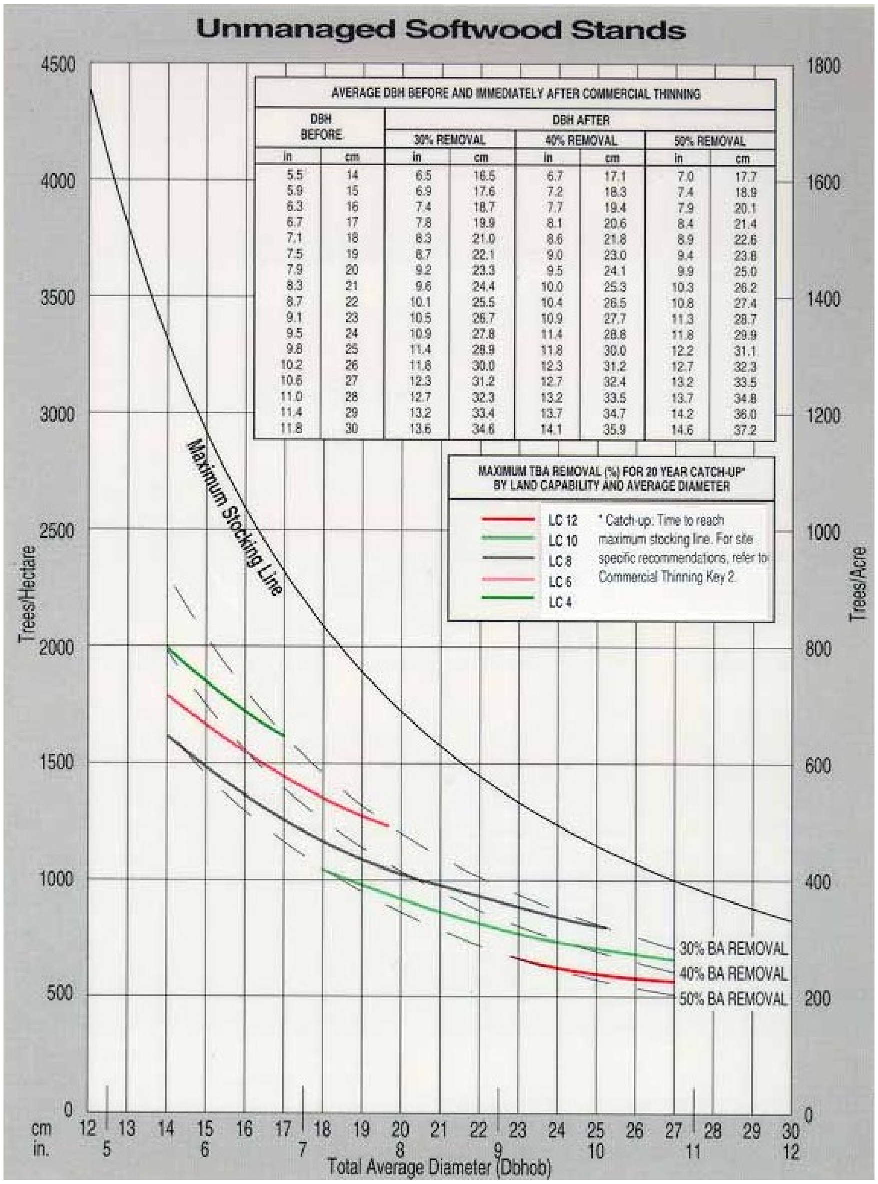


Figure 2-2 – Land capability for Nova Scotia softwoods (NSDNR 1993a)

Fully stocked unmanaged natural stands always have the maximum number of trees per hectare, or maximum stocking, that the LC will allow. The number of trees per area of land is sometimes referred to as density. The maximum stocking, or maximum density, depends on d^{θ} where θ stands for softwood (S) or hardwood (H). If we plot the maximum number of trees for all diameters for a given LC, we obtain a curve such as the one in figure 2-3 which is referred to as the maximum stocking line.



AVERAGE DBH BEFORE AND IMMEDIATELY AFTER COMMERCIAL THINNING

DBH BEFORE		DBH AFTER					
		30% REMOVAL		40% REMOVAL		50% REMOVAL	
in	cm	in	cm	in	cm	in	cm
5.5	14	6.5	16.5	6.7	17.1	7.0	17.7
5.9	15	6.9	17.6	7.2	18.3	7.4	18.9
6.3	16	7.4	18.7	7.7	19.4	7.9	20.1
6.7	17	7.8	19.9	8.1	20.6	8.4	21.4
7.1	18	8.3	21.0	8.6	21.8	8.9	22.6
7.5	19	8.7	22.1	9.0	23.0	9.4	23.8
7.9	20	9.2	23.3	9.5	24.1	9.9	25.0
8.3	21	9.6	24.4	10.0	25.3	10.3	26.2
8.7	22	10.1	25.5	10.4	26.5	10.8	27.4
9.1	23	10.5	26.7	10.9	27.7	11.3	28.7
9.5	24	10.9	27.8	11.4	28.8	11.8	29.9
9.8	25	11.4	28.9	11.8	30.0	12.2	31.1
10.2	26	11.8	30.0	12.3	31.2	12.7	32.3
10.6	27	12.3	31.2	12.7	32.4	13.2	33.5
11.0	28	12.7	32.3	13.2	33.5	13.7	34.8
11.4	29	13.2	33.4	13.7	34.7	14.2	36.0
11.8	30	13.6	34.6	14.1	35.9	14.6	37.2

Figure 2-3 – Graph indicating maximum stocking for any diameter in an unmanaged softwood stand (NSDNR 1993a)

The concepts of maximum stocking and crown closure are closely related. The crown of a tree includes all branches, twigs and leaves extending from the trunk of the tree and, together, the crowns of all the trees on the stand form a more or less continuous cover that is referred to as the canopy (Canadian Council of Forest Ministers 2010). Crown closure (cc) is a measure of the proportion of the ground covered by the vertical projection to the ground of the crown of all the trees on the stand (Canadian Council of Forest Ministers). If the vertical projections of all the trees on the stand cover 100% of the ground, which means there are no open areas in the forest canopy, the stand is said to have reached 100% crown closure. The maximum stocking line in figure 2-3 refers to 100% crown closure for any diameter and indicates the maximum number of trees ($maxtrees^{\theta}$) a stand can support for a given diameter, land capability and species. The graph in figure 2-3 shows an average for SW in N.S. and is the one used in this research. The diameter of a tree grows faster when the trees have more space to grow hence a cc less than 100% means the diameter of the trees grows faster but there are fewer trees per hectare because there is more space between them. The effects of having a cc < 100% are discussed in section 2.6.

One more variable is needed to properly describe an unmanaged natural stand. This variable is presented here but applies to all types of stands. Stocking, not to be confused with maximum stocking from figure 2-3, is a qualitative expression of the adequacy of tree cover on an area, in terms of crown closure, number of trees, basal area, or volume, in relation to a pre-established norm (Canadian Council of Forest Ministers 2010). For our purposes, stocking is the proportion of the area of a stand that can grow trees. A stand may have wide open areas that aren't growing any trees for a number of reasons such as swamps, silvicultural activity and rocky outcrops. Those areas not growing any trees are added together for the entire stand and subtracted from the total area of the stand and this yields the stand stocking as a proportion. Stand stocking will be used when calculating yield in section 2.8.

Crown closure (cc) and stocking go together to describe a stand. A stand with 40% stocking and 100% crown closure isn't the same stand as 40% crown closure on a 100% stocked stand. The first stand has 60% of its surface grouped in wide open areas with no growing trees while the rest of the stand is supporting as many trees as it can (100% cc) for the average diameter of the growing trees. The second stand has trees spread out on its entire surface without any big open areas but the trees are spread apart in such a way that only 40% of the ground is covered by the vertical projection of the crowns of the trees. These two stands will not grow the same way as cc plays a big role in determining the diameter growth of trees on a stand with less than 100% cc.

Up to now, we have only discussed one management option or decision for one type of stand: what age to perform a regeneration harvest on a natural unmanaged red spruce stand at which point another growth cycle begins. With the addition of crown closure as a variable, we can start discussing managed softwood stands and the management options that either lead to or are applicable to managed stands.

2.4 Managed Stand

The term "density" presented in the previous section refers to the number of trees per hectare of land and is a function of d^{θ}, cc and stocking, and is possibly a consequence of the history of what has been done to the stand since the last regeneration harvest. In fully stocked unmanaged natural stands, the density is assumed to be equal to the maximum stocking where, in managed stands, the density can be below maximum stocking. The maximum stocking line in figure 2-3 is an average value over the forest region therefore we assume that the stand being studied is an average stand and that the maximum stocking line applies. In this section, we will discuss the difference between unmanaged and managed stands and present new management options and an expanded stand definition.

A managed stand can be described as a stand that has had its cc reduced below 100% or below the maximum stocking line at least once during its growing cycle in order to encourage accelerated diameter growth. The objective of a managed stand is to have trees that will grow well for the LC, that have good economic value and that are far enough apart to encourage good diameter growth. In this work, pre-commercial thinnings (PCT) and commercial thinnings (CT) are ways of getting managed stands.

Managed stands are more complex to model and manage than unmanaged natural stands because of more complicated growth and yield models and of the additional decision possibilities associated with a PCT or a CT. All stands, managed or unmanaged, are divided into 5 categories or treatment types (TRT) depending on the type of silvicultural treatment that has been applied to the stand. They are presented in table 2-2. Treatment type 1 stands are those discussed in the previous section and only occur when the management decision is to let the stand naturally regenerate after a regeneration harvest without any silvicultural intervention.

Table 2-2 – Treatment types

Treatment	Description
1	Unmanaged natural stand
2	Pre-commercially thinned natural stand
3	Unthinned plantation
4	Commercially thinned plantation
5	Commercially thinned natural stand

In order to explain the four other main treatment types in table 2-2, we now present descriptions of PCT and CT. PCT is a thinning that does not yield trees of commercial value and is usually designed to improve crop spacing and to improve growth, quality, and percentage of desirable trees (Canadian Council of Forest Ministers 2010). For example, an unmanaged natural softwood stand with an average diameter at breast height (d^S) of 5cm will have approximately 21000 trees per hectare. This yields a space between trees (SP^S) of 0.68m. That spacing is measured from tree trunk to tree trunk. Add the crown to those trees and the stand is impassable and doesn't allow any room for the trees to grow until some of them die and make room for the others. PCT only applies to unmanaged natural stands. Herbicides and/or cutting trees are the two most common techniques used to release stands (Smith et al. 1997). This discussion is limited to the importance of PCT as a silvicultural treatment and does not expand to a discussion of the techniques used in such treatments. In reference to the treatment types in table 2-2, an unmanaged natural stand is a treatment type 1 (TRT=1). After applying a PCT to a TRT=1 stand, it becomes TRT=2.

In commercial thinning, the removal of trees is delayed until the stand has enough trees of marketable value so that releasing the stand not only gives more room for the remaining trees to grow but the trees being removed can be sold to create revenue (Canadian Council of Forest Ministers 2010). The following two variables are critical when making commercial thinning (CT) decisions for single-species even-aged stands: what percentage of the stand basal area to remove ($BARem$) and what type of CT to apply ($THINTYP$).

The balance between percentage removed and percentage remaining is critical. There has to be enough volume of wood products removed by the CT operation to be economically viable but there must also be enough trees left behind to ensure that at least one more commercial operation will be economically fruitful and that the stand is protected from natural disasters which are discussed in section 2.9.

There are many types of CT but this discussion is limited to three types: thinning from above or selection thinning, thinning from below or low thinning, and thinning across the diameter distribution (Canadian Council of Forest Ministers (2010), Pelkki (1999), Smith et al. (1997), Mäkinen and Isomäki (2004a), Mäkinen and Isomäki (2004b)). The type of CT refers to which size of tree will be targeted first when removing a given percentage of the BA. The % of BA to remove ($BARem$) is transformed into an amount in m^2 referred to as the target BA removal. Thinning from above means removing the largest trees first until the target BA removal is reached. In thinning from below, the smallest commercially viable trees are removed until the target BA removal is reached. The third type of CT requires dividing the trees into groups by diameter size and making sure to remove the same number of trees from each group until the target BA removal has been reached. With these three types of CT, the removal of trees is done in such a way that cc remains uniform over the entire area of the stand. The detailed methodology for calculating the effect of commercial thinning on the average diameter of the stand, on crown closure and on softwood percentage is briefly discussed in section 2.8 with detailed equations and examples presented in chapter 3.

We can now discuss the remaining treatment types in table 2-2. An un-thinned plantation (TRT=3) means that the trees on the stand were planted and the stand was treated in such a way as to ensure that it remains a plantation, by the removal of competing vegetation for example, but no CT was applied to the stand since it was planted. It`s assumed that there is no unexpected mortality and that all planted trees survive and grow to be mature trees. When a CT is applied to a plantation, it goes from TRT=3 to TRT=4. When a CT is applied to an unmanaged natural stand (TRT=1) or a pre-commercially thinned natural stand (TRT=2), it becomes a commercially thinned natural stand (TRT=5). Figure 2-1 and table 2-1 are based on data for an unmanaged natural red spruce stand which is only one of the softwood species growing in Nova Scotia. Figure 2-2 presents data that is generalized for all softwood (SW) species in the province. Similar data exists for hardwoods (HW). HW stands can be defined the same way SW stands were defined in this chapter. Hardwood plantations aren't grown commercially in N.S. because of the costs associated with protecting them from the fauna as the leaves, twigs and buds are a source of food for many species. Natural stands will grow with a mix of SW and HW and the mix depends on a long list of factors, soil characteristics and weather being two of the most important. In order to properly define an unmanaged natural stand that includes both species, pct^S is added and is generally defined to mean the proportion of the total stand basal area that is SW. Therefore, $1 - pct^S$ is the proportion of the stand basal area that is HW. What this means in actual volumes of wood products will be discussed in section 2.8. With the addition of SW percentage as a variable, a few more details can be added for PCT and CT.

When doing a PCT, there are 3 options: leaving the mix of species as is where pct^S remains unchanged, preferring SW, or preferring HW. Preferring one of the two species means that during the operation of releasing the stand, if the operator has to choose between the removal of a SW or a HW tree, the preferred species will remain and the other will be removed. Modelling the latter two options of PCT, in this work, means removing all SW or all HW and spacing the remaining trees according to the guidelines for PCT published by NS DNR.

When doing a CT in a stand with two species, the total BA removal amount in m^2 must be divided between SW and HW. This is referred to as the BA removal split % ($BARem_{split}$). For example, if $30m^2$ of basal area is to be removed and $BARem_{split}$ is 80% SW, we would remove $24m^2$ of SW and $6m^2$ of HW.

After a regeneration harvest, the forest manager can do a fill planting on the stand followed by early competition control. The Canadian Council of Forest Ministers defines fill planting as the planting of trees in areas of inadequate stocking to achieve the desired level of stocking, either in plantations or areas of natural regeneration, and early competition control as a treatment designed to reduce the competitive effect of undesirable vegetation threatening the success of the regeneration of desirable tree species. Doing a fill planting followed by early competition control will ensure the stand recovers as a fully stocked natural unmanaged stand in the period immediately following a regeneration harvest. This management option applies to any stand after a regeneration harvest regardless of the past history of the stand.

The distinctions made between treatment types in table 2-2 are important because stands grow differently depending on how they were treated since the last regeneration harvest so they need to be kept separate. The next four sections present the procedures for growing and calculating the yield from a forest stand.

2.5 Regeneration Period

Before discussing growth models, the period of time between the removal of all trees from a stand and the beginning of growth of a new stand needs to be discussed. After a regeneration harvest or a natural disaster, a stand doesn't always recover quickly unless money is invested in preparing the stand and trees are planted to ensure regeneration of the stand at the next time period. Without planting, the small trees living under the canopy of the mature stand, which are called advanced regeneration and are counted on to help the stand recover its growing stock, can sustain damage from a regeneration harvest or a natural disaster and affects the capacity of the stand to recover after such events and for the trees to reach breast height. In this research, we consider four regeneration states of age 0 (R_1, R_2, R_3, R_4). These four states have an increasing probability of regenerating and containing growing stock at the next time period. When a stand is in one of these regeneration states at time t and the decision is to do nothing and let it grow, one of two states may occur at time t+5:

i) The stand still doesn't contain any growing stock but it has a higher probability of containing growing stock at time t+10
ii) The stand is equivalent to a 5 year old natural stand and grows accordingly.

A stand R_k can remain in a non-regenerated condition for 4 - k periods. The probability that a stand R_k will regenerate is PR_k and the probability it proceeds to state R_{k+1} is 1 - PR_k where the PR_k are increasing with k and PR_4 = 1. One of these states occurs after a regeneration harvest or natural disaster as a function of many factors such as the type of equipment used for the regeneration harvest and stand characteristics, as well as the type and severity of the natural disaster that has affected the stand.

At each subsequent time period after the stand has entered a regeneration state, the stand will progress from one regeneration state to another with an increasing probability of regeneration until the stand contains growing stock and is no longer in a regeneration state.

Figure 2.4 shows the probabilities of regeneration for each regeneration state. It should be interpreted as follows: when a stand is in state R_1 at time t, it has PR_1 probability of being a fully stocked 5 year old natural stand at time t+5 and 1 - PR_1 probability of being in a regeneration state with a higher probability of regeneration at time t+5.

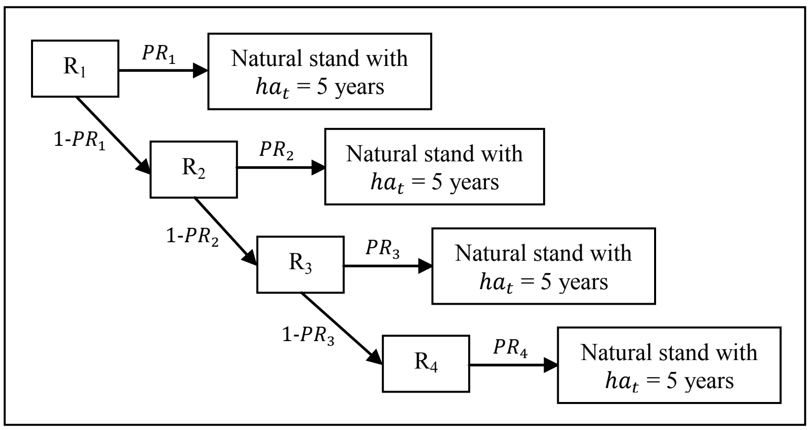


Figure 2-4 – Transition probabilities of the regeneration states after regeneration harvest

States don't automatically enter a regeneration state at R_1. For example, after a regeneration harvest, the stand enters state R_2 if no planting or competition control is applied to the stand. This is done because the presence of advanced regeneration on the stand raises the probability of regeneration. On the other hand, a high intensity fire destroys advanced regeneration which puts the stand in state R_1 in the period immediately following the fire. A regeneration state doesn't occur until 5 years after the regeneration harvest or natural disaster. Therefore, R_1 occurs 5 years after a stand that has been cut or destroyed. In the worst case scenario, a stand goes through the 4 regeneration states in succession starting with R_1 and 25 years elapse from harvest/natural disaster to beginning or growth.

Using example probabilities PR_1 = 30%, PR_2 = 50%, PR_3 = 70% and PR_4 = 100%, we can calculate the probability of the stand taking between 1 and 5 periods to regenerate. There is 0% probability that the stand will regenerate after 1 period which we note as P(X>1) = 1 where X is the number of periods before growing stock appears on the stand after a regeneration harvest or natural disaster. With the probability that state R_1 will contain growing stock at the next period set at 30%, we can write P(X=2) = 0.3 and P(X≥3) = 0.7. Using the same logic, we conclude the following probabilities:

P(X=3) = 0.7 × 0.5 = 0.35
P(X≥4) = 1 – 0.3 – 0.35 = 0.35
P(X=4) = 0.35 × 0.7 = 0.245
P(X≥5) = 0.35 – 0.245 = 0.105

We can't have more than 5 periods of delay before regeneration so P(X=5) = 0.105. The number of periods of delay can be combined with the probabilities of occurrence to calculate the expected number of periods of delay before a natural unmanaged stand has growing stock after a regeneration harvest or natural disaster.

$$\bar{X} = 2\times0.3 + 3\times0.35 + 4\times0.245 + 5\times0.105 = 3.155$$

The decision maker can, at any point in the regeneration process, plant trees on the stand which would guarantee growing stock on the stand at the next time period. In practical terms, any decision maker wishing to use this type of approach will need to have a clear understanding or definition of what R_1, R_2, R_3 and R_4 stands look like in order to determine the state of the stand at decision time.

2.6 Growing a Single-Species Forest Stand

The terms growth and yield are often used together but they are two separate terms. Growth refers to the change, over time, of average stand diameter, crown closure and species percentage. Height is always a function of age and site index (SI^{θ}) and doesn't depend on TRT. Therefore, height is not used as a stand variable in any of the growth models. Equivalently, we could have used height and site index and treated age as a derived variable. The yield of a stand refers to the volume of wood products that are available for harvest on a stand at any given time. The yield equations are the same for all TRT types but differ between SW and HW because of the tree shape characteristics. Yield models will be presented in section 2.8.

NS DNR has collected and analyzed data from permanent sample plots spread across the province over a period of 40 years (at 5 year intervals). This data has been used to create equations that define the diameter growth of HW and SW stands for each of the treatment types presented in table 2-2 (NSDNR (1993b), O'Keefe and McGrath (2006)). Stands of treatment types 3 and 4 are pure SW stands without HW. Stands of TRT types 1, 2 and 5 can include SW and/or HW.

In most stand level growth models, site index (SI^{θ}) is an important factor. In any given stand, there may be a site index for SW and one for HW. In this work, growing a stand means using the equations to determine the values of the stand variables 5 years in the future. This section presents procedures for growing single-species stands for all TRT types. The next section discusses mixed-species stand growth and the next chapter presents some modeling results.

In order to simplify the discussion yet show relationships between variables more clearly, the following set of variables is used in stand growth and yield modeling.

cc_t	crown closure fraction at time t
st_t	stocking (% area forested) at time t
pct_t^{θ}	percentage of total basal area of the stand at time t that is species type θ
d_t^{θ}	stand quadratic mean diameter at time t for species type θ (centimeters)
ha_t	height age for stand at time t

The t subscript in the variables above represents time. NS DNR did their data collection, at 5 year intervals, over a 40 year period. Therefore, the growth equations are built for 5 year intervals and t+5 means 5 years have elapsed.

The age discussed so far in this chapter refers to the height age. It`s the actual age of the stand since it`s last regeneration harvest and is the one used to calculate the height of the dominant trees on the stand using the site index curves. Diameter age or fake age

(fa_t^θ) is used when calculating diameter growth for certain types of managed stands. Fake age isn't the actual age of the stand but rather an age that represents the current state of the stand which may be the result of a series of silvicultural operations that modified diameter and spacing since the last regeneration harvest. With either fa_t^θ or ha_t, we can write $age_t + 5 = age_{t+5}$.

Once growing stock is established on the stand, growth of the stand can be calculated using the models in this section. The growth models discussed in this section can be divided into 5 sets based on the stand characteristics and they are presented in table 2-3. For stands of TRT types 2, 3, 4 and 5, spacing (SP_t^θ) and species are used to determine which set of equations to use for growth calculations. SP_t^θ is calculated using equation (2-1).

$$SP_t^\theta = \frac{BA_t^\theta}{3.1415926 \times \left(\frac{d_t^\theta}{200}\right)^2} \qquad (2\text{-}1)$$

Functions f_i presented in this section and the next are taken from NS DNR research reports and can be found in appendix A.

Table 2-3 – Sets of forest stand growth equations

Set 1	TRT = 1 (non-species specific)
Set 2	TRT types 2, 3, 4, 5 with $SP^S \geq$ 3.1m (softwood only)
Set 3	TRT types 2, 4, 5 with $SP^S <$ 3.1m (softwood only)
Set 4	TRT = 3 with $SP^S <$ 3.1m (softwood only)
Set 5	TRT types 2, 5 (hardwood only)

Set 1:

This set includes all unmanaged natural stands which are the easiest growth to calculate as they are always at 100% crown closure. Therefore the number of trees on the stand depends only on d_t^θ which is a function of ha_t and SI^θ. Growing the stand is simply a matter of adding 5 years to the current age (ha_{t+5}). All other stand variables are dependent on the new age ha_{t+5}. Equations (2-2), (2-3) and (2-4) are used to calculate d_t^θ which is required for yield calculations.

$$dht_t^{\theta} = f_1(ha_t, SI^{\theta}) \quad (2\text{-}2)$$

$$aht_t^{\theta} = f_2(dht_t^{\theta}) \quad (2\text{-}3)$$

$$d_t^{\theta} = f_3(aht_t^{\theta}, SI^{H}) \quad (2\text{-}4)$$

Set 2:

This set includes softwood stands of TRT types 2, 3, 4 and 5 that have $SP_t^S \geq 3.1m$. Rather than one equation, calculating d_{t+5}^S for this set requires a sequence of preliminary values which are calculated with the equations presented in appendix A. Using d_t^S and the appropriate maximum stocking line, $maxtrees_t^{\theta}$ is obtained. The stand crown closure, SI^S, d_t^S and $maxtrees_t^{\theta}$ are used to calculate d_{t+5}^S.

$$d_{t+5}^S = f_4(d_t^S, SI^S, maxtrees_t^{\theta}, cc_t) \quad (2\text{-}5)$$

Set 3:

This set includes softwood stands of TRT types 2, 4 and 5 that have $SP_t^S < 3.1$m. We calculate a fake age (fa_t^S) corresponding to the observed spacing, diameter and site index and it may differ from the actual age of the stand (ha_t^S). For a given site index and spacing, f_5 is the inverse of f_6.

$$fa_t^S = f_5(SP_t^S, d_t^S, SI^S) \quad (2\text{-}6)$$

5 years of growth are added to fa_t^S and the new diameter is calculated with fa_{t+5}^S.

$$d_{t+5}^S = f_6(fa_{t+5}^S, SP_t^S, SI^S) \quad (2\text{-}7)$$

Set 4:

This set includes softwood stands of TRT type 3 that have $SP_t^S < 3.1m$. Plantations (TRT=3) are similar to natural stands in that the stand grows differently if it's been thinned but plantations don't contain any hardwood so there is only one set of growth equations. The diameter growth of a plantation with SP_t^S < 3.1m can't be calculated directly with one equation. Rather d_{t-2}^S and d_{t+3}^S are calculated and the difference between the two values is the diameter growth for the next 5 years.

$$d_{t+3}^S = f_6(ha_{t+3}^S, SP_t^S, SI^S) \quad (2\text{-}8)$$
$$d_{t-2}^S = f_6(ha_{t-2}^S, SP_t^S, SI^S) \quad (2\text{-}9)$$
$$d_{t+5}^S = d_t^S + d_{t+3}^S - d_{t-2}^S \quad (2\text{-}10)$$

Set 5:

This set includes all hardwood stands of TRT types 2 and 5 regardless of spacing. The hardwood diameter growth model doesn't explicitly depend on spacing therefore there is only one diameter growth model (eq. 2-11).

$$d_{t+5}^H = f_7(d_t^H, BA_t^H, SI^H) \quad (2\text{-}11)$$

The last step in the process for any single-species stand of treatment types 2, 3, 4 and 5 is to calculate crown closure (cc_{t+5}) which doesn't have a species indicator. The growth models assume that crown closure is the same for both species. Using d_{t+5}^{θ} and the appropriate maximum stocking line, $maxtrees_{t+5}^{\theta}$ is obtained. Using equations (2-12) and (2-13), $TFREQ_{t+5}^{\theta}$ and cc_{t+5} are calculated.

$$TFREQ_{t+5}^{\theta} = f_8(d_{t+5}^{\theta}) \quad (2\text{-}12)$$
$$cc_{t+5} = \frac{TFREQ_{t+5}^{\theta}}{maxtrees_{t+5}^{\theta}} \times 100 \quad (2\text{-}13)$$

The next section discusses how the growth of two species in a single stand is modeled in this thesis.

2.7 Proposed Method for Growing a Mixed-species Forest Stand

Up to this point in chapter 2, all growth models presented are those developed by NS DNR. They are divided into sets for easier presentation. In this section, we propose a new method for combining growth models for more than one species into a single stand mixed-species growth model. It follows directly from the models presented up to this point and is one of the contributions of this thesis. Some results of applying this modelling approach to typical stands found in Nova Scotia are presented in chapter 3.

The previous section divided the growth models into 5 sets and, with the exception of set 1 which deals only with natural unmanaged stands, they are all single species diameter growth models. In set 1, diameter and height growth depend only on age and site index of the stand. In that case, any information related to species mix on the stand will be used solely for volume calculation and are not covered by the modelling approach proposed here. Sets 2 to 5 deal with managed natural stands and plantations and are divided according to species and/or spacing. With single species stands, the equations in the previous section can be applied directly without considering the mixed-species approach proposed here. When dealing with a mixed-species stand, the sets of growth equations in the previous section all apply as presented but we work on the basis that each species grows as if it were alone on a smaller stand. In this section, we explain the proposed approach of how to calculate the respective spacing for each species type when they are growing together on the same stand, how to grow each of them according to their respective growth equations and how to put them back together to calculate the new values of the stand variables.

The main observation that drives our approach is that trees, whether softwood or hardwood, experience the same crown closure. This crown closure creates an effective spacing for each species. This then can drive all the models. In chapter 3, the proposed methodology is illustrated by way of some examples.

The proposed approach can be summed up by the following steps:

Step 1: Calculate the fraction of the stand covered by each species type;

Step 2: Calculate the respective spacing for each species type. This is done based on the fact that crown closure is the same for each species type;

Step 3: Given the spacing for each species type, calculate diameter growth using the respective growth models;

Step 4: Combine the two species types to calculate the stand characteristics of the new stand.

What follows is a detailed explanation of each step with accompanying diagrams and equations where appropriate.

Step 1: Calculate the fraction of the stand covered by each species type

The procedure starts with the calculation of the maximum stocking basal area for each species type, were each a pure single-species stand, using the function below:

$$BAFULL_t^{\theta} = maxtrees_t^{\theta} \times 3.1415926 \times \left(\frac{d_t^{\theta}}{200}\right)^2$$

Using the equations for $maxtrees_t^{\theta}$ published by NS DNR (NSDNR (1993b), O'Keefe and McGrath (2006)) which depend on d^{θ}, we determine that a pure single-species SW stand has approximately twice as much BA as a pure single-species HW stand ($BAFULL^S$ = 60m^2 vs. $BAFULL^H$ = 30m^2). With this difference of maximum BA for the two species, the fraction of the ground area of the stand covered by each species type in a mixed-species stand isn't the same as pct^S and pct^H, which are the percentages of stand total basal area for each species type. Therefore, we define $fraction^{\theta}$ as the fraction of ground area of the stand covered by species θ to yield pct^{θ}. We require that

$$\sum_{\theta} fraction^{\theta} = 1$$

With two species, the solution is easy: $fraction^S$ is the solution to

$$pct^S = \frac{fraction^S \times BAFULL^S}{fraction^S \times BAFULL^S + fraction^H \times BAFULL^H}$$

where pct^S is given as a stand variable. The solution is:

$$fraction_t^S = \frac{1}{\left\{\left(\frac{1}{BAFULL_t^H}\right) \times \left[\left(\frac{BAFULL_t^S}{pct_t^S}\right) - BAFULL_t^S\right]\right\} + 1}$$

and $fraction_t^H = 1 - fraction_t^S$, which are then used to calculate spacing in step 2.

Step 2: Calculate spacing between the trees

In section 2.3, crown closure was defined as a measure of the proportion of the ground covered by the vertical projection to the ground of the crown of all trees on the stand. In this step of the proposed approach, we take advantage of the fact that crown closure is the same for both species which leads to the calculation of spacing as explained below.

Given the number of trees on the stand for each species and the area of the stand covered by each species, we can calculate the spacing between SW and HW trees as they are perceived by the individual trees on the stand. It is this spacing that is used in the diameter growth models from the previous section.

If crown closure < 100%, the actual number of trees on a pure single-species stand ($TFREQ_t^\theta$) is a proportion of $maxtrees_t^\theta$ according to the following relationship:

$$TFREQ_t^\theta = maxtrees_t^\theta \times cc$$

On a mixed-species stand, a fraction of the stand is covered by each species type so we adjust the number of trees as follows:

$$TFREQ_t^\theta = fraction_t^\theta \times maxtrees_t^\theta \times cc$$

Given that a stand is defined as being one hectare in size and that a hectare is 10,000m^2 (100m × 100m), we calculate the spacing for each species as follows:

$$SP_t^\theta = \sqrt{\frac{10000 \times fraction_t^\theta}{TFREQ_t^\theta}}$$

Given the spacing for each species type, we proceed to step 3 where diameter growth is calculated.

Step 3: Grow the species types according to their respective growth equations

What follows is an explanation of the adjustments that need to be made to each set of equations in section 2.6 for combining SW and HW in the same stand. Equations from set 2 are used when $SP^S \geq 3.1$m but the growth equations don't explicitly depend on spacing. In this case, the spacing calculated above is only used to determine whether the equations from set 2 should be used. The same goes for set 5. In sets 3 and 4, the diameter growth equations explicitly depend on SP^S and SP^H. Once the spacing for each species has been calculated as described in the previous step, growth equations in sets 3 and 4 are applied as described in the previous section to determine diameter growth hence the new diameters for SW and HW.

With spacing for both species types known, the appropriate set of growth equations is chosen and SW and HW diameter growth is calculated.

Step 4: Combine the species to calculate the values of the stand variables for the new stand

Given the diameter growth calculated in the previous step, we add it to d_t^θ and calculate d_{t+5}^θ which is used to calculate $maxtrees_{t+5}^\theta$ using maximum stocking lines published by NS DNR. The following equations are then used to calculate the stand characteristics post-growth.

$$TFREQ_{t+5}^{\theta} = fraction_t^{\theta} \times maxtrees_{t+5}^{\theta} \times \frac{cc_t}{100}$$

$$BA_{t+5}^{\theta} = TFREQ_{t+5}^{\theta} \times 3.1415926 \times \left(\frac{d_{t+5}^{\theta}}{200}\right)^2$$

$$pct_{t+5}^{\theta} = \frac{BA_{t+5}^{\theta}}{BA_{t+5}^{S} + BA_{t+5}^{H}}$$

$$cc_{t+5} = \frac{TFREQ_{t+5}^{S}}{maxtrees_{t+5}^{S}} \times fraction_t^{S} + \frac{TFREQ_{t+5}^{H}}{maxtrees_{t+5}^{H}} \times fraction_t^{H}$$

The next section explains how to calculate the volume of wood and the amount of wood products on a stand.

2.8 Calculating Yield for a Forest Stand

Yield is the volume of wood products removed from a stand through a combination of commercial thinning (CT) and/or regeneration harvests and, in this work, is measured in m^3 and transferred to dollars. For evaluating the monetary value of wood products, they are separated into two groups: MV_t^{θ} and SV_t^{θ}.

The procedure differs between CT and regeneration harvests. For a regeneration harvest, all wood products are removed and revenue from the wood products and the cost of cutting and removing the wood can be calculated. There is no change in d_t^{θ} or BA_t^{θ} to calculate because all trees are removed and growth of the stand starts over at age zero. The steps described below for calculating yield take this into consideration.

Step 1: For a regeneration harvest, go to step 3. In the case of CT, calculate the amount of BA removed from applying CT to the stand using equations (2-22a), (2-22b) and (2-23) below.

$$\widetilde{BA}_t^{S} = BA_t^{S} - (BA_t^{S} + BA_t^{H}) \times \frac{BARem}{100} \times \frac{BARem_{split}}{100} \quad (2\text{-}22a)$$

$$\widetilde{BA}_t^{H} = BA_t^{H} - (BA_t^{S} + BA_t^{H}) \times \frac{BARem}{100} \times \left(1 - \frac{BARem_{split}}{100}\right) \quad (2\text{-}22b)$$

$$\overline{BA}_t^\theta = BA_t^\theta - \widetilde{BA}_t^\theta \tag{2-23}$$

The notation over BA in $\widetilde{BA}_t^\theta$ indicates the resulting value after CT but before a period of growth. The notation over BA in $\overline{BA}_t^\theta$ indicates the amount removed during CT. Go to step 2.

Step 2: Calculate the change in stand average diameter due to CT by using the equations presented in section 3-2 and appendix D. Add or subtract the diameter change to d_t^θ to yield $\tilde{d}_t^\theta$. Go to step 4.

Step 3: Set $\overline{BA}_t^\theta = BA_t^\theta$, $\bar{d}_t^\theta = d_t^\theta$ and $\overline{TFREQ}_t^\theta = TFREQ_t^\theta$. Go to step 5.

Step 4: Complete preliminary calculations of values needed for yield calculation using equations (2-24) to (2-26) and go to step 5.

$$T\widetilde{FREQ}_t^\theta = \frac{\widetilde{BA}_t^\theta}{3.1415926 \times \left(\frac{\tilde{d}_t^\theta}{200}\right)^2} \tag{2-24}$$

$$\overline{TFREQ}_t^\theta = TFREQ_t^\theta - T\widetilde{FREQ}_t^\theta \tag{2-25}$$

$$\bar{d}_t^\theta = 200 \times \sqrt{\frac{\overline{BA}_t^\theta}{3.1415926}} \tag{2-26}$$

Step 5: Complete yield calculations that lead to MV_t^θ and SV_t^θ. Those calculations are shown in detail in Appendix A as they are not done with one equation but rather by a series of long procedures (f_9) which differ between natural and plantation stands. The inputs required are indicated below.

$$f_9(stocking, \bar{d}_t^\theta, aht_t^\theta, \overline{BA}_t^\theta, \overline{TFREQ}_t^\theta) \tag{2-27}$$

Natural disasters have an impact on how stands grow and how much wood can be harvested from them so they are presented in the next section and the effects on growth and yield are discussed.

2.9 Natural Disasters

Natural disasters are incorporated into the models because they are an important source of uncertainty faced by decision makers. Three major types of natural disasters are simulated: hurricanes, forest fires and insect outbreaks. When a stand is affected by a natural disaster, we model it as if it may or may not succumb to the disaster. If it doesn't succumb, no damage is done and the stand remains unchanged. If it does succumb, we model this as if it will be completely wiped out and the value of the wood products on the stand would only be enough to pay for the cost of removing them, therefore the stand has no salvage value. We could have modeled it as if removal of knocked down trees could be done at a profit. In this case, the profit would be calculated as if it had occurred subsequent to a CT or regeneration harvest action. After succumbing, the probability of regenerating rapidly as a healthy natural stand depends on the type of natural disaster that occurred and the intensity of that disaster. The probability of ending up in one of the three regeneration states (R_1, R_2, R_3) at the beginning of the next period following a natural disaster is the joint probability of any one of the three natural disasters occurring at a given intensity and sending the stand to that state. The probability of not succumbing is $1 - \sum$(probabilities of succumbing).

Before proceeding to the details of each type of natural disaster, we discuss the general approach taken to simulate natural disasters. Fires and hurricanes are defined by their size and return interval as well as their probabilities of occurrence at any one of three intensity levels. If a fire or hurricane occurs, its intensity will impact the regeneration state in which the stand ends up in the next period. In the case of insect outbreaks, they are modelled as if only the return interval matters. Therefore, if an outbreak occurs, there is only one intensity level and if the stand succumbs, it will end up in the regeneration state with the lowest probability of regeneration.

Table 2-4 shows basic parameters for natural disasters used in this work. These values are used as reasonable estimates for the forested region in the west of Nova Scotia simply as a demonstrative example and aren't based on any known data for the region. These values are used to calculate the probability that a natural disaster will occur and that a stand will be affected.

Table 2-4 – Basic parameters for natural disasters in this work

Approximate forested area in the west of Nova Scotia (hectares)	1,691,300
Average number of fires per year	3.5
Average size of a fire (hectares)	10,000
Return interval of major hurricanes (years)	50
Average area of wind for a major hurricane (hectares)	400,000
Return interval of major insect outbreaks (years)	50

The probabilities of occurrence at any given level of intensity for fires and hurricanes will be given in the two next sections. In addition to the probabilities of occurrence and the intensity for each type of natural disaster, we need to discuss how each stand will resist to each type and intensity of natural disaster. In this thesis, this depends on stand characteristics where larger trees have better resistance to fire but lower resistance to hurricanes and vice-versa for small trees. In this work, the intensity of natural disasters is used as one way of characterizing the transition to the different regeneration states subsequent to the occurrence of natural disasters. Large trees have a higher probability of succumbing to insect outbreaks. The reasons for these levels of susceptibility and the specific probabilities are discussed in the next three sections. The susceptibility values are chosen as illustrative examples and not by analysing any data. They are chosen to demonstrate the approach and could be adjusted to suit a different scenario.

2.9.1 Hurricanes

Modelling hurricanes is a matter of determining the probabilities that a stand will end up in any one of the regeneration states in any given period. This is done by multiplying three probabilities together:

- Probability that a hurricane arrives and a stand is affected
- Probability a given intensity hurricane occurs if one occurs
- Joint probability a stand succumbs if a hurricane occurs

Hurricanes are characterized by their return interval or inter-arrival time in years, the average area of wind in hectares and three levels of intensity with a joint probability of 100%. The first step is to calculate the probability a hurricane will arrive in any given period. Many studies have characterized the arrival process of a hurricane as a Poisson process (Brodin and Rootzén (2009), Cox et al. (2004)). The mean of the Poisson process (E(X)) is expressed as

$$E(X) = \gamma = \frac{length\ of\ study\ period}{return\ interval} \quad (2\text{-}28)$$

The conditional probability a stand is affected by a hurricane if it occurs is calculated.

$$P(\text{stand affected} \mid \text{hurr. occurs}) = \frac{avg.\ area\ of\ wind}{size\ of\ study\ area} \quad (2\text{-}29)$$

The forested portion of the west of the province is considered to be the study area. The probability a stand will be affected in any period can be calculated by multiplying the probabilities in equations (2-28) and (2-29) with each other.

The general practice when referring to hurricanes is to describe them by their intensity levels. Three intensity levels are used: low, medium and high. Each intensity of hurricane has its own probability of occurrence if a hurricane occurs. These probabilities are given in table 2-5 along with the resulting state in brackets. The lower intensity hurricane is modelled as resulting in a worst regeneration state as it is assumed that the stand will have trees that aren't completely knocked down and will block sun from getting to the small regeneration present under the canopy of the stand before the hurricane.

Table 2-5 – Example probabilities of different intensity hurricanes occurring

Probability of low intensity hurricane if it occurs (new state = R_1)	0.5
Probability of medium intensity hurricane if it occurs (new state = R_2)	0.3
Probability of high intensity hurricane if it occurs (new state = R_3)	0.2

The conditional probability a stand succumbs if a hurricane occurs, uses the probabilities in table 2-5 and the probabilities of succumbing, or susceptibility, given in figure 2-5.

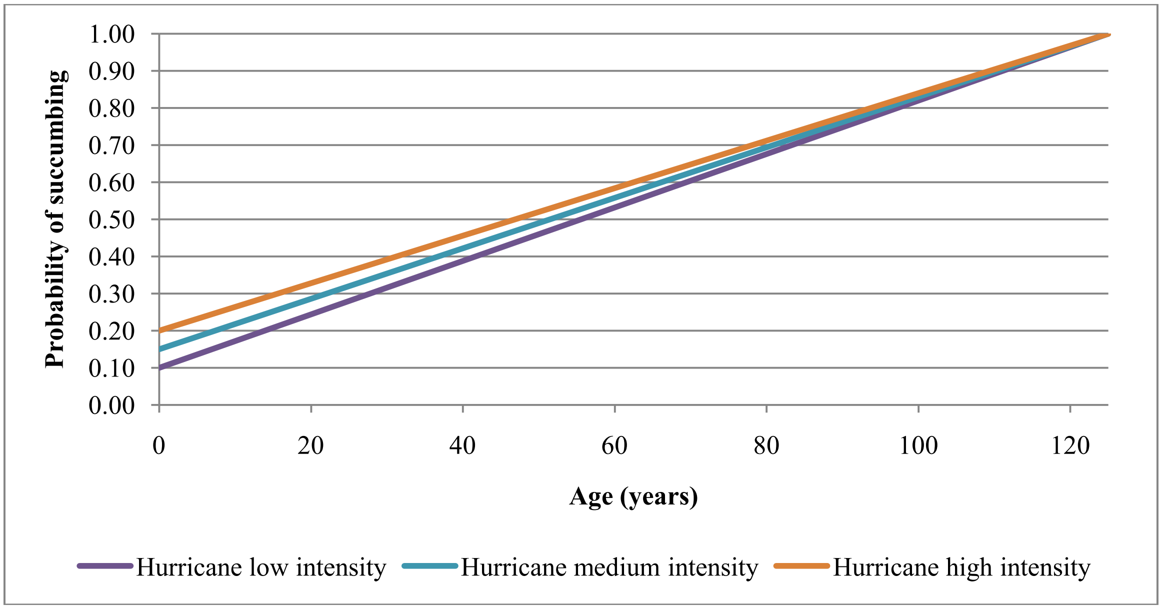


Figure 2-5 – Probabilities of succumbing, or susceptibility, to hurricanes of different intensities

The height, diameter and crown closure of a stand affect its susceptibility to a hurricane (Duryea and Kampf (2008), Gilman et al. (2008)). In order to qualify this effect, we use susceptibility or the probability a stand succumbs if a hurricane occurs. Figure 2-5 shows the three levels of intensity for hurricanes and the corresponding probabilities of succumbing for stands of varying ages. In this thesis, the probabilities of succumbing only depend on age for hurricanes although diameter and height could also have been used, as age, diameter and height are very closely correlated for all stand types. Crown closure has not been used as a factor in the susceptibility of trees to hurricanes.

Probabilities of susceptibility range from 10% for young stands where trees are less than 1 cm average diameter to 100% for very old stands with tall large trees. The examples in table 2-6 are for unmanaged natural stands and are taken from figure 2-5.

Table 2-6 – Example age dependent probabilities of succumbing to a hurricane of low, medium or high intensity for an unmanaged natural stand

Hurricane Chart			
Age	Low int.	Med. Int.	High int.
10	0.172	0.218	0.264
15	0.208	0.252	0.296
20	0.244	0.286	0.328

The example below shows how to calculate the probability that a 15 year old natural unmanaged stand will succumb to a hurricane and end up in state S_{10}. A low intensity hurricane is the only one that can send a stand to the regeneration state R_1.

Return interval of major hurricanes = 50 years

Average area of wind = 400 000 ha

Size of study area = 1 691 300 ha (estimated forested portion of N.S.)

E(X) = $\gamma = \frac{5}{50} = 0.1$

Probability hurricane arrives in period = 1 – P(X=0)

$$= 1 - \frac{e^{-\gamma}\gamma^{x}}{x!} = 1 - \frac{e^{-0.1}0.1^{0}}{0!}$$

= 0.0952

Conditional probability stand affected by hurricane = $\frac{400\ 000}{1\ 691\ 300}$ = 0.2365

Prob. hurricane arrives and stand affected = 0.0952 × 0.2365 = 0.0225

Probability of succumbing and ending up in R_1 = P (R_1 | hurricane) =

(Prob. hurricane arrives and stand affected) ×

(Prob. of low intensity hurricane if it occurs) ×

(Joint conditional prob. stand succumbs if hurricane occurs) =

0.0225×0.5×(0.208×0.5+0.252×0.3+0.296×0.2) = 0.0027

We could calculate the probabilities of ending up in state R_2 and R_3 for the same stand by using the probabilities of occurrence of medium and high intensity hurricanes given in table 2-5 and replacing them in the equation above in place of the probability of occurrence of a low intensity hurricane. This procedure needs to be repeated for any stand, regardless of treatment type, for which probabilities of succumbing to hurricanes are required.

2.9.2 Forest Fires

Modeling forest fires is treated in a similar manner as modeling hurricanes. Many studies characterize the occurrence of a forest fire as a Poisson process (Cunningham and Martell (1973), Martell and Sun (2008)). Equations (2-30) and (2-31) reflect the nature of forest fires given the occurrence rate follows a Poisson process.

$$E(X) = \gamma = (\text{length of study period}) \times (\text{\# fires per year in study area}) \quad (2\text{-}30)$$

$$P(\text{stand affected} \mid \text{fire occurs}) = \frac{(\text{avg. fires per year}) \times (\text{avg. size of each fire})}{\text{size of study area}} \quad (2\text{-}31)$$

The probability a stand is affected in any period is calculated by multiplying the probabilities in equations (2-30) and (2-31) with each other. The rest of the procedure for calculating the probability of ending up in a regeneration state in any period due to a forest fire is identical to the procedure for hurricanes. The probabilities presented in table 2-7 are slightly different than those for hurricanes.

Table 2-7 – Example probabilities of different intensity fires occurring

Probability of low intensity fire if it occurs (new state = R_3)	0.3
Probability of medium intensity fire if it occurs (new state = R_2)	0.5
Probability of high intensity fire if it occurs (new state = R_1)	0.2

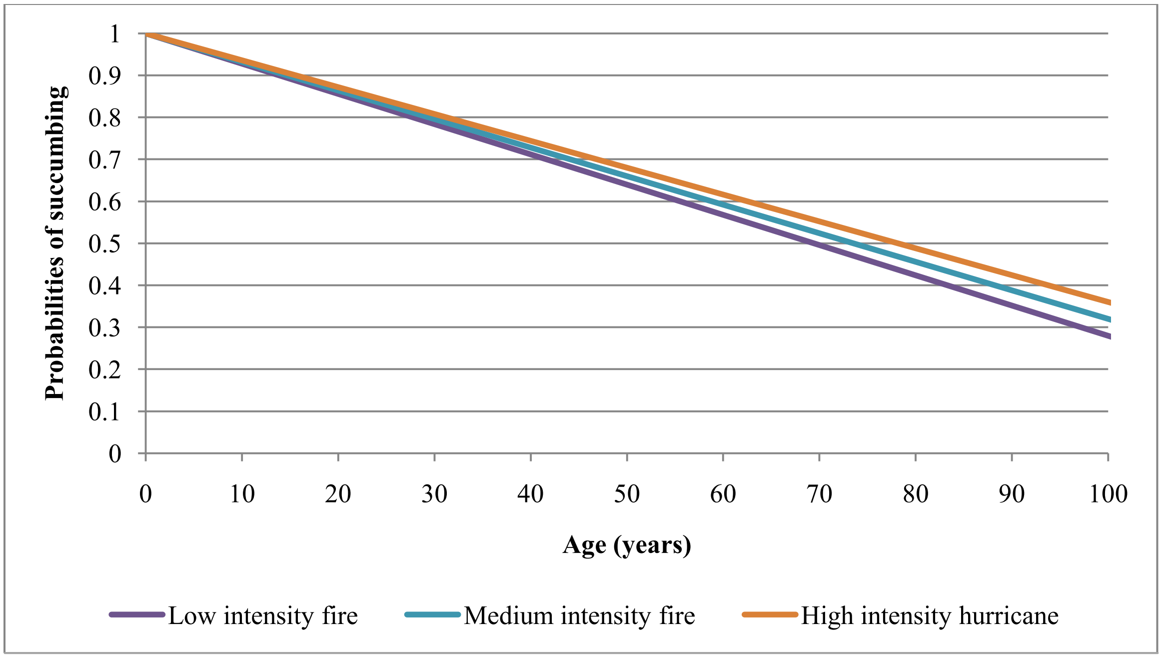


Figure 2-6 – Probabilities of succumbing to fires of different intensities

Figure 2-6 shows the probabilities of succumbing, or susceptibility, for a range of ages. The susceptibilities are reversed when compared to hurricanes because small trees don't have a thick bark to protect them from the intense heat of a fire therefore we model a young stand as having a higher probability of ending up in a worst regeneration state. These probabilities are a function of age but they could also have been given as a function of diameter as it's closely related to age for all treatment types.

Table 2-8 – Example age dependent probabilities of succumbing to a fire of low, medium or high intensity for an unmanaged natural stand

Fire Chart			
Age	Low int.	Med. int.	High int.
10	0.928	0.932	0.936
15	0.892	0.898	0.904
20	0.856	0.864	0.872

The examples given in table 2-8 are taken from figure 2-6 and are used in the following example. This example shows the steps in calculating the probability of a 15 year old stand succumbing to a forest fire and ending up in state R_1. A high intensity fire is the only one that can result in a stand going to a regeneration state R_1.

Average number of major forest fires per year in study area = 3.5

Average size of each fire = 2857.1 ha

Size of study area = 1 691 300 ha

E(X) = γ = 5 × 3.5 = 17.5

Probability fire occurs in period = 1 – P(X=0)

$$= 1 - \frac{e^{-\gamma}\gamma^{x}}{x!} = 1 - \frac{e^{-17.5}17.5^{0}}{0!} = 1$$

Conditional prob. stand burned by fire = $\frac{5\times3.5\times2857.1}{1\ 691\ 000}$ = 0.0296

Prob. fire occurs and stand is burned = 1 × 0.0296 = 0.0296

Probability of succumbing and ending up in R_1 = P (R_1 | fire) =

(Prob. fire occurs and stand is burned) ×

(Prob. of high intensity fire if it occurs) ×

(Joint cond. prob. stand succumbs if fire occurs) =

0.0296×0.2×(0.892×0.3+0.898×0.5+0.904×0.2) = 0.0053

The probabilities of ending up in regeneration states R_2 and R_3 are also calculated the same way as is described in the previous section.

2.9.3 Insect Outbreaks

The probability of occurrence is calculated the same way as it is for hurricanes therefore equation (2-28) is used for γ. The occurrence of damaging insect outbreaks has been modeled using a Poisson process in some cases (Batabyal and Beladi 2009). There are two major differences with insect outbreaks in comparison with the other two natural disasters. First, there isn't any conditional probability the stand will be affected. If an outbreak occurs, the stand will be affected therefore

P (stand affected | insect outbreak occurs) = 1

Second, with insect outbreaks, it's assumed that if the stand succumbs, the damage will be such that the stand will end up in the worst possible state for natural regeneration (R_1) with 0% probability of ending up in state R_2 or R_3. In other words, there aren't different levels of intensity for insect outbreaks.

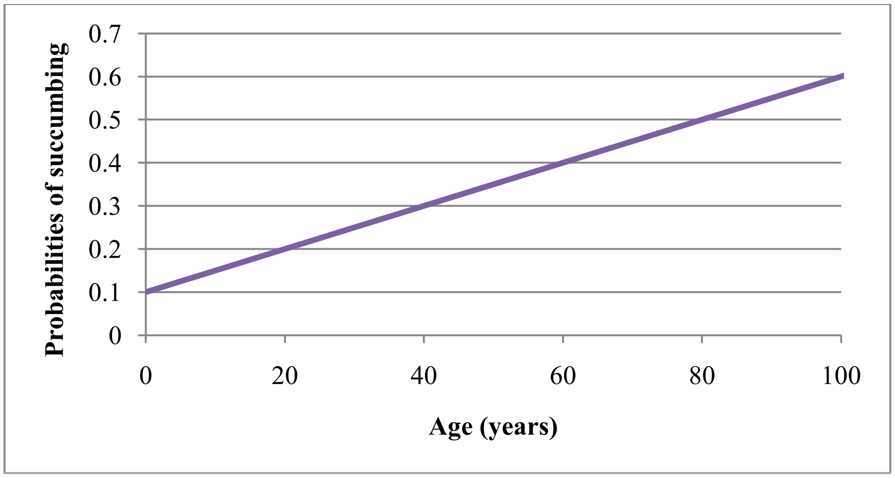


Figure 2-7 – Probabilities of succumbing to insect outbreaks

Figure 2-7 shows age dependent probabilities of succumbing to an insect outbreak. The values shown in table 2-9 are taken from figure 2-7. Larger trees are modeled as having a higher probability of succumbing because they offer better food and shelter for invading insects.

Table 2-9 – Example age dependent probabilities of succumbing to an insect outbreak for an unmanaged natural stand

Age (years)	0	5	10	15	20	25	30	35
Susceptibility	0.1	0.125	0.15	0.175	0.2	0.225	0.25	0.275

The example below shows steps in calculating the probability that a 15 year old stand will succumb to an insect outbreak and end up in state R_1.

Return interval of major insect outbreak = 30 years

$E(X) = \gamma = \frac{5}{30}$=0.1666

Probability insect outbreak occurs in period = 1 – P(X=0)

$$= 1 - \frac{e^{-\gamma}\gamma^{x}}{x!} = 1 - \frac{e^{-0.1666}0.1666^{0}}{0!} = 0.1535$$

Probability of succumbing and ending up in R_1 = P (R_1 | outbreak) =

(Prob. insect outbreak occurs) ×

(Prob. stand succumbs if insect outbreak occurs) =

0.1535×0.175 = 0.0268

We need the joint probability that a stand will end up in one of the regeneration states in the time period immediately following a hurricane. For a 15 year old unmanaged natural stand, given the probabilities of ending up in regeneration state R_1 calculated as examples for each natural disaster, we have

P (R_1) = P (R_1 | hurricane) + P (R_1 | fire) + P (R_1 | insect outbreak)

= 0.0027 + 0.0053 + 0.0039 = 0.0119

In order to calculate the probability that no natural disaster will occur and affect the stand, we need to calculate the same probabilities for R_2 and R_3 for the same stand and then calculate the probability that the stand doesn't succumb to a natural disaster.

P (not succumbing) = 1 - P (R_1) - P (R_2) - P (R_3)

The procedure is repeated for any stand where these probabilities are needed. The next chapter presents examples of growth and yield models for mixed-species stands. Variable removal percentages for commercial thinning are also discussed and examples are presented.

Chapter 3: Mixed-species Forest Stands - Some Results from Models

The previous chapter presented models for growth and yield of mixed-species stands as well as for variable thinning models. Important assumptions were made for the models to hold. This chapter is divided into three sections with a focus on presenting computer results for the forest stand management models discussed in the previous chapter. The models are tested to show that they perform reasonably under the assumptions made.

In order to show that the models developed and presented here perform reasonably, they are tested for stands with 0%, 10% and 20% HW as well as 0%, 10% and 20% SW and results are shown in section 3.1. The results are compared with NS DNR single-species models for validation. If the models produce reasonable results for the stands listed above, this at least suggests the results will be reasonable for other stands. This is as strong a claim as can be made because models for growing mixed-species stands don't exist therefore aren't available for comparison with the proposed approach.

Sections 3.2 and 3.3 discuss the development and application of CT thinning in more detail than is currently available from NSDNR. Section 3.2 shows simulation results for the development of the variable removal methodology used for commercial thinning (CT) in this research. The development of the method is explained and an example is given of its application to a mixed-species stand. Section 3.3 explains how the methodology developed in section 3.2 is applied to calculate the yield for a CT operation on a mixed-species stand.

3.1 Mixed-species Growth Models

Detailed growth models are presented in sections 2.6 and 2.7 and functions are detailed in appendix A. The first part of this section compares the proposed model in section 2.7 with NS DNR single-species models for validation. The second part demonstrates long-term performance for mixed-species stands and demonstrates the difference between plantations and natural stands.

3.1.1 Comparison with Single-Species Models

We recall that, given a uniform stand, every tree in the stand sees the same crown closure (cc). But with different maximum stocking basal areas, the spacing isn't the same for each species. Furthermore, SW grows faster than HW. Therefore SW stands reach full stocking faster than HW stands when starting at the same diameter and same cc.

In order for the proposed mixed-species growth model to accurately calculate the growth of each species in a mixed-species stand, the model must accurately represent SW and HW growth in pure stands. The proposed approach uses cc and species percentage to calculate the spacing between trees for each species. When a stand contains only one species, the spacing calculation is done using equation (2-1) and each species grows as in table 3-1 which shows the growth of pure SW and HW stands starting from the same d^{θ} and the same cc. The cc columns clearly show the difference in the rates at which the pure SW and HW stands reach 100% crown closure. In a mixed-species stand that combines SW and HW, the presence of SW will speed up the rate at which the HW portion of the stand reaches 100% crown closure therefore reducing its gain in diameter growth as opposed to a pure HW stand. The presence of HW in a mixed-species stand will slow the rate at which the SW portion of the stand reaches 100% crown closure therefore accelerating diameter growth of SW as opposed to a pure SW stand. In other words, adding HW to a mixed-species stand reduces total stand BA, for the same cc, which raises spacing between trees and encourages faster diameter growth. These findings are consistent with other results reported in the literature (Andreassen and Tomter (2003) and Huang and Titus (1995)). Table 3-1 will serve as the benchmark for comparison in order to validate the mixed-species model.

Tables 3-2 and 3-3 show the effect of 10% and 20% HW in a mixed-species stand. As expected, with the same starting d^{θ} and cc as table 3-1, the effect on diameter growth is exactly as described in the beginning of this section. The presence of 10% HW in a mixed-species stand gives SW a 2.16% gain in diameter growth over a 20 year period (21.25cm vs. 20.80cm). The presence of 20% HW gives SW a 2.71% gain in diameter growth over the same 20 year period (21.38cm vs. 20.80cm) when compared with the pure SW stand. The pure SW stand reaches 100% crown closure in 20 years and the presence of 10 and 20% HW only has a small effect on the rate at which the stand reaches 100% crown closure. There is not enough HW BA to have a strong influence on SW growth. These findings are consistent with the expected effect of the presence of small amounts of HW to an almost pure single-species SW stand. The smaller full stocking basal area and larger spacing of HW has the effect of creating a little more space for the SW trees to grow when the HW is present in small amounts in a predominantly SW stand.

Table 3-1 – Growth for pure SW and HW stands, $SI^S = SI^H = 16$m

$Year$	d^S	d^H	cc SW	cc HW	BA^S	BA^H
0	15.00	15.00	60	60	35.05	17.57
5	16.84	15.95	74	67	44.16	19.87
10	18.38	16.80	88	73	52.64	22.05
15	19.69	17.58	99	78	60.39	24.13
20	20.80	18.29	100	84	61.22	26.12
25	21.89	18.94	100	89	61.67	28.03
30	22.95	19.56	100	94	62.09	29.88
35	24.00	20.13	100	99	62.49	31.66
40	25.03	20.68	100	100	62.87	32.38
45	26.04	21.21	100	100	63.23	32.63

Table 3-2 – Growth for a mixed-species stand with 10% HW, $SI^S = SI^H = 16$m

$Year$	*Softwood*		*Hardwood*		cc	BA^S	BA^H
	d^S	pct^S	d^H	pct^H			
0	15.00	90	15.00	10	60	28.69	3.19
5	16.97	91	15.95	9	74	36.74	3.60
10	18.64	92	16.72	8	87	44.30	3.96
15	20.05	92	17.38	8	98	51.26	4.28
20	21.25	92	17.95	8	100	52.68	4.41
25	22.40	92	18.51	8	100	53.08	4.45
30	23.52	92	19.06	8	100	53.45	4.49
35	24.63	92	19.60	8	100	53.81	4.53
40	25.71	92	20.14	8	100	54.14	4.57
45	26.77	92	20.68	8	100	54.46	4.60

When we study stands that are predominantly HW with small proportions of SW, the effect on diameter growth is exactly as described in the beginning of this section. The presence of 10% SW in a mixed-species stand causes a 0.16% loss of diameter growth for HW over a 20 year period (18.26cm vs. 18.29cm). The presence of 20% SW causes a loss of 0.33% in hardwood diameter growth over the same 20 year period (18.23cm vs.

18.29cm) when compared with a pure HW stand. The pure HW stand reaches 100% crown closure in 40 years whereas the presence of 10 and 20% SW slightly reduces that time. Although the time to reach 100% crown closure is slightly affected, there is not enough BA^S present to have a strong influence on hardwood diameter growth.

Table 3-3 – Growth for a mixed-species stand with 20% HW, SI^S = SI^H = 16m

Year	*Softwood*		*Hardwood*		cc	BA^S	BA^H
	d^S	pct^S	d^H	pct^H			
0	15.00	80	15.00	20	60	23.39	5.85
5	16.97	82	15.95	18	73	29.95	6.61
10	18.67	83	16.74	17	84	36.22	7.28
15	20.12	84	17.41	16	95	42.10	7.88
20	21.38	84	18.00	16	100	44.81	8.40
25	22.52	84	18.56	16	100	45.15	8.48
30	23.65	84	19.11	16	100	45.47	8.56
35	24.75	84	19.65	16	100	45.76	8.63
40	25.83	84	20.19	16	100	46.05	8.71

The findings in table 3-4 and 3-5 are consistent with the expected effect of the presence of small amounts of SW to an almost pure single-species HW stand. The larger full stocking basal area and smaller spacing of SW has the effect of removing growing space for the HW trees when the SW is present in small amounts in a predominantly HW stand.

Table 3-4 – Growth for a mixed-species stand with 10% SW, SI^S = SI^H = 16m

Year	*Softwood*		*Hardwood*		cc	BA^S	BA^H
	d^S	pct^S	d^H	pct^H			
0	15.00	10	15.00	90	60	1.85	16.65
5	16.97	11	15.95	89	67	2.37	18.82
10	18.80	12	16.80	88	74	2.90	20.87
15	20.47	13	17.56	87	80	3.45	22.82
20	22.01	14	18.26	86	86	3.98	24.67
25	23.41	15	18.90	85	91	4.50	26.44
30	24.68	15	19.50	85	96	4.97	28.14
35	25.82	15	20.06	85	100	5.19	29.44
40	26.88	15	20.60	85	100	5.22	29.68

Table 3-5 – Growth for a mixed-species stand with 20% SW, $SI^S = SI^H$ = 16m

Year	*Softwood*		*Hardwood*		cc	BA^S	BA^H
	d^S	pct^S	d^H	pct^H			
0	15.00	20	15.00	80	60	3.90	15.62
5	16.97	22	15.95	78	68	5.00	17.65
10	18.78	24	16.79	76	75	6.12	19.57
15	20.44	25	17.55	75	81	7.25	21.37
20	21.94	27	18.23	73	88	8.35	23.07
25	23.30	28	18.86	72	94	9.42	24.69
30	24.52	28	19.45	72	99	10.13	26.25
35	25.63	28	19.99	72	100	10.32	26.80
40	26.69	28	20.53	72	100	10.38	27.02

The method proposed in section 2.7 for combining single species stand level growth models produces reasonable results for the stands tested and is consistent with the results in the sources cited earlier which at least suggests that the results will be reasonable for other stands.

3.1.2 Mixed-species Examples

We now present three examples to demonstrate the application of the proposed model to managed and unmanaged mixed-species stands. The first example is for TRT=1 only. The focus of this example is to show the relationship between stand variables in an unmanaged natural mixed-species stand over time. The second example uses a commercially thinned natural stand (TRT=5) and demonstrates the application of the proposed methodology to managed mixed-species natural stands. The last example is used to demonstrate the difference between plantations and natural stands.

Unmanaged mixed-species natural stand

The growth of unmanaged natural mixed-species stands depends only on the number of years since the last final harvest (ha_t) and the percentage of softwood naturally present in the stand (pct^S). Table 3-6 shows an example of a stand that starts with ha_t = 30 years and pct^S = 50%. Stand stocking is 100%.

Table 3-6 – Example of growth for a natural untreated stand with stand stocking = 100% and SI^S = SI^H = 16m at age 50 years

$Year$	Age	Softwood		Hardwood		$fraction^S$	$fraction^H$	BA^S	BA^H
		d^S	pct^S	d^H	pct^H				
0	30	7.3	50	5.9	50	0.32	0.68	14.75	14.75
5	35	8.3	49	6.9	51	0.32	0.68	15.09	15.54
10	40	9.4	49	8.0	51	0.32	0.68	15.38	16.22
15	45	10.4	48	9.0	52	0.32	0.68	15.64	16.82
20	50	11.3	48	9.9	52	0.32	0.68	15.87	17.36
25	55	12.2	47	10.8	53	0.32	0.68	16.07	17.82
30	60	13.0	47	11.6	53	0.32	0.68	16.25	18.24
35	65	13.8	47	12.4	53	0.32	0.68	16.41	18.61
40	70	14.6	47	13.1	53	0.32	0.68	16.55	18.95
45	75	15.3	46	13.8	54	0.32	0.68	16.68	19.24
50	80	15.9	46	14.4	54	0.32	0.68	16.80	19.51
55	85	16.5	46	15.0	54	0.32	0.68	16.90	19.75
60	90	17.1	46	15.5	54	0.32	0.68	16.99	19.96

Based on the example in table 3-6, starting at age 30 and for rest of the life of the stand, the softwood trees always have a larger diameter than the hardwood trees but the diameters essentially gain the same amount at each time period. Based on the method proposed in section 2.7, the fraction of the stand covered by each species will not change over the life of the stand, as long as the stand starts with cc = 100%, as evidenced by the results shown in table 3-6. Furthermore, all stands in this thesis are even-aged throughout the entire growth cycle of the stand which means that, by definition, all trees on the stand are essentially the same age. According to this definition, no new younger trees will grow in to replace the ones currently growing until after a regeneration harvest. Therefore, it makes sense for the space occupied by each species to remain unchanged when the stand is at full stocking. Because the trees grow at different rates and $maxtrees^S \neq maxtrees^H$ for a given diameter, and based on the relationship between species percentage and $fraction_t^\theta$ given in section 2.7, pct^θ drifts slightly from the initial values but the change isn't significant.

Managed natural stands

An example of a commercially thinned natural stand with 60% SW basal area and 15cm diameter for SW and HW is shown in table 3-7. The example in table 3-8 has the same species split but HW has an initial diameter of 9.4 cm.

Table 3-7 – Example of growth for a stand with TRT = 5 and SI^S = SI^H = 16m at age 50 years with identical starting diameters

Year	*Softwood*		*Hardwood*		$fraction^S$	$fraction^H$	cc	BA^S	BA^H
	d^S	pct^S	d^H	pct^H					
0	15.0	60	15.0	40	0.429	0.571	60	15.05	10.03
5	16.8	63	15.9	37	0.457	0.543	70	18.96	11.34
10	18.5	65	16.8	35	0.479	0.521	79	22.82	12.53
15	19.9	66	17.5	34	0.496	0.504	88	26.55	13.62
20	21.2	67	18.1	33	0.510	0.490	96	30.09	14.63
25	22.3	67	18.7	33	0.510	0.490	100	31.55	15.37
30	23.4	67	19.2	33	0.510	0.490	100	31.76	15.51
35	24.4	67	19.8	33	0.510	0.490	100	31.96	15.65

Table 3-8 – Example of growth for a stand with TRT = 5 and SI^S = SI^H = 16m at age 50 years with different starting diameters

Year	*Softwood*		*Hardwood*		$fraction^S$	$fraction^H$	cc	BA^S	BA^H
	d^S	pct^S	d^H	pct^H					
0	15.0	60	9.4	40	0.394	0.606	60	13.81	9.21
5	16.8	60	10.5	40	0.402	0.598	73	17.40	11.41
10	18.4	61	11.3	39	0.409	0.591	85	20.80	13.42
15	19.8	61	12.1	39	0.415	0.585	95	23.98	15.26
20	20.9	61	12.8	39	0.415	0.585	100	25.41	16.30
25	22.0	61	13.4	39	0.415	0.585	100	25.59	16.55
30	23.1	61	14.0	39	0.415	0.585	100	25.77	16.78
35	24.1	60	14.6	40	0.415	0.585	100	25.93	17.00

There are two noticeable differences between these examples and the last example in the previous section. First, crown closure increases steadily until it reaches 100% in 25 years in table 3-7 and 20 years in table 3-8. The fraction of a stand covered by each species changes until the stand reaches crown closure at which point it stops changing. pct^{θ} changes only slightly as it did in the previous example. Second, the diameter is the same for SW and HW in table 3-7 which could not occur in an unmanaged natural stand. Because these stands may have been treated since the last final harvest, the diameter growth could have been altered from its natural state.

In table 3-7, SW and HW have the same initial diameter. The result is a rapid change in pct^{θ} until crown closure is reached which is caused by the faster SW diameter growth and larger number of SW trees on the stand. In table 3-8, there are more hardwood trees because of the smaller d^{H}. This results in a higher $fraction^{H}$ and a higher $TFREQ^{H}$ which allows the HW to retain its fraction of the stand area. The larger $TFREQ^{H}$ allows the HW to gain basal area faster in table 3-8 compared to table 3-7.

Comparison of plantations and natural stands

In forests that are managed for maximum economic gain, forest managers often opt for plantations over natural stands when choosing what type of stand will replace another after a regeneration harvest. This will be discussed in detail in chapter 5 when the results of the optimization are presented. The last example in this section isn't used to show the progression of a stand over time but rather to show the difference between natural stands and plantations which may serve to illustrate why many jurisdictions around the world are seeing natural forests being replaced by plantations when the main management objective is of a purely economic nature. Tables 3-9 and 3-10 show the contrast between natural and plantation stands. These values are taken directly from the NS DNR growth and yield models.

Table 3-9 – Example of an unmanaged natural stand (TRT = 1), SI^{S} = SI^{H} = 16m at age 50 years, *ha* = 70 years

Softwood		*Hardwood*		$fraction^{S}$	$fraction^{H}$	BA^{S}	BA^{H}
d^{S}	pct^{S}	d^{H}	pct^{H}				
15.0	100	0	0	1.000	0.000	51.12	0

Table 3-10 – Example of an non-commercially thinned plantation (TRT = 3), SI^{S} = SI^{H} = 16m at age 50 years, *ha* = 70 years

Softwood		*Hardwood*		$fraction^{S}$	$fraction^{H}$	BA^{S}	BA^{H}
d^{S}	pct^{S}	d^{H}	pct^{H}				
15.0	100	0	0	1.000	0.000	58.30	0

Plantations don't contain hardwoods in N.S. therefore the comparison is done with a single-species SW natural stand. The main difference is the amount of BA in a plantation compared to a natural stand. The trees in plantations are planted in a very organized pattern with very little wasted space which allows the plantation to have a slightly higher number of trees per hectare at full stocking when compared to natural stands. Essentially, for the same diameter, $maxtrees^S$ for a plantation is higher than $maxtrees^S$ for a natural stand which yields a higher BA for a plantation.

3.2 Commercial Thinning Methods

The growth and yield models of the NS DNR only model thinning from below. These regression models were developed using data from Nova Scotia`s fully stocked research permanent sample plots (PSP) collected over a 15 year period where there is only data available for removal from below therefore it is currently the only method of simulating commercial thinning in Nova Scotia forests (NSDNR (1993b), O'Keefe and McGrath (2006)). For the purpose of this work, models are required for other removal types and this section details the methodology used to develop those models.

Previous work (Gunn et al. 2000) has developed a family of Weibull distributions whose parameters depend only on d^θ. In that work, the authors show that the diameter distribution of trees in a stand after thinning is not readily distinguishable from the diameter distribution before the thinning. We take advantage of this property and use the Weibull simulation built for the work by Gunn et al. (2000) to simulate commercial thinning for SW and HW, from above and below, for a range of quadratic mean diameters and basal area removal percentages. The results of those simulations are then used to create an approach to calculating the diameter change for a range of species, diameters, removal types and percentages.

What follows is a step by step brief overview of the method used to develop the CT models used in the DP model.

<u>Step 1:</u> Choose the average stand diameter and CT parameter values to simulate;

<u>Step 2:</u> Create a stand with randomly distributed trees where the trees are created by randomly simulating their individual diameters based on the

Weibull distribution. The number and size of trees on the stand are functions of the average stand diameter;

Step 3: CT is applied to the stand created in step 2 by removing trees according to the CT parameter values being simulated;

Step 4: The post-CT stand average diameter is recorded for future retrieval along with the pre-CT diameter and CT parameter values used in step 3;

Step 5: Steps 1 through 4 are repeated for the range of diameters and CT parameters required in the DP model;

Step 6: Repeat for a reasonable number of replications. The data collected in steps 1 through 5 for all replications is used to determine the parameters of the linear equation fit on the data.

Steps 1 through 5 make up one replication of the simulation and the choice of the number of replications is explained in step 6 below. The 6 steps are described in greater detail below. These steps are applied for one type of CT, either thinning from above or from below, and for each species separately.

Step 1: Choose the average stand diameter and CT parameters

For a single simulated stand, a combination of quadratic mean diameter (d^{θ}) and basal area removal percentage $(BARem)$ is chosen. For the purpose of this work, simulations are done for each combination of species and thinning type with d^{θ} ranging from 14 to 35 cm in 1 cm increments and $BARem$ of 0.05 to 0.4 in 0.05 increments. Stands with $d^{\theta} < 14$cm have very little commercial value therefore they are not simulated. In commercial timber operations in Nova Scotia, stand average diameters of 35 cm are rare thus it has been chosen as an arbitrary maximum diameter. Removing less than 5% of the wood on a stand during a CT wouldn't remove enough timber to be economically sustainable and removing more than 40% would leave the stand with very little protection against the wind. The combination of 22 values of d^{θ} with each of the 8 values of $BARem$ gives 176 combinations. Combined together, the 176 combinations represent one simulation replication.

Step 2: Create a stand

A stand containing T trees is created using the method that follows. The model developed by Gunn et al. (2000) gives the equations below for creating the individual trees on a stand. A random number between 0 and 1 is picked from a uniform distribution and used to determine the diameter of a tree d_i based on a Weibull distribution for d^θ. The parameters used in the d_i equation are all dependent on d^θ. Parameters ξ, η, and λ are different for hardwood and softwood.

$$d_i = \lambda + \eta\big(-Log(1-u)\big)^{1/\xi}$$

Softwood: $\lambda = 0.75 \times (0.374 \times d^S + 0.004 \times (d^S)^2)$

$\eta = -0.2877 + 0.7784 \times d^S$

$\xi = 3.7143 - \frac{2.344}{(d^S)^{0.333}}$

Hardwood: $\lambda = 0.75 \times (0.4382 \times d^H)$

$\eta = -0.0898 + 0.7195 \times d^H$

$\xi = 2.8128 - \frac{1.5701}{(d^H)^{0.5}}$

During each replication, a virtual parcel of land is divided into 128 cells where each cell corresponds to a manageable cell size for the worker doing the commercial thinning as decisions about which trees to remove need to be made among the trees that are in sight of the worker. For each simulated stand, the stand starts at 100% crown closure with T trees where $T = maxtrees^\theta$ for the given d^θ and the trees are randomly distributed among the 128 cells and then sorted by diameter inside each cell.

Step 3: Apply Commercial Thinning

In this step, thinning is done either from above or from below, and for SW and HW. In the case of thinning from above, it is applied to the stand created in step 2 by removing the largest tree from each cell starting from cell 1 and moving through subsequent cells until the desired basal area has been removed from the stand. If the desired basal area removal hasn't been reached after having gone through the 128 cells, the process restarts at cell 1. When thinning from below, the same process is followed except that the smallest tree in each cell is removed. The spatial distribution of the trees in a cell forces workers to remove trees that normally would stay or leave trees that would normally be removed. For example, when doing thinning from above, if two big trees are side by side, both wouldn't be removed in subsequent commercial thinnings (CT) because it would leave a big opening in the stand. During the simulation of each stand, 20% of the trees that would normally be removed are randomly skipped. Once the desired basal area has been removed, the new number of trees and the new stand diameter are calculated.

Step 4: Record data for future retrieval

In this step, d^{θ} before CT, d^{θ} after CT and $BARem$ are stored to be used in step 6. This step is done for all combinations of species type, CT type, d^{θ} and $BARem$.

Step 5: Repeat steps 1 through 4

Steps 1 through 4 are repeated 30 times each for the combinations described in step 1 where 30 is a large enough number of replications given the small amount of variability shown in figure 3.1 for each combination of d^{θ} and $BARem$.

Step 6: Create CT linear model

A total of 5280 stands are simulated (30 replications × 22 diameters × 8 $BARem$ values). Figure 3-1 shows the 5280 simulations for thinning from below for SW in three dimensions: d^{S} before CT, d^{S} after CT and $BARem$. It shows that there is a linear relationship between d^{S} and $BARem$ and the new diameter after CT for thinning from below for SW. Similar results were found for thinning from below for HW as well as for thinning from above for both species types. Results of those simulations are presented in appendix D.

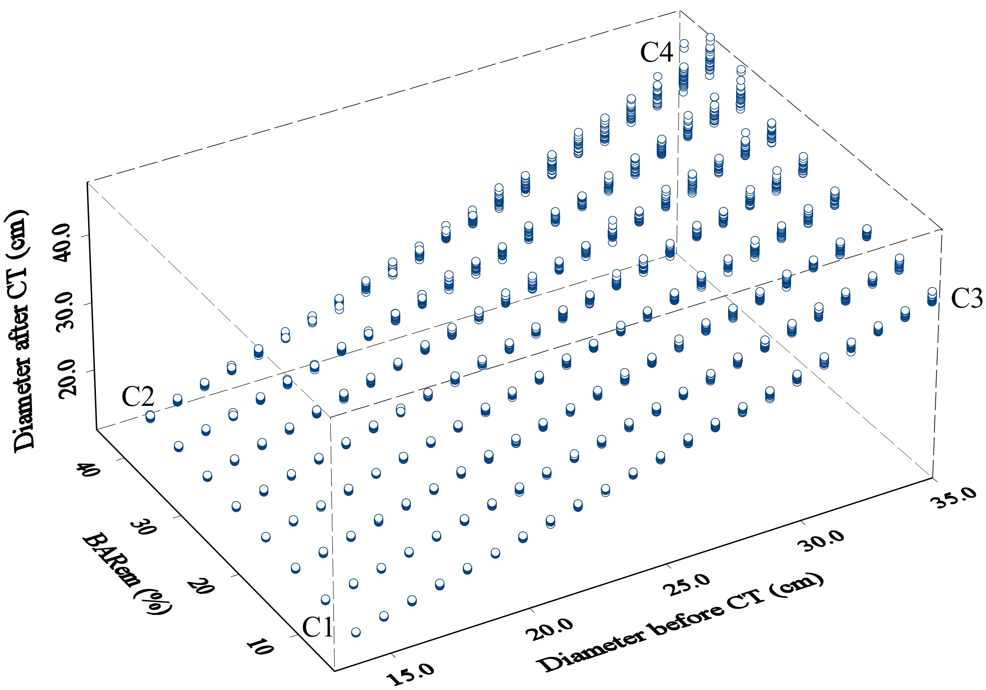


Figure 3-1 – Plot of 5280 diameter change simulations for CT from below for SW

For the implementation in the DP model, we take advantage of the linear relationship shown in figure 3-1 and fit a linear model to the data to determine the diameter change for a stand given d^{θ} and $BARem$. The fitting is done on a plane where the four corners have (x,y,z) coordinates shown in table 3-11.

Table 3-11 – (x,y,z) coordinates for the plane shown in figure 3-1

	Coordinates		
Corner	x (d^S before CT)	y ($BARem$)	z (d^S change after CT)
C1	14 cm	5%	0.35 cm
C2	14 cm	40%	1.89 cm
C3	35 cm	5%	1.70 cm
C4	35 cm	40%	9.46 cm

The z coordinates are the average values for the d^S change for all simulation replications where d^S before CT and $BARem$ are equal to those shown in table 3-11. Equations (3-1) to (3-3) are standard linear interpolation equations. Equation (3-1) calculates the diameter change for d^S_{Pre-CT} = 14cm and a given value of $BARem$ between 5% and 40%. Equation (3-2) calculates the diameter change for d^S_{Pre-CT} = 35cm for the same $BARem$ as above. Equation (3-3) interpolates between the values calculated in equations (3-1) and (3-2) for a given d^S between 14cm and 35cm.

14Rem = ($BARem$ - 0.05) / 0.35 * (14_high - 14_low) + 14_low (3-1)
35Rem = ($BARem$ - 0.05) / 0.35 * (35_high - 35_low) + 35_low (3-2)
$\tilde{d}^S$ = d^S+ (d^S - 14) / 21 * (35Rem - 14Rem) + 14Rem (3-3)

where 14_low – Average diameter change for $BARem$ = 5% and d^S = 14 cm
14_high – Average diameter change for $BARem$ = 40% and d^S = 14 cm
35_low – Average diameter change for $BARem$ = 5% and d^S = 35 cm
35_high – Average diameter change for $BARem$ = 40% and d^S = 35 cm
14Rem – Average diameter change for a given $BARem$ with d^S = 14 cm
35Rem – Average diameter change for a given $BARem$ with d^S = 35 cm
$\tilde{d}^S$ is the average diameter of the stand immediately following the CT

An example is given below for a softwood stand with thinning from below with d^S = 21.5 cm and $BARem$ = 35%.

14_low = 0.35 cm, 14_high = 1.89 cm, 35_low = 1.70 cm, 35_high = 9.46 cm
14Rem = (0.35 - 0.05) / 0.35 * (1.89 – 0.35) + 0.35 = 1.67 cm
35Rem = (0.35 - 0.05) / 0.35 * (9.46 – 1.70) + 1.70 = 8.35 cm
$\tilde{d}^S$ = 21.5 + (21.5 - 14) / 21 * (8.35 – 1.67) + 1.67 = 25.56 cm

In this example, d^S rises by 4.06 cm due to the thinning from below. NS DNR has published a diameter adjustment equation for thinning from below so it is useful to compare it to the method presented above. Equation (3-4) is used by NS DNR for thinning from below.

$$NEWDIAM^S = 0.00437 \times d^S \times BARem + 0.25 + d^S \quad (3\text{-}4)$$

There aren't any readily available testing procedures for comparing a regression model to a data set if the underlying data for the regression model isn't known. Therefore, plotting the proposed method and the NS DNR regression equation on the same graph and calculating the largest difference between the two curves for a typical range of diameters and $BARem$ encountered in the DP model is a reasonable method of comparison.

Figures 3-2a and 3-2b plot the proposed model and the NS DNR model for SW, removal from below and $BARem$ values of 10%, 20%, 30% and 40%. For example, the largest difference between the proposed method and the NS DNR model for SW with $BARem$ = 20% is at d^S = 35 cm where the NS DNR model yields a new diameter of 38.31 cm and the proposed model gives 40.02 cm which is a difference of 1.71 cm or 4.5%. For the purposes of this thesis, we are treating this error as acceptable.

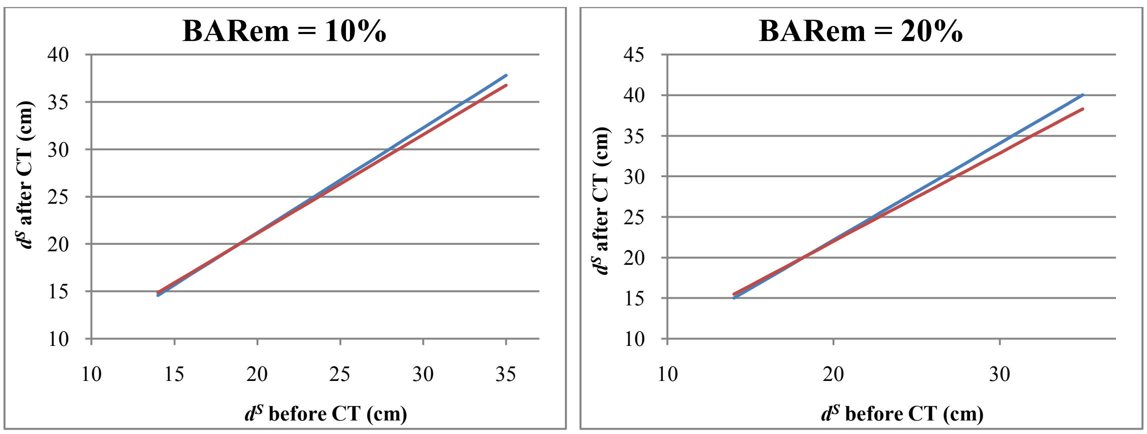


Figure 3-2a – Comparison of proposed CT diameter adjustment model and equivalent NS DNR model for 10% and 20% basal area removal from below for SW – proposed model in blue and NS DNR model in red

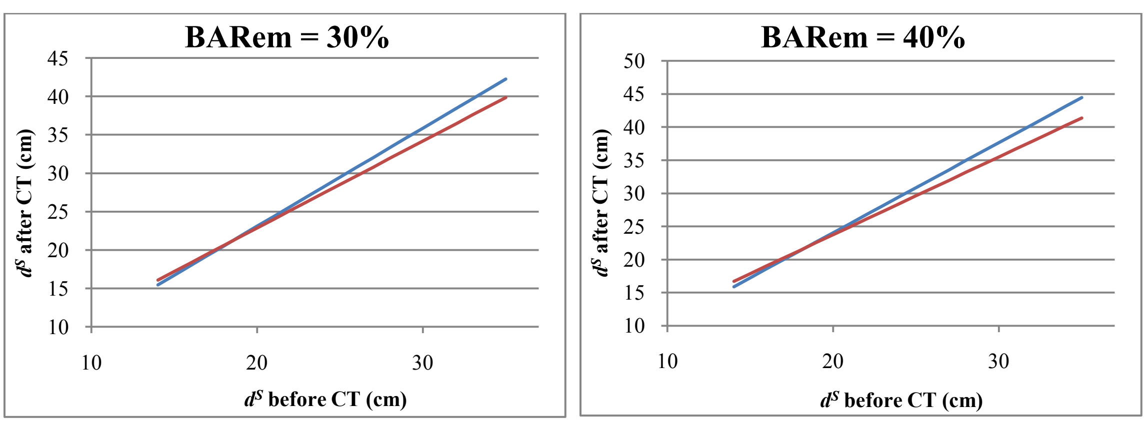


Figure 3-2b – Comparison of proposed CT diameter adjustment model and equivalent NS DNR model for 30% and 40% basal area removal from below for SW – proposed model in blue and NS DNR model in red

An example of the application of the proposed thinning model is given in table 3-12. In a stand with SW and HW, it is assumed that the change in diameter for each species is the same as if it were two smaller single-species stands with the same diameter and basal area. Table 3-12 shows the results, immediately before and after thinning, of a mixed-species stand thinned from below with 30% $BARem$, split 75/25 between softwood and hardwood. This stand has 47.3 m^2 of total basal area before thinning and 30% of that is removed from the stand for a total removal of 14.2 m^2. The basal area removed has to be split between SW and HW as follows: 10.6 m^2 SW and 3.6 m^2 HW. The actual removal amounts represent 32% of the SW on the stand and 25% of the hardwood on the stand before the thinning. These actual $BARem$ percentages are used to calculate d^S and d^H as well as BA^S and BA^H immediately following thinning. The updated value for pct^S is calculated using the new values of BA^θ.

Table 3-12 – Example of the use of the proposed method for calculating the changes in d^θ, pct^S and BA^θ that result from thinning from below in a mixed-species stand where $BARem$ = 30% and $BARem_{Split}$ = 75% SW

Before thinning					After thinning				
d^S	d^H	pct^S	BA^S	BA^H	d^S	d^H	pct^S	BA^S	BA^H
20 cm	18 cm	70%	33.1 m^2	14.2 m^2	23.3 cm	20.3 cm	67.8%	22.4 m^2	10.6 m^2

3.3 Yield From Commercial Thinning

All commercial thinnings have the same type of result regardless of the type of stand so only one example is presented here. The example is for a 60 year old unmanaged natural stand with stand stocking = 100%. The stand has a site index of 16m for both species at ha = 50 years and pct^S = 75%. The diameters and basal areas before thinning are d^S = 17.41 cm, d^H = 15.38 cm, BA^S = 32.92 m^2 and BA^H = 10.97 m^2. Table 3-13 shows the resulting stand when we apply $BARem$ = 30% split 50/50 between softwood and hardwood. Table 3-14 shows results for the same split but with $BARem$ = 40%.

Table 3-13 – Example of a CT on an unmanaged natural stand (TRT = 1), $SI^S = SI^H$ = 16m, ha = 60 years, pct^S = 75%, $BARem$ = 30%, $BARem_{Split}$ = 50% (MV^θ is in m^3)

Thinning	$\tilde{d}^S$	$\tilde{d}^H$	$\widetilde{pct}^S$	$\tilde{cc}$	MV^S	SV^S	MV^H	SV^H	$\bar{d}^S$	$\bar{d}^H$
From below	18.62	20.90	99%	48.9%	106.78	80.46	39.77	26.91	13.80	15.00
Across all diameters	17.39	15.18	99%	49.5%	119.00	109.13	39.95	27.72	17.39	15.18
From above	16.61	11.37	99%	49.9%	125.86	125.86	40.23	29.05	23.32	15.48

Table 3-14 – Example of a CT on an unmanaged natural stand (TRT = 1), $SI^S = SI^H$ = 16m, ha = 60 years, pct^S = 75%, $BARem$ = 40%, $BARem_{Split}$ = 50% (MV^θ is in m^3)

Thinning	$\tilde{d}^S$	$\tilde{d}^H$	$\widetilde{pct}^S$	$\tilde{cc}$	MV^S	SV^S	MV^H	SV^H	$\bar{d}^S$	$\bar{d}^H$
From below	18.93	0.00	100%	44.3%	144.95	112.68	41.96	29.12	14.20	15.18
Across all diameters	17.39	0.00	100%	44.8%	158.67	145.51	41.96	29.12	17.39	15.18
From above	16.35	0.00	100%	45.1%	167.56	167.56	41.96	29.12	22.88	15.18

In both tables, as the diameter of the trees removed ($\bar{d}^\theta$) goes up, the average diameter of the trees left behind ($\tilde{d}^\theta$) goes down. Thinning from below removes the smaller trees therefore the trees that are left on the stand will be larger than the ones left behind after thinning from above. Merchantable volume (MV) and sawtimber volume (SV) are also higher when we remove the larger trees even though the amount of basal area removed is the same. The size of trees removed accounts for the difference in volume for the same amount of basal area removed. The last two columns in tables 3-13 and 3-14 show the diameter of the trees that were removed and the results are as expected. Thinning from above removes larger trees than thinning from below. The stand has 75% softwood before thinning therefore the removal of 30% of the total stand basal area with a 50/50 split leaves very little hardwood on the stand. The removal of 40% of the total stand basal area takes all the hardwood from the stand and this is reflected in the value of $\overline{pct}^\theta$ in both tables. With $BARem$ = 40% and the removal of all hardwood from the stand, MV^H, SV^H and $DIAMREM^H$ are the same for all types of thinning. The new crown closure is lower with 40% basal area removal than it is for 30% removal because more trees have been removed therefore leaving bigger gaps in the canopy of the stand.

After thinning, all of the stands in tables 3-13 and 3-14 grow according to the equations presented in section 2.7. As an example of this growth, table 3-15 shows 35 years of growth for the first stand in table 3-13. Without commercial thinning, the original stand would have grown as shown in table 3-16.

Table 3-15 – 35 year growth of the resulting stand after thinning from below – first stand shown in table 3-13

	Softwood		Hardwood		$fraction^S$	$fraction^H$	cc	BA^S	BA^H
Year	d^S	pct^S	d^H	pct^H					
0	18.62	99	20.90	1	0.979	0.021	49	32.36	0.33
5	20.69	99	21.93	1	0.982	0.018	60	39.97	0.36
10	22.45	99	22.79	1	0.983	0.017	69	47.02	0.39
15	23.94	99	23.53	1	0.984	0.016	78	53.47	0.41
20	25.20	99	24.19	1	0.985	0.015	86	59.28	0.44
25	26.29	99	24.78	1	0.986	0.014	93	64.49	0.46
30	27.22	99	25.32	1	0.986	0.014	99	69.13	0.48
35	28.01	99	25.82	1	0.986	0.014	100	70.08	0.49

Table 3-16 – 35 year growth of the original stand described at the beginning of section 3.3

	Softwood		Hardwood		$fraction^S$	$fraction^H$	cc	BA^S	BA^H
Year	d^S	pct^S	d^H	pct^H					
0	17.4	75	15.4	25	0.628	0.372	100	32.92	10.97
5	18.6	75	16.3	25	0.628	0.372	100	33.29	11.17
10	19.7	75	17.1	25	0.628	0.372	100	33.62	11.35
15	20.8	75	17.9	25	0.628	0.372	100	33.91	11.51
20	21.8	75	18.6	25	0.628	0.372	100	34.18	11.65
25	22.7	75	19.3	25	0.628	0.372	100	34.42	11.77
30	23.6	74	19.9	26	0.628	0.372	100	34.63	11.89
35	24.4	74	20.4	26	0.628	0.372	100	34.82	11.99

Thinning from below created a stand with only 1% hardwood basal area. With BA^H = 0.33 m^2 and d^H = 20.9 cm, there are only 10 hardwood trees left on the stand after commercial thinning. Commercial thinning opened up the stand and created space for the trees to grow more quickly. The difference between tables 3-15 and 3-16 is worth noting. As expected, diameter growth is higher when the stand cc is below 100%. The difference in stand total basal area after 35 years is very high with 46.81m^2 for the untreated stand and 70.57m^2 for the commercially thinned stand. The difference is easily explained. The untreated stand is close to its maximum stand basal area because table 3-16 starts with a 60 year old stand. It remains at or near the same total stand basal area for 35 years with very little gain. The trees are getting bigger but the number of trees is decreasing. In the commercially thinned case, the stand is well below crown closure which means the trees are gaining significantly more diameter growth but the number of trees stays steady with large gains in total stand basal area. The other critical difference is that the commercially thinned stand has 99% softwood which, in a pure stand, has a much higher maximum total stand basal area than hardwood.

A natural question arising from the results in table 3-13 is how does the stand grow after thinning from above? Table 3-17 shows 35 years of growth for the third stand in table 3-13. The main difference between thinning from below and thinning from above is in the rate at which the crown closure reaches 100%. The stand that was thinned from above reaches 100% crown closure earlier at which point diameter growth slows and the stand as a whole gains less basal area. The difference isn't significant but after 35 years of growth, the stand that was thinned from below has 1.13 m^2 more total stand basal area than the stand that was thinned from above. This is explained by the fact that thinning from above leaves the smaller trees which grow faster and close the canopy of the stand slightly faster than the larger trees left behind with thinning from below. The slight delay in reaching 100% crown closure causes the stand to gain a little more basal area in the case of thinning from below.

Starting with a stand that has 100% SW, as is the case in table 3-18, yields slightly higher total basal area after 35 years of growth (71.01 m^2 vs. 70.57 m^2) when compared to the case with 99% SW shown in table 3-15. As discussed earlier, adding a small amount of hardwood to the stand raises spacing between trees which yields a smaller number of trees per hectare, which in turn translates to a slightly lower stand basal area.

Table 3-17 – 35 year growth of the resulting stand after thinning from above – third stand shown in table 3-13

	Softwood		Hardwood		$fraction^S$	$fraction^H$	cc	BA^S	BA^H
Year	d^S	pct^S	d^H	pct^H					
0	16.61	99	11.37	1	0.976	0.024	50	32.40	0.33
5	18.59	99	12.54	1	0.977	0.023	62	40.59	0.40
10	20.26	99	13.51	1	0.977	0.023	72	48.22	0.46
15	21.68	99	14.34	1	0.978	0.022	82	55.19	0.52
20	22.88	99	15.06	1	0.978	0.022	91	61.49	0.57
25	23.91	99	15.71	1	0.979	0.021	98	67.14	0.62
30	24.79	99	16.30	1	0.979	0.021	100	68.50	0.64
35	25.63	99	16.87	1	0.979	0.021	100	68.79	0.65

Table 3-18 – 35 year growth for a stand with the same initial d^S and cc as the first stand in table 3-15 but with d^H = 0 and pct^S = 100%

	Softwood		Hardwood		$fraction^S$	$fraction^H$	cc	BA^S	BA^H
Year	d^S	pct^S	d^H	pct^H					
0	18.62	100	0	0	1.0	0.0	49	33.01	0
5	20.70	100	0	0	1.0	0.0	60	40.78	0
10	22.44	100	0	0	1.0	0.0	69	47.96	0
15	23.93	100	0	0	1.0	0.0	78	54.50	0
20	25.19	100	0	0	1.0	0.0	86	60.40	0
25	26.26	100	0	0	1.0	0.0	93	65.68	0
30	27.19	100	0	0	1.0	0.0	99	70.36	0
35	27.97	100	0	0	1.0	0.0	100	71.01	0

3.4 Concluding Remarks

Development of the mixed-species model used in this thesis is based on the idea that all trees on a stand, regardless of species type, experience crown closure as being the same. The examples presented in this chapter rely on the fact that crown closure is directly related to spacing between the trees on a stand and, in the growth and yield models published by NS DNR, spacing is the major driving force of individual SW and HW diameter growth models. The models and examples in this chapter have led to an important principle: slow growing species with a low maximum density such as hardwood, when mixed with faster growing higher density species such as softwood, will give the faster growing species more room to grow by slowing the rate at which the stand reaches crown closure thus resulting in a gain in diameter growth. The reverse of this principle also holds. As discussed in section 3.1, these observations were previously made by Andreassen and Tomter (2003) and Huang and Titus (1995) which further supports the claim that the mixed-species models developed and used in this work are reasonable.

Chapter 4: Dynamic Programming Approaches to the Individual Forest Stand Optimization Problem

The problem statement and the context of this thesis were presented in chapter 1. Stochastic dynamic programming was established as the method of choice for optimizing mixed-species forest stand management policies when facing uncertain outcomes. This chapter discusses DP as it applies in this research. Specifically, the forest stand management problem has a large dimensional state space with a mix of discrete and continuous state variables. Because the cost and profit functions and the dynamics are complex, no analytical solution is possible. Thus we resort to numerical methods. We would like to understand infinite horizon policies. The method we have chosen to explore this is value iteration. However, since some of our state variables are continuous, we cannot examine all states in attempting to create the cost-to-go function. The approach we are using is to attempt to calculate the cost-to-go function at a given stage of the algorithm at a finite set of state points and then attempt to interpolate or approximate these cost-to-go values with a continuous function on the continuous portion of the state space.

The focus in this chapter is on the critical issues related to developing a DP model in the context of individual forest stand management: stochastic processes, cost-to-go function approximations, policy simulation, rate of convergence and termination criteria for the value iteration DP algorithm, and choosing appropriate values for the discount factors. But first, the DP problem is presented. The notation and presentation format used in this chapter is borrowed from Bertsekas (2000).

4.1 Dynamic Programming Basics

The combination of continuous variables such as age, d^{θ}, pct^{θ}, cc and market prices is referred to as state x_k which is an element of space S_k. The evolution of the state is influenced by decisions u_k made at discrete instances of time and by random disturbances w_k that are observed after the decisions have been made. This system has the form:

$$x_{k+1} = f_k(x_k, u_k, w_k) \quad k = 0, 1, \ldots, \text{N-1} \tag{4-1}$$

where k indexes discrete time and is referred to as periods

x_k is a combination of the state of the forest stand F_k which is dependent upon past management of the stand, and the state of the market prices of wood products M_k which is independent of F_k ($x_k \epsilon\, S_k$)

u_k is the decision variable or control to be selected at period k $\left(u_k \epsilon\, U_k(x_k)\right)$

w_k is a random parameter that depends on x_k and u_k ($w_k \epsilon\, D_k$)

N is the number of time periods

f_k is a set of functions that describes the system and how x_k (F_k and M_k) transitions occur in response to u_k and w_k.

The control u_k is constrained to take values in a given nonempty set $U_k(x_k)$ which depends on the current state x_k; that is, $u_k \epsilon\, U_k(x_k)$ for all $x_k \epsilon\, S_k$ and k. S_k is the set of feasible states. It is a combination of continuous and discrete variables and will be discussed in greater detail later. The random disturbance w_k is characterized by a probability distribution P(• | x_k, u_k). Disturbances that may affect state transitions include natural disasters, uncertain regeneration after a regeneration harvest and random market prices.

The yield equations presented in chapter 2 are used to calculate expected revenue by combining volume of wood products removed from the stand, after taking action u_k in forest state F_k, with random market prices M_k. The cost of obtaining the wood products is subtracted from the expected revenue to yield expected profit. The expected profit is additive in the sense that the expected profit gained at period k, denoted by $g_k(x_k, u_k, w_k)$, accumulates over time. The total expected profit is given by:

$$E\left\{g_N(x_N) + \sum_{k=0}^{N-1} g_k\,(x_k, u_k, w_k)\right\}$$

where $g_N(x_N)$ is the terminal profit gained from the forest and market states at the end of the planning horizon and where the expectation is with respect to the joint probability of the random disturbances such as natural disasters, market fluctuations and uncertain regeneration which will be discussed in detail in this chapter. Therefore, a problem can be formulated as the optimization of the expected total profit where the optimization is done over the set of controls $(u_0, u_1, \ldots, u_{N-1})$ and each control is selected with full knowledge of x_k. The literature refers to g_k as the cost function therefore we will refer to g_k as the cost function in this thesis to retain consistency even though the objective is maximization of profit.

We are interested in closed-loop forest stand optimization which needs to find a set of functions $\mu_k, k = 0, \ldots, N$-1 that maps state x_k into management decisions u_k so as to maximize profit. The meaning of μ_k is that, for each k and each possible state x_k,

u_k = $\mu_k(x_k)$ = the management option that should be applied to the state at period k if the state is x_k according to the set of functions μ_k

and is such that $\mu_k(x_k) \epsilon\, U_k(x_k)$ for all $x_k \epsilon\, S_k$. The sequence of functions $\pi = (\mu_k, \ldots, \mu_{N-1})$ is referred to as a policy or control law. Given an initial state x_0, an admissible policy π and functions g_k, k = 0, 1, …, N, we can write

$$J_\pi(x_0) = E\left\{g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, \mu_k(x_k), w_k)\right\}$$

In the DP literature, $J_\pi(x_0)$ is generally referred to as the cost function or value function with respect to state x_0 for policy π. Here, it is referred to as the cost function to retain consistency even though g_k is a profit. An optimal policy π^* is one that maximizes the cost function

$$J_{\pi^*}(x_0) = \max_{\pi \epsilon \Pi} J_\pi(x_0)$$

where Π is the set of all admissible policies. Note that π^* is associated with x_0 so the optimal cost depends on x_0.

$$J^*(x_0) = \max_{\pi \epsilon \Pi} J_\pi(x_0) \tag{4-2}$$

J^* assigns to each initial state x_0 the optimal cost $J^*(x_0)$ and is referred to as the optimal cost function. Equation (4-2) is referred to as the basic problem.

DP rests on a very simple idea, the principle of optimality (Bellman 1957). The principle of optimality states the following fact:

> Let $\pi^* = \{\mu_0^*, \mu_1^*, \dots, \mu_{N-1}^*\}$ be an optimal policy for the basic problem in equation (4-2). Assume that, when using π^*, a given forest stand and market state x_τ occurs at period τ. Consider the sub-problem whereby we are at x_τ at period τ and wish to maximize the "cost-to-go" function from period τ to period N
>
> $$E\left\{g_N(x_N) + \sum_{k=\tau}^{N-1} g_k(x_k, \mu_k(x_k), w_k)\right\}$$
>
> Then the truncated policy $\{\mu_\tau^*, \mu_{\tau+1}^*, \dots, \mu_{N-1}^*\}$ is optimal for this sub-problem.

To see this, if the truncated policy wasn't optimal, the policy for the basic problem could be modified by using the optimal truncated policy from period τ onwards, contradicting the assumption of optimality for the basic problem. The above stated fact is intuitive but, nevertheless, fundamental to the DP algorithm stated below.

> For every initial state x_0, the optimal cost-to-go $J^*(x_0)$ of the basic problem is equal to $J_0(x_0)$, given by the last step of the following algorithm, which proceeds backwards in time from period $N-1$ to period 0
>
> $$J_N(x_N) = g_N(x_N) \tag{4-3}$$

$$J_k(x_k) = \max_{u_k \epsilon U_k(x_k)} \left[\mathop{\mathrm{E}}_{w_k} \{ g_k(x_k, u_k, w_k) + \alpha J_{k+1}\big(f_k(x_k, u_k, w_k)\big) \} \right]$$

$$k = 0,1,\ldots,N\text{-}1 \qquad (4\text{-}4)$$

where the expectation is taken with respect to the probability distribution of w_k, which depends on x_k and u_k, and α is the discount factor. Furthermore, if $u_k^* = \mu_k^*(x_k)$ maximizes the right side of eq. (4-4) for each x_k and k, the policy $\pi^* = \{\mu_0^*, \mu_1^*, \ldots, \mu_{N-1}^*\}$ is optimal. For a mathematical proof of this proposition, see (Bertsekas 2000).

The formulation above is strictly a finite horizon framework which has applications in numerous fields. Solution methods for this algorithm are well covered in (Bertsekas 2001). We now formulate the basic problem as an infinite horizon discounted problem. The discount rate α is discussed in detail in section 4.3. Infinite horizon DP problems can be formulated many ways. Several of them are presented in Bertsekas (2001). Two reasons support the use of the formulation used in this thesis: the profit per stage is finite and discounting leads to the development of optimal policies that are useful in forest stand management.

In the infinite horizon framework, by letting $N \to \infty$, there is essentially no difference in being in a given state at stage k or stage $k + 1$. In this framework, it makes sense to talk about a stationary policy of the form $\pi = \{\mu, \mu, \ldots\}$ that depends only on the state and not the period. For every stationary policy μ, the associated cost function satisfies

$$J_\mu(x) = \mathop{E}_{w}\{g(x, \mu(x), w) + \alpha J_\mu\big(f(x, \mu(x), w)\big)\} \text{ for all } x \in S$$

We are looking for a stationary policy μ which we state as being optimal if $J_\mu(x) = J^*(x)$ for all states x. The DP algorithm in equations 4-3 and 4-4 can be restated in an infinite horizon discounted form as follows:

$$J_k(x_k) = \max_{u_k \epsilon U_k(x_k)} \left[\operatorname*{E}_{w_k} \left\{ g_k(x_k, u_k, w_k) + \alpha J_{k+1}\left(f_k(x_k, u_k, w_k)\right) \right\} \right] \quad (4\text{-}5)$$

The formulation in equation (4-5) is used when probabilities are realized after the decision is made. Were it the other way around, the DP algorithm would have the form $J_k = \mathrm{E}[\max\,(g_k + \alpha J_{k+1})]$ which is not used in this work. Because the variables used to describe the state of the individual forest stand are mostly continuous and g_k and f_k are complicated, no analytical solution is possible and we resort to numerical methods. In order to solve the DP algorithm with numerical methods and find the optimal stationary policy, the state space must be discretized in some way. For a complex state space, numerical execution of the DP algorithm may be time consuming but DP is the only general approach for sequential optimization under uncertainty (Bertsekas 2000).

Assuming g_k is the bounded profit per stage, we will discuss two numerical approaches to solving the basic problem in its infinite horizon form. The first approach, value iteration, is essentially the DP algorithm for a finite number of periods N with N allowed to increase. In the limit, this DP yields the optimal profit function and optimal policy. The second approach, policy iteration, generates a sequence of stationary policies, each with improved profit over the preceding one. With a finite number of states and controls, policy iteration converges in a finite number of iterations but when the number of states and controls is large, solving the linear system in the policy evaluation step of the policy iteration approach can be time consuming. Value iteration is the only option investigated for solving the DP algorithm in this thesis and it presents some challenges.

For the value iteration approach, we have opted to evaluate the CTG function at a finite number of discrete states that are chosen to reasonably represent the continuous state space S where the subscript k is dropped because the state space is independent of time. S is divided into five subsets S_n, n = 1,…,5, based on the treatment types discussed in chapter 2. The discrete states i chosen to represent S_n are referred to as evaluation states and they form a finite set S_n^{Eval} ($i \in S_n^{Eval}$). Taking action u in state i leads to state j, and the set of all states j is referred to as S_n^{Result} where $S_n^{Eval} \neq S_n^{Result}$ although there may be states that appear in both sets. The states in each subset are defined by a set of variables that adequately represent the complexity of the stands in their respective subset and are shown in table 4-1. The number of discrete evaluation states in each subset is given in brackets after the subset indicator for the base case scenario. The higher number of discrete evaluation states in subsets 2 and 5 reflect the elevated level of variability and complexity within those particular subsets. An example list of states in S_n^{Eval} is given in appendix C.

Treatment type 1, unmanaged natural stands, becomes S_1^{Eval} and these states are defined by age and stocking. The data collected to create the growth equations was collected at 5 year intervals and the maximum MAI for a typical stand in Nova Scotia occurs at well below 100 years of age so discrete states are created to cover the range from 0 to 95 years. Stocking was defined in chapter 2 and we have chosen to study natural stands with stocking values of 75% and 100%. Given 19 ages, each with two stocking percentages, we get 38 states. The base case scenario is described at the beginning of chapter 5.

Table 4-1 – Description of the evaluation states for the 5 subsets.

Subset	Variable	Min. value	Max. value	Explanation
S_1 *(38)*	ha *(years)*	0	95	Max MAI for typical stands in N.S. is reached between 50 and 80 years – MAI for all stands is on the decline at age 95
	st	75%	100%	Forest stands are on homogenous parcels of land therefore stands are normally chosen to not include non-productive land (stocking can be less than 100%)
S_2 *(275)*	ha *(years)*	5	95	Same explanation as S_1 for maximum value. NS DNR recommends a minimum height for pre-commercial thinning which is reflected by the minimum age
	d^S*(cm)*	5.3	30.8	d^S is a function of ha (minimum ha for softwood is 15 years), pct^S and cc
	d^H*(cm)*	1.1	20.7	d^H is a function of ha (minimum ha for hardwood is 5 years), pct^S and cc
	pct^S*(%)*	0	100	Stands range from 100% SW to 100% HW
	cc *(%)*	5.7	100	Recommended spacing for a pre-commercial thinning in N.S. is between 2.1m and 3.0m depending on the species – this yields a very low cc in young stands

Table 4-1 Continued…

Subset	Variable	Min. value	Max. value	Explanation
S_3 *(76)*	ha *(years)*	5	95	Same explanation as S_1 for maximum value. A plantation is guaranteed to produce growing stock in the first period after planting therefore a plantation has a minimum age of 5 years (one period).
	d^S *(cm)*	0	28.1	Minimum d^S is zero because the growing stock on a plantation stand has not reached breast height at 5 years. The maximum depends on ha and the *initial planting density*
	Initial planting density	1000	4000	These minimum and maximum values are well below and well above current practice in N.S. and serve the purpose of studying other management possibilities
S_4 *(90)*	ha *(years)*	10	100	Same explanation as S_1 for maximum value. The minimum age is slightly below the age that would yield the minimum recommended diameter for CT according to NS DNR.
	d^S *(cm)*	3.9	33.4	d^S is a function of ha and cc
	cc *(%)*	40	100	This range reflects the NS DNR maximum recommended removal percentage for CT
S_5 *(648)*	ha *(years)*	25	105	Same explanation as S_1 for maximum value. NS DNR recommends a minimum diameter for commercial thinning which is reflected by the minimum age
	d^S *(cm)*	8.5	37.2	d^S is a function of ha, pct^S and cc
	d^H *(cm)*	6.8	28.3	d^H is a function of ha, pct^S and cc
	pct^S *(%)*	0	100	Stands range from 100% SW to 100% HW
	cc *(%)*	40	100	This range reflects the NS DNR maximum recommended removal percentage for CT

The control space is also continuous which we have chosen to approximate by choosing a finite set of discrete controls. At each iteration of the DP algorithm, the expected CTG must be calculated for each evaluation state i by optimizing over the controls u in $U(i)$, for each i in each subset $S_n^{Eval}, n = 1,\ldots,5$. The discrete set of controls $U(i)$ must be kept small so the number of calculations required at each iteration doesn't get out of hand.

As discussed in chapter 2, some actions will result in a state transition between two subsets therefore the discrete set of evaluation states within each subset must be chosen carefully. Figure 4-1 illustrates all possible state transitions between the 5 subsets. The arrows indicate possible transitions which depend on u_k. For example, a stand in subset S_2 can go to subset S_5 if a CT is applied to it but it can't go back to subset S_2 without first getting a regeneration harvest which takes it to subset S_1 and then being pre-commercially thinned. Subsets S_1 and S_3 can be reached from any subset by doing a regeneration harvest and either by letting the stand naturally regenerate (S_1) or by planting trees (S_3). Therefore, figure 4-1 indicates all possible single-stage state transitions for the five subsets.

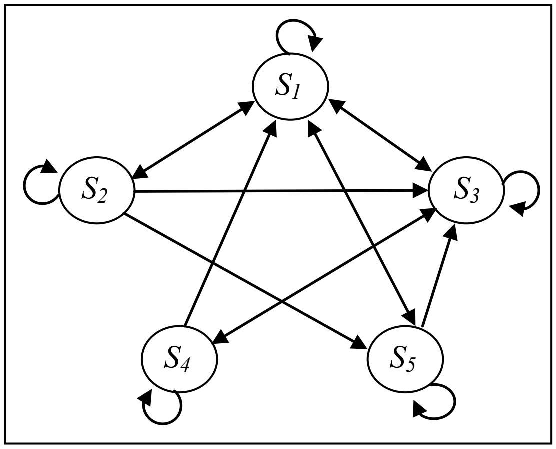


Figure 4-1 – Representation of single-stage state transitions for subsets $S_1, \ldots S_5$

As observed by Gunn (2005):

> "An interesting dichotomy exists in DP models in forest stand management. On the one hand, we find models with complex state variables but deterministic dynamics and prices. On the other, we find models with stochastic prices and/or dynamics but very simple state representation. We generally don't find both."

The general interest of this work is to build models that yield optimal policies with the complex state space and dynamics discussed above. Many studies use a small finite set of discrete values of one or two continuous state variables where the set of discrete values is chosen to best represent the range of values most likely to occur for each state variable. Most of them have a single state descriptor such as age (Brodie and Kao 1979), volume (Schreuder 1971) or basal area (Chen et al. 1980).

Many studies in forestry have focussed on simple state descriptions and, in addition, all state transitions are deterministic and the set of controls is limited to a few discrete control options. With deterministic state transitions, forward DP recursions can be used and the reader is directed to these studies as examples of this approach (Amidon and Akin (1968), Brodie and Kao (1979), Haight et al. (1985), Arthaud (1986), Pelkki (1997), Pelkki and Arthaud (1997), Pelkki and Kirillova (2004)). These papers have a common thread: they seek to develop optimal thinning strategies, optimal growing stock and/or optimal rotation ages in a deterministic setting. Because these researchers were only considering deterministic problems, the use of a forward formulation made it easier to search for an optimal network path by reducing the number of states to consider.

Although forward recursions are not used in this research, an important concept coined as "neighborhood storage" by Brodie and Kao (1979) is a good precursor to our discussion of approximate DP. Brodie et al. (1978) acknowledge that the continuous nature of growth models may dictate a growth increment that leads to a value for a state variable that isn't included in the finite set of discrete evaluation states, which they call nodes, chosen to represent the continuous variable in the model. The dotted lines in figure 4-2 illustrate period to period transitions that don't take state i_3 to one of the discrete states in the finite set S^{Eval} at period $k+1$.

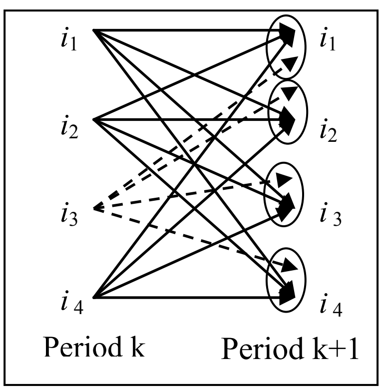


Figure 4-2 – State transition illustration

The four decisions $u \epsilon U(i_3)$ applicable to state i_3 result in a transition to states that aren't part of S^{Eval}. The ovals in figure 4-2 represent neighbourhoods. In order to gain precision, Brodie et al. (1978) want to eliminate the need to round values of state variables computed from continuous functions to the nearest node. When solving the DP algorithm, the CTG function values for all state transitions that take the state to a given neighbourhood are compared together and the best is kept while the others are discarded.

The authors acknowledge that this process may eliminate network paths that may become optimal in later stages of the algorithm but state that this effect is minor if the neighbourhoods are small. However, smaller neighbourhoods mean more variables and the problem of a high number of nodes and of computational intensity remains.

This discussion of forward recursion is used to illustrate the challenge of numerically solving DP problems with continuous variables. Success of the "neighbourhood storage location" concept requires consistency (Bertsekas 2000) as does any approach that attempts to find good approximations to the actual CTG functions.

The approach taken in this thesis is to calculate J_k for all evaluation states i in each subset S_n and then fit a continuous function on these values, one for each subset. Fitting these functions has proved very challenging. The three function fitting approaches, or approximation architectures, used are described in detail in section 4.4. These continuous functions are used at the next iteration of the DP algorithm to approximate the CTG function value for the continuous portion of the state space between evaluation states. The elements of S^{Result} are part of the continuous portion of the state space between evaluation states. This process is repeated at each iteration of the DP algorithm. The detailed steps of the value iteration procedure are described in section 4.6.

The following section describes the stochastic processes associated with the random disturbances w_k.

4.2 Stochastic Processes

For discussion purposes, we recall the system equation $x_{k+1} = f_k(x_k, u_k, w_k)$. The random disturbances $w_k \epsilon D_k$, $k = 0,1, \ldots$ are characterized by probabilities $P(\cdot\,|x_k, u_k)$ where $P(w_k|x_k, u_k)$ is the probability of occurrence of w_k, when the current state is x_k and the current action is u_k. In this thesis, D_k are finite sets. We also recall that x_k has a forest state component F_k and a market state component M_k.

As discussed earlier, when using DP to find optimal management policies for individual forest stand modeling, we have a system with a finite number of evaluation states. For each evaluation state, we have a finite number of decisions and, for each decision, a finite number of stochastic outcomes. Thus, there are a finite number of result states j that we can reach from a given evaluation state i. Therefore, the random disturbances in this system can be represented in terms of their probabilities of transition between states. In other words, we need to know $p_{ij}(u,k)$ which is the conditional probability, at period k, that the next state will be j given that the current state is i and control u is chosen. It is given by

$$p_{ij}(u,k) = P\{x_{k+1} = j | x_k = i, u_k = u\}.$$

If we assume that the system is stationary, that is the transition possibilities, the profit per stage and control sets do not change from stage to stage, then the transition probabilities can be defined by

$$p_{ij}(u) = P\{j | i, u\}.$$

Using this definition of the transition probabilities and denoting by $g(i,u)$ the expected profit per stage at evaluation state i when control u is applied, where the random process w is implicit based on the definitions above, the infinite horizon DP algorithm given by equation (4-5) can be re-written in discrete form as

$$J_k(i) = \max_{u \epsilon U(i)} \left\{ \sum_j p_{ij}(u) \left[g(i,u) + \alpha J_{k+1}(j) \right] \right\} \tag{4-6}$$

In order to compute $J_k(i)$, the transition probabilities $p_{ij}(u)$ must be further discussed. There are many ways for stochastic processes to enter into forest stand management. In this research, two processes in particular are discussed: the state of the market (M) and the state of the forest stand (F), and these two processes are assumed to be independent of each other. Lembersky and Johnson (1975) described it best:

> "The joint distribution of these processes is the product of an action dependent forest stand transition probability and an action independent market transition probability."

If we express $p_{ij}(u)$ in terms of these two independent stochastic processes, we can rewrite the transition probabilities as a product of the two processes:

$$p_{ij}(u) = P(M_{k+1}|M_k) \times P(F_{k+1}|F_k, u) \tag{4-7}$$

where F_k and M_k are components of evaluation state i and F_{k+1} and M_{k+1} are components of result state j. The nature of the stochastic processes in equation (4-7) is such that any future state is independent of any past state and only depends on the present state. Therefore, the expected CTG value for any result state j is the expected future profit created from choosing a control while being faced with random state transitions. The next two sections discuss in detail the two stochastic processes in equation (4-7).

4.2.1 Market State Dynamics

A review of the literature in this area has yielded a number of studies where the prices are correlated between time periods and a similar number where they are not. Whether the prices between periods are correlated or not will affect the type of DP model used and the policies that are derived from it. Haight and Holmes (1991) is a good example of this effect. Stationary price series can be modeled using Markov Decision Processes with an infinite horizon DP optimization. For non-stationary prices, finite horizon DP with prices modeled as an autoregressive or Brownian motion process are necessary. Because the focus is on the use of infinite horizon models, and because the growth equations are based on five year growth increments, the 5 year joint discrete probability mass function $P(M_{k+1}|M_k)$ for the prices of the main products in this study will be modelled using Markov processes.

Of great significance is the timing of the decision relative to observing the market prices for the products. The timing of the decision, before or after the market prices have been observed, changes the structure of the DP algorithm and likely the resulting policy.

As stated earlier, in all cases, we assume prices are independent of the forest state and the decision taken in the previous stage. Very few studies consider making a decision before observing market prices because it takes flexibility away from the decision maker. When the decision is put off until after the market state and forest state are observed, the additional information can lead to higher returns.

Yoshimoto and Shoji (1998) consider the case where the decision maker isn't given the opportunity of making a decision with full knowledge of the current market state. Plantinga (1998) also considers the cases where a decision is made with and without observing the market before hand. The paper goes on to explain the differences between the maximization problems for each of these cases. In each case, an option price is calculated. The option price indicates the price above which harvest is done in the current period and below which it is delayed until the next period. The study by Plantinga highlights the importance of considering the timing of the decision.

The two studies referenced above highlight the fact that using pricing information to make better decisions is much more desirable. There is an extensive literature in the area of price modeling of commodities and this literature is filled with studies where the decision maker is allowed to observe the market state before making a decision. These studies span every decade since the 1970's and table 4-2 shows a list of papers in forestry that consider stochastic prices with stationary random processes where prices are independent of previous price information and states the authors' reason for using them.

Table 4-2 – Studies that use stationary random price models and the reasons stated for their use

Studies	Reason for use
Brazee and Mendelsohn (1988)	Review of data has shown it is an appropriate model
Haight (1990)	Difficult to obtain data
Haight and Smith (1991)	Difficult to obtain data
Forboseh et al. (1996)	Complexity of autoregressive and non-stationary price models
Forboseh and Pickens (1996)	Assumption made without justification
Lu and Gong (2003)	Relatively long decision intervals

This table doesn't present a comprehensive review of papers in the area but rather a sampling of the work that has been done in this area over the last three decades. The objective of this study is not to undertake an extensive review of price models and to model all of them but rather to develop a methodology with sufficient flexibility to allow for the implementation of a variety of price models. As our focus is on infinite horizon models, a stationary price model is used. Prices are based on the Normal distribution where, at period $k + 1$, prices that are closer to the mean of the distribution, have a higher probability of occurrence. The prices and probabilities used in this study are presented in section 5.1. In addition, the DP algorithm is structured so that the decision is made with full knowledge of the current forest and market states.

4.2.2 Growth and Yield Dynamics

Although the literature on stochastic prices exists in abundance, the uncertainty in the growth and yield of individual forest stands hasn't garnered nearly as much attention. The stochastic growth literature can be divided into two main groups: discrete and continuous. As discussed earlier, in forest stand management, discrete evaluation states are chosen to numerically solve the DP recursion. Therefore, the review presented here will concentrate on stochastic growth in the discrete context.

In the discrete context, most studies have generally concentrated on developing a small number of states in order to simplify numerical calculations and simply demonstrate the optimization approach. Lembersky and Johnson (1975) defined 48 stand indicants which were combinations of diameter at breast height (d) and number of trees per hectare. They used a Markov decision process algorithm to solve the model for an optimal policy which "assigns an action to every possible combination of observed states and periods". In order to develop their state transition probabilities, the equivalent of $P(F_{k+1}|F_k, u)$, the authors relied on data from an experimental forest. The authors do a wonderful job of developing and solving the model but the size of their model, and therefore the number of states, is limited by the computer processing power available at the time. Carlsson (1992) has access to much more computing power but still uses state variables that don't adequately model the complexity of a forest stand. By limiting the state variables to stand age and volume of the two species in his study, his model lacks information critical to decision making at a level of detail required here. Although the state space is restricted as is acknowledged by the author, the state transition matrices for price and stand states are well stated.

Buongiorno authors or co-authors three papers which use Markov decision process models to develop optimal stand management policies (Rollin et al. (2005), Buongiorno (2001), Buongiorno and Zhou (2005)). In the first study, only 6 stand states are used and the resulting management policy is well defined for the 6 states. Unfortunately, a management policy based on only 6 states does not contain enough information to represent growth processes in the detail required in this study. The other two papers expand on the state space somewhat by having two species of trees, three product classes per species and two levels for each class. The result is 64 stand states with the accompanying transition probability matrices for all state combinations (64 x 64 = 4096) but the resulting state space is still very sparse.

All of the papers listed above have taken the same approach to stochastic growth. They establish a list of states and define matrices that give transition probabilities between stand states. A similar approach is used in this study but the continuous nature of forest stand variables requires a slight modification. In the case of natural disasters, the first source of uncertainty, the transition probability matrix is replaced by a set of transition probability rules, or regression equations, that are continuous. These transition probability rules are explained in section 2.9. The second source of uncertainty, regeneration uncertainty after a regeneration harvest or natural disaster, is represented by the probabilities described in section 2.5. A third source of uncertainty, natural variation in the growth of a forest stand, is implicit in the regression equations used to calculate its growth therefore it doesn't need to be incorporated into the transition probability rule or represented by a Markov process. $P(F_{k+1}|F_k, u)$ is the joint probability of the first two sources of GNY uncertainty stated above. The first source, natural disasters, needs to be considered every time an action is taken and a state transition occurs from an evaluation state to a result state. There is always a small possibility that any stand will succumb to a natural disaster and be sent to a regeneration state. The second source of uncertainty only applies when the stand is in a regeneration state.

Given the timing of the decision relative to observing the forest and market states, the profit function $g(i,u)$ becomes a deterministic value and the DP algorithm in equation (4-6) is restated as

$$J_k(i) = \max_{u \in U(i)} \left\{ g(i,u) + \alpha \sum_j p_{ij}(u) \left[J_{k+1}(j) \right] \right\} \tag{4-8}$$

where i is in S^{Eval} and the j are in $S^{Results}$. In section 4.4, we will discuss how this equation is modified in order to account for the continuous state space.

4.3 Setting the Value of the Discount Rate

The choice of value of the discount rate heavily influences the outcome of any optimization attempt (Bateman and Lovett (2000), Meilby et al. (2001), Ward et al. (2004)) yet there is no consensus in the literature on how the discount rate should be chosen or even what that rate represents in real world applications.

Davis and Johnson (1987) propose that the discount rate used in forest management is the equivalent of the guiding interest rate used by government, corporations and individuals when making important economic decisions. The rest of the discussion in this section will be closely based on the terminology in Davis and Johnson's text. The guiding rate can be described as the highest return on investment that could be obtained if the money being invested in a forestry project was invested in other ventures. This guiding rate includes three elements: pure rate, inflation rate and risk rate. The pure rate is the rate of using money over time such as bank interest rate or long term interest rate that could be obtained in another investment. The inflation rate is the rate at which the value of money or buying power decreases on a yearly basis. The risk rate is an indication of the uncertainty of the future. The higher the risk of the investment in forestry, the higher is the risk rate. If an individual or corporation views the future return on a forestry investment as uncertain with a relatively high risk of natural disaster or market uncertainty for example, a higher risk component will be added to the guiding rate in order to give more importance to immediate or short term returns and less to long term returns. The sum of the three rates gives the guiding rate. A high guiding rate means greater discounting of future returns to the present. This puts more emphasis on immediate satisfaction and less on future gains. The discount rate is therefore, in a sense, an indication of time preference of money.

Of course, governments, corporations and individuals have different guiding rates. Individual land owners are normally not interested in long term investments which they see as being risky. Thus, they have a high risk rate which translates to a high guiding rate. A forestry investment in their case will likely be short lived and the proceeds from harvesting will be used to help pay for other investments such as children's education, retirement funds or house renovations. Large corporations normally have a lower guiding rate as they have capital for long term investment. Some corporations don't mind long term investment as long as the risk is relatively low. Thus their guiding rate will be low as long as the upper management of the corporation sees investment in forestry as a low risk. Government will normally have very low guiding rates as they have provincial or national responsibility. They will normally have low pure rates such as government bonds and low risk rates that lead to long term low risk forest production on crown land.

Samuelson (1976) asks: "What interest rate is appropriate for forestry?". This paper by Samuelson has, arguably, been the most influential paper in the forest economics literature. However, in his paper, Samuelson states that he does not want to pronounce himself on the exact value to be used for discount rates in forestry. What he does do is give a thorough description of the questions that need to be considered when deciding the value of the discount factor. Samuelson states that one of the things we need in order to discuss forest economics correctly is to concentrate on the real rate of interest or the actual interest rate on money minus the presumed known rate of overall price inflation. Prices used in the model must be free of inflation in order to use an inflation-free discount rate. But there are other complications. Marginal tax rates, capital-gains tax treatments and income tax laws modify the real taxation rate to be applied to economic calculations. Much work must be done by individuals, corporations and governments to develop a reasonable valuation of the real rate of interest that incorporates all of these elements.

Samuelson discusses at length the impact of the chosen policies on long term sustainability of the forest. Because government controls harvesting on a major portion of land in our country, the policies implemented by the government will have a large effect on long term sustainability of our forests. These policies are a direct result of the discount rate used in the analysis of forestry models. How much wealth or forests should be used to lower the federal debt that is passed on to the future generations and how much should be left for future generations to enjoy? This is, essentially, a social question and the discount rate used in both cases will be different. Historically, governments have owned much timber land and have, for the most part, managed it conservatively. Were it not for this, there would be much less forests in the North America today. Samuelson states that "were the government to rent out public land to the private lumber companies at the maximum economic return competition will establish, this is a sure prescription for future chopping down of trees". Essentially, the only certain conclusion from this work is that the discount rate used in forestry economics must be chosen to represent the economic situation of the decision maker doing the study. There is no value that fits every situation. Essentially, the management policy that results from the discount rate being used should be one with which the decision maker is comfortable.

To the contrary, Pearse (1967) argues that private companies need to use a discount rate equivalent to the rate of interest that could be obtained if they directed it to its highest alternative use. He goes on to say that if this isn't the case "society as a whole suffers from the misallocation of economic resources". In a purely economic sense, Pearse may be right but he does not discuss the impact these rates would have on the structure or age distribution of the forest. High discount rates are going to create short rotation periods or young rotation ages. Young rotation ages can't be employed perpetually by all stakeholders in forestry without destroying all old growth forests and reducing all forests to young stands that offer little habitat to a lot of wildlife. This is one of the reasons that discount rates are often discussed in the context of social responsibility. Is it socially responsible to use high discount rates when we know what the effects could be on the structure of the forest and the habitat of wildlife? Paradoxically, is it reasonable to expect corporations to get involved in forestry at a much lower rate of return than they could get elsewhere? Most people agree that we need to continue the exploitation of our most abundant renewable resource. How we go about this isn't as unanimous.

The reasons for choosing discount rates seem to be as numerous as the researchers that are studying their impact on forestry management. The examples given here support the argument that no single rate will fit every situation. Brukas et al. (2001) show results that indicate that the discount rate should depend on the species being studied. With slower growing species such as some hardwoods, a lower discount rate should be used to ensure that future revenues are not discounted too much while the opposite is true for faster growing species. They report discount rates that range between 4% and 8% in the USA and then state that the "opportunity cost of capital should not be that different between various regions of the USA".

De Graaf et al. (2003) study the forest industry in Amazonia. They compare the net present value for different management options using discount rates of 5, 8, 10, 15 and 20%. In one case, there was a 97% reduction in annual value between 5% and 20% discount rates. This study clearly shows the need to properly choose this rate. Ashton et al. (2001) studying forests in Sri Lanka use real rates of 4% and 6%. The higher rate includes a 2% risk rate. Some economists would probably consider this risk rate relatively low considering the short-term concession system and unstable rights to harvest present in Sri Lanka as described by the authors. Hanewinkel (2001) studies forestry in Germany and uses very low discount rates of 1 and 3%. The forests in his study are state owned national forests and the discount rate employed gives a measure of the importance of the time element involved (Davis et al. 2005). Hence, governments, with their provincial and national responsibilities, often use low discount rates because, in general, they have the lowest time preference of any stakeholders in forestry.

Many studies have taken an empirical approach to determining socially acceptable discount rates. In these studies, multiple scenarios were developed based on time and money preferences and respondents were asked to choose which scenarios they would prefer. In some instances, respondents were asked to consider themselves as government agencies and choose what is appropriate for a government to do. Discount rates weren't explicitly stated but they were implied by the answers given by the respondents because the scenarios had been developed with different discount rates. Results vary greatly but generally respect the notions put forward in (Samuelson 1976). Here are some examples of those studies.

Luckert and Adamowicz (1993) ask respondents to choose between scenarios where they are in control of a forest. They are then asked to choose between the same scenarios if the forest is managed by a public resource management agency and that the revenues accrue to the state. These two scenarios are repeated with the resource in question being stocks and bonds. Implied discount rates are inflation free and risk free. For publicly managed goods, more respondents tend to avoid scenarios with high implicit discount rates and to choose scenarios with constant flows of cash with their implied 0% discount rates. Another telling result is that respondents seem to be more likely to choose the high discount rate for stocks and bonds than for forests. Although this study has many shortcomings acknowledged by the authors and the authors caution the reader to read the results with skepticism, the results point toward the fact that individuals, and therefore society, tend to see natural resource management under a different light than other investments. This is an important consideration in this study. Results show that respondents may have an aversion to instability and that the rapidly changing times we are going through may be causing a perception that natural resources are a safer investment than stocks and bonds.

In another study of comparison of scenarios by respondents, Pope and Perry (1989) administer questionnaires to business and natural science students and ask them to pick from 5 alternative income streams based on two scenarios. The first scenario is one where the student receives an endowment which will be passed on to their heirs. The second scenario assumes the endowment is publicly owned and managed. For the privately owned and managed scenario, the authors report that "most respondents preferred a relatively low level of resource depletion at least in terms of its income-producing potential as opposed to business students who, as a group, demonstrated a significantly higher preference for more rapid depletion of the resources, suggesting relatively high discounting of the future". These results show that the point of view of the decision maker can potentially have a major impact on the management decision.

Lumley (1997) conducts a study among farmers in the Philippines that mainly discusses income and borrowing. Philippine society is extremely poor and lending institutions aren't regulated like they are in well developed countries. It is generally accepted that time preference for money is affected by age, education and wealth. Young, educated, rich people tend to have more of what they need at the any given time and tend to better understand that the future may bring good things. Yet, in this study, the poorest farmers who pay the highest interest on their loans which are taken out for food and farming investment have, on all sites studied, lower time preferences than the richest farmers. A relationship seems to exist between soil conservation and income. On all sites but one, the poorest farmers adopted soil conservation in a higher percentage than the richest farmers. Lumley states that we lack the data and knowledge to provide definitive answers to explain this behaviour but that perhaps ethics plays a role in decision making.

In a study by Taylor et al. (2003), researchers measured respondents' preference for timber harvesting or recreation in the Rose Creek Educational Forest in Alberta. The researchers choose several levels of discounting for each of these options and present the respondents with management options without indicating the discounting associated with each option. Respondents appear to be more accepting of uneven flows in recreation services, the level of recreation services available to the public may vary over time, than they are of uneven flows of timber harvests. This may arise because they feel forests provide many ecosystem benefits and thus they are unwilling to deviate significantly from even flow for timber. Generally, results in this study indicate that respondents prefer lower discount rates which translate to lower harvests and more recreation areas as long as the lower harvests are consistent.

Samuelson (1976) aptly concludes that it is unlikely that there is a simple answer to the question "what interest rate is appropriate for forestry?" We use discount rates in forestry essentially to bias current versus future profits. Another view is that we can guard against uncertainty. The computations carried out and discussed in this thesis can be seen as a way of studying these tradeoffs.

4.4 Approximating the CTG Function

The idea of neighbourhood storage locations in forward DP can be viewed as a piecewise constant approximation of the CTG function which has important drawbacks such as those discussed in section 4.1. Nonetheless, the idea of function approximation is a central topic in approximate dynamic programming but it would make sense to use continuous approximations if the state variables are continuous. We now discuss the challenges associated with continuous approximations of the CTG function.

These approximations are done through the use of approximation architectures such as multiple linear regression, radial basis functions and distance weighted interpolation. In particular, we replace $J_{k+1}(j)$ with $\tilde{J}_{k+1}(j,r)$ where r is a vector of parameters that are optimized to minimize the difference between $J_{k+1}(j)$ and $\tilde{J}_{k+1}(j,r)$ at a given set of evaluation states which depend on the approximation architecture being used. We can rewrite the DP algorithm as

$$J_k(i) = \max_{u \epsilon U(i)} \left\{ g(i,u) + \alpha \sum_j p_{ij}(u) \, [\tilde{J}_{k+1}(j,r)] \right\} \tag{4-9}$$

$J_k(i)$ implicitly depends on r but it is omitted in $J_k(i)$ to simplify notation. Depending on the type of approximation architecture being used, the vector of parameters r may be pre-computed once before the optimization begins and remain unchanged for the duration of the optimization, or it may be necessary to recompute r at each iteration of the DP algorithm. In the latter case, the architecture in question uses the values of $J_k(i)$ as inputs in the calculation of r.

Regardless of the approximation architecture being used, calculating r always requires the use of the values of the variables that define the discrete evaluation states. The range of values for each variable is shown in table 4-1. Scaling values of the independent variables when fitting functions on scattered data is a common practice. Here, all values for the variables that define the discrete evaluation states are scaled to take on values between 0 and 1. This is done by dividing the discrete values for each variable by the largest value of that variable in table 4-1. For example, a stand from subset S_2 with age = 40 years would have its age scaled to 0.42 (40/95) where 95 is the maximum age for any discrete evaluation state in S_2. Unless specifically stated otherwise, when referring to the distance in this section, we are referring to this scaled distance.

Approximation architectures can be divided into two broad classes: non-averager methods and averager methods (Gordon 1995). Non-averager methods approximate the CTG function by fitting a function ($\tilde{J}$) at some or all of the discrete evaluation states in each S_n^{Eval}, n=1,…,5, which we refer to as basis points, and minimizing some measure of the error between J and $\tilde{J}$ at those basis points. The quality of the fit is a function of the complexity and spatial distribution of the basis points and the ability of the architecture to approximate the shape of J. Multiple regression and radial basis functions are examples of non-averager methods. Averager methods only use information about the spatial distribution of the basis points in the calculation of the parameter vector r, or what is commonly referred to as the vector of weights. Distance Weighted Interpolation is an example of an averager method. The next two sections discuss these methods in detail.

4.4.1 Non-Averager Methods

Non-averager methods may be further classified as linear or non-linear. A linear architecture is of the general form:

$$\tilde{J}(i,r) = \sum_{m=0}^{M} r_m \varphi_m(i) \tag{4-10}$$

where r_m, $m = 0, \dots, M$, are the components of the parameter vector r, M is the number of elements in r, and φ_m are fixed easily computable functions and are referred to as the basis functions (Bertsekas and Tsitsiklis 1996). The training data pairs $\big(i, J(i)\big)$ are fit using a linear architecture by minimizing

$$\sum_{i}\left(J(i)-\sum_{m}\left(r_m\varphi_m(i)\right)\right)^2$$

over the vector r. Non-linear architectures are not discussed here.

Two non-averager methods are used: multiple regression and radial basis functions (RBF). Multiple regression is well known for its properties of fitting scattered data if the underlying model is known or can be approximated. Therefore, we must assume that we know something about the shape of the function we are trying to approximate. This shape is represented by the terms in the regression model which take the following form $y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \psi$ where X are the regressors, β are the unknown parameters and ψ is a vector of errors in approximation rather than the random errors as in the usual regression model. Each discreet state in S^{Eval} is defined by 2, 3 or 5 variables and linear and non-linear combinations of those variables can be used as regressors. Techniques such as the method of least squares can be used to determine the value of the coefficients of the equation, or vector of parameters, and an analysis of the residuals and the ANOVA information can give some indication of the validity of the model. In matrix form, the multiple regression model can be stated as $y = X\beta + \psi$ where y is the response, X is a matrix of regressors, β is a vector of unknown parameters and ψ is a vector of errors. Using the method of least squares to minimize ψ, we can calculate $\beta = (X'X)^{-1}X'y$. In reference to equation (4-10), $\beta = r$, $\varphi = X$ and $y = \tilde{J}(i,r)$. TRT type 4 stands are described using 3 variables: age, d^S and cc. The proposed regression model for TRT = 4 has nine terms and the choice of the terms is discussed in section 5.2.1. They are: age, d^S, cc, age^2, ${d^S}^2$, cc^2, $age \times d^S$, $age \times cc$, $d^S \times cc$. For this set of terms and for TRT = 4, constructing the X matrix means adding a row to the matrix for every evaluation state for TRT = 4 with the values of the entries in each row equal to the values of the nine terms above calculated using values of the stand variables for each of the evaluation states.

Radial basis functions are typically used to build up function approximations and we need to approximate the cost-to-go function values at states $j \in S^{Result}$. Here, basis function values are calculated at a finite number of states i_m which we call centers and which are chosen from the set of states S^{Eval}. Section 5.2.2 gives examples using two sets of centers. The basis functions are radially symmetric about these centers which means their values depend only on the Euclidian distance from each center to each of the evaluation states, and on the shape of the basis function. Buhmann (2000) states many positive properties of RBF's one of which is its attribute of being universally applicable regardless of the dimension of the problem which makes it an attractive method for forestry where we find multi-dimension continuous state spaces. The interpolant $\tilde{J}(j,r)$ is used to approximate $J(j)$ and is of the form:

$$\tilde{J}(j,r) = \sum_{m=1}^{M} r_m \varphi(|j - i_m|_p) \qquad (4\text{-}11)$$

where i_m are the centers chosen from the set S^{Eval}, j is the discrete result state for which we want to approximate the CTG function, φ is the radial basis function, r_m are reals, M is the number of centers, and p is the dimension of the Euclidian norm. The summation in equation (4-11) is done over all centers. Four of the most mentioned forms of the radial basis function are the thin plate spline $\varphi = \delta^2 \log \delta$, the multiquadric $\varphi = (\delta^2 + \omega^2)^{1/2}$, the inverse multiquadric $\varphi = (\delta^2 + \omega^2)^{-1/2}$, and the Gaussian $\varphi = e^{-\omega\delta^2}$ where ω is a tunable shape parameter (Buhmann (2000), Fasshauer (1997), Rippa (1999)). The first two are probably the best known and most often used versions of the RBF and are the ones used in this research.

In order to calculate the elements r_m and ensure exactness of the method, we will force $\tilde{J} = J(i_n)$ at all states $i_n \epsilon S^{Eval}$ where $J(i_n)$ are the CTG function values at those states. The first step is to create a set of equations, one for each i_n, using equation (4-12).

$$J(i_n) = \sum_{m=1}^{M} r_m \varphi(|i_n - i_m|_2) \text{ for } n = 1 \dots m \qquad (4\text{-}12)$$

We know φ and we use $p = 2$ thus we have M unknowns, the r_m, and M equations giving a linear system $A \times r = f$, where the elements of the $M \times M$ square A matrix are given by $\varphi(|i_n - i_m|_2)$ for $n = 1 \ldots m$, and the M elements of vector f are the values of $J(i_n)$. If A is invertible, we have $r = f \times A^{-1}$ which yields the weights r_m.

We then use equation 4-11 and calculate the M distances between state j and each center i_m using the Euclidian norm. The chosen RBF is applied to each of those M distance values and, when multiplied with its weight r_m and summed for m = 1,…,M, we obtain the approximation $\tilde{J}(j, r)$ for state j. Micchelli (1986) gives general conditions of φ that ensure nonsingularity of A. The two functions chosen for implementation both meet those conditions. Here is an example of the calculation of A where the three states in table 4-3 are taken from S_3^{Eval}. The maximum values for each variable in S_3^{Eval}, used for scaling purposes, are age = 95, d^S = 28.7 and initial planting density = 4000.

Table 4-3 – Three states taken from S_3^{Eval}

age (years)	d^S (cm)	Initial planting density (trees / hectare)
20	11.3	1000
25	13.6	1750
30	15.7	2500

If we use $\varphi = \delta^2 \log \delta$ as a radial basis function where $\delta = |i_n - i_m|_2$, we can create a matrix with 9 elements in a 3x3 layout where the elements on the diagonal of the matrix are all 0 and the first element of the second row of the A matrix (A_{21}) is

$$\delta = \left(\frac{20\text{-}25}{95}\right)^2 + \left(\frac{11.3\text{-}13.6}{28.7}\right)^2 + \left(\frac{1000\text{-}1750}{4000}\right)^2 = 0.044$$

$$A_{21} = 0.044^2 \log 0.044 = \text{-}0.00263$$

All of the elements of A can be calculated in this manner.

The results in chapter 5 show that the matrix inversion discussed above can be challenging when the dimension of A is large. It may be advantages to only use a fraction of the centers to calculate the r_m values, in which case we have what we call a reduced basis RBF which means a reduction in the number of centers through which $\tilde{J}$ is forced to pass. In chapter 5, we investigate this and discuss the effect of reducing the number of centers, if any, on the accuracy of the RBF function approximation architecture.

An important component of the radial basis function is the tunable shape parameter ω. Rippa (1999) discusses several algorithms that may be used to set a good value for ω. In essence, the algorithms minimize the root mean square (RMS) of the difference between the interpolant $\tilde{J}$ and the function J. This process is iterative and the value of ω is influenced primarily by the shape of the function to be approximated. Although there are differences in RMS between different values of ω in Rippa's work, no convincing arguments are given that would justify the extra investment into a complex scheme of determining the optimal value of ω when there are simpler methods (Hardy (1971), Franke (1982)). Section 5.2.2 shows results that demonstrate that fairly simple choices of ω give satisfactory results.

Gordon (1995) shows that convergence isn't guaranteed when using non-averager methods. The main reason is the uncertain behaviour of the approximation architecture between basis points when fitting a cost function with a high dimensional state space. There is no implied upper or lower bound when approximating with non-averager methods. However, in control theory, bounding the response of a system to a given input is standard practice (Farrell and Polycarpou 2006). For example, electric motor control requires algorithms that track the power output of a motor in response to an input such as the accelerator pedal of a vehicle. This is done to avoid overshooting in order to ensure smooth acceleration of the vehicle. A similar concept is applied in this thesis in order to create adaptive upper and lower bounds on the approximation of J at the result states j. They are based on the idea of adaptive approximation based control where the bounds of the approximation are taken from the values of the CTG function in the neighbourhood of the result state for which the CTG is being approximated. The neighbourhood of result state j is made up of evaluation states from the same treatment type. These evaluation states are divided into 2^N subsets where N = number of variables that define j. The evaluation state from each subset which is in closest proximity to result state j is included in its neighbourhood. The following algorithm describes the process of defining the neighbourhood and calculating the bounds.

Step 1: Start with a given result state $j \epsilon S_n^{Result}$ for which we need to define CTG approximation bounds. Set $d(i_m)$ = 0 for m = 1 to M where M = number of states in S_n^{Eval}. Set $d^{min}(POS_a)$ = ∞, for a = 1 to 2^N where N = the number of variables that define state j. Set m = 1 and go to step 2.

Step 2: Retrieve $i_m \epsilon S_n^{Eval}$ from memory where i_m is an evaluation state from the same TRT type as state j. Determine the subset (POS_a) for state i_m in N-space relative to state j. For illustrative purposes, if N = 2, we have 4 possible subsets for state i_m relative to state j as shown in the figure below.

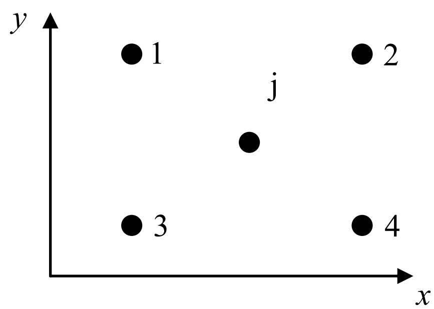


Figure 4-3 – Illustrative example of neighbourhood around state j formed by points 1 to 4

The Cartesian coordinates for points 1 to 4 are $[x_i, y_i]$, i = 1…4. The four possible positions for state i_m relative to state j in terms of its $[x, y]$ coordinates are:

$$POS_1\colon [x_1 < x_j, y_1 > y_j]$$
$$POS_2\colon [x_2 > x_j, y_2 > y_j]$$
$$POS_3\colon [x_3 < x_j, y_3 < y_j]$$
$$POS_4\colon [x_4 > x_j, y_4 < y_j]$$

In the context of this thesis, the coordinates are the values of the variables for states i_m and j. The procedure naturally extends to any value of N. If all 2^N subsets are non-empty, the neighbourhood creates a convex hull which includes result state j. Go to step 3.

Step 3: For each evaluation state in each subset POS_a where a = 1,…, 2^N, calculate the Euclidian distance between states i_m and j using scaled values (0-1) of the state variables:

$$d(i_m) = |i_m - j|_2$$

If $d(i_m) < d^{min}(POS_a)$ where $d^{min}(POS_a)$ is the minimum distance recorded for POS_a, we set $d^{min}(POS_a) = d(i_m)$. $CH(\,)$ is the vector of evaluations states, one from each subset POS_a, that make up the neighbourhood that includes result state j. Set $CH(j, POS_a) = i_m$ and go to step 4.

Step 4: Set $m = m + 1$. If m = M, end procedure. Otherwise, go to step 2.

The steps in the algorithm above are repeated for all result states before the DP optimization begins. At the end of each iteration of the DP algorithm, starting with iteration 1, the following procedure is applied for each result state j.

Step 1: Given a state $j \epsilon S_n^{Result}$, set a = 1 and k = 1. Set $\tilde{J}_{k+1}^U(j) = 0$ and $\tilde{J}_{k+1}^L(j) = LARGE$ where $LARGE \geq$ largest value of $J_k(i)$ for all states $i \epsilon S_n^{Eval}$. Go to step 2.

Step 2: Set $i = CH(j, POS_a)$ and retrieve $J_k(i)$. Go to step 3.

Step 3: If $J_k(i) > \tilde{J}_{k+1}^U(j)$ then $\tilde{J}_{k+1}^U(j) = J_k(i)$. If $J_k(i) < \tilde{J}_{k+1}^L(j)$ then $\tilde{J}_{k+1}^L(j) = J_k(i)$. Go to step 4.

Step 4: If $a = 2^N$ where N is the number of variables that define j, end the procedure. Otherwise, set $a = a + 1$ and go to step 1.

The preceding two algorithms describe how to determine the neighbourhood for result state j and how to calculate CTG function approximation bounds for j. When solving the DP algorithm, the CTG function approximation for state j is adjusted as required according to what follows: if $\tilde{J}_{k+1}(j) > \tilde{J}_{k+1}^U(j)$, then $\tilde{J}_{k+1}(j) = \tilde{J}_{k+1}^U(j)$.

Similarly, if $\tilde{J}_{k+1}(j) < \tilde{J}^L_{k+1}(j)$, then $\tilde{J}_{k+1}(j) = \tilde{J}^L_{k+1}(j)$. Otherwise, $\tilde{J}_{k+1}(j)$ isn't adjusted.

The procedures described above can be used with MR and RBF's. However, as well see in chapter 5, the implementation of the RBF approximation architecture in this work doesn't require the use of CTG approximation bounds. The DP algorithm maximizes $J_k(i)$ at each iteration by choosing the action that yields the largest CTG function value. $\tilde{J}_{k+1}(j)$ is an element of $J_k(i)$ therefore if its value is over or under evaluated, it may have an impact on the optimal policy. Results in section 5.2.1 show some examples of the application of the bounds and the resulting adjustments applied to the approximated CTG function.

4.4.2 Averager Methods

The method proposed in this section is derived from Sheppard (1968). With averager methods, a decision must be made about which evaluation states will be part of the set of states, or basis points, used to approximate the CTG function for result state j. Those states are chosen based on their proximity to j and the set is referred to as M_j. A weight r_{ij} is calculated for each element of M_j and we have

$$\sum_{i \epsilon M_j} r_{ij} = 1$$

In our context, the CTG function approximation is then given by

$$\tilde{J}_{k+1}(j, r) = \sum_{i \epsilon M_j} r_{ij} \left(J_k(i) \right)$$

where r is the vector of parameters used to fit the CTG function, the elements r_{ij} of the parameter vector are generally calculated based on the distance between j and each individual basis point, and $J_k(i)$ are the cost-to-go values for the evaluation states or basis points in M_j from the previous iteration of the DP algorithm. With averager methods, the

CTG function approximation $\tilde{J}_{k+1}(j, r)$ is implicitly bounded above and below because the sum of the weights r_{ij} is always 1.

Distance weighted interpolation (DWI) is the averager method used in this thesis. There are many DWI weighting schemes but the first step in the approximation of the CTG function for result state $j \in S_n^{Result}$ is the calculation, for each j, of the distance between j and all discrete evaluation states $i \in S_n^{Eval}$ using the Euclidean norm $d_{ij} = |i - j|_2$ where the value of each defining variable for states i and j are scaled to a value between 0 and 1 as described in the beginning of this section. The number of elements in $M_j = v$ and doesn't have to be equal to the number of evaluation states in S_n^{Eval}. If v is smaller than the number of states in S_n^{Eval}, the v elements of M_j are chosen based on the distances d_{ij} where closer proximity is preferred. The results in section 5.2.3 support the use of a reduced basis.

Then, v weight factors $weight_{mj}$ are calculated for each result state j as follows:

$$weight_{mj} = \left(\frac{\left(\sum_m d_{mj}\right) - d_{mj}}{\left(\sum_m d_{mj}\right) * d_{mj}} \right)^2 \text{ where } m = 1,\ldots,v$$

and finally, the weights r_{ij} are calculated and used for the approximation.

$$r_{ij} = \frac{weight_{mj}}{\sum_m weight_{mj}} \text{ where } m = 1,\ldots,v$$

The idea is to use a small number of states v to do the approximation in order to reduce the influence of states with much higher or much lower CTG values that are relatively far away from the result state j for which the CTG function value is being approximated. For discussion and comparison, the results in section 5.2.3 use 2, 10 and 40 weights for the approximation. These values were chosen to represent three distinct cases. Using 2 weights means restraining the function approximation to CTG values of the evaluation states in the immediate neighbourhood around the result state being approximated. Using 40 weights allows the approximation to adapt to large fluctuations of the CTG function values at the evaluation states beyond the immediate neighbourhood of the result state being approximated. The impact of using a varying number of weights is discussed in section 5.2.3. One advantage of the averager method is that weights can all be pre-computed before the optimization begins and stored for retrieval at every iteration of the optimization because we can calculate all result states j for all combinations of i and its applicable actions u.

The distance $d_{ij} = |i - j|_2$ is critical to the implementation of DWI as an approximation architecture for DP. In this work, where discrete evaluation and result states are defined by up to five variables that take on a wide range of values, scaling of the distance between points is critical to the success of the method and results in section 5.2.3 support this claim.

4.5 Rate of Convergence and Termination Criteria

Bertsekas and Tsitsiklis (1996) discuss the fact that infinite horizon approximate value iteration can have convergence issues if the approximation architecture doesn't closely represent the intermediate CTG functions obtained in the course of the value iteration algorithm. The presence of implicit bounds in averager methods and calculated bounds in non-averager methods ensures that cost-to-go function approximations on the continuous portion of the state space are consistent with the cost-to-go function values at the evaluation states. Because approximate value iteration additively builds approximations to the CTG function values at the evaluation states, inconsistent cost-to-go function approximations on the continuous portion of the state space could prevent the DP algorithm from converging in a finite number of iterations.

As discussed in Bertsekas (2001), the value iteration algorithm will yield a stable policy before the CTG value converges to the optimal value J^*. Therefore, it is useful to discuss what type of termination criteria should be used in value iteration.

In order to lighten notation in this section, we introduce the transform T which should be viewed as a mapping that transforms the function J on S into the function TJ on S where

$$TJ(i) = \max_{u \epsilon U(i)} \left[g(i,u) + \alpha \sum_{j=1}^{n} p_{ij}(u)[\tilde{J}(j,r)] \right]$$

We know that, in the case where exact values of $J(i)$ can be obtained and for all i,

$$\lim_{k \to \infty} (T^k J)(i) = J^*(i)$$

Furthermore, the error sequence $|T^k J(i) - J^*(i)|$ is bounded by a constant multiple of α^k. The reader is directed to Bertsekas (2001) for the mathematical proof.

Without discussing the details of the proof, it follows that the upper (c_k^U) and lower (c_k^L) bounds on the change of CTG function value for all evaluation states between iterations of the DP algorithm are given by

$$c_k^L = \frac{\alpha}{1-\alpha} \min_{i \epsilon S^{Eval}} [T^k J(i) - T^{k-1} J(i)]$$
$$c_k^U = \frac{\alpha}{1-\alpha} \max_{i \epsilon S^{Eval}} [T^k J(i) - T^{k-1} J(i)]$$

In our DP model, the value iteration algorithm is terminated when $c_k^U - c_k^L \leq \varepsilon$ where ε is the stopping criterion. ε is chosen so that the CTG value of the DP algorithm doesn't converge before the policy has converged and is small enough so that a very good approximation of $J^*(i)$ can be obtained. If $J_\mu(i)$ is the CTG function value for state i and policy μ, then we can write:

$$\max_i \left[J_\mu(i) - J^*(i) \right] \leq \varepsilon$$

Therefore, when $c_k^U - c_k^L$ is sufficiently small, value iteration is terminated and μ is the approximation we obtain for the optimal policy.

In order to verify that the policy obtained from this algorithm indeed yields a CTG value that is near optimal, μ can be simulated to obtain bounds of the CTG function value for the simulated policy given an initial state i. These bounds should contain $J_\mu(i)$ in order to confirm that the simulated μ is in fact a good approximation for μ^*. In section 4.7, policy simulation is discussed in more detail and examples are given.

It is clear that the discount factor, which was discussed in section 4.3, influences the bounds c_k^U and c_k^L. The next section explains how the DP algorithm is implemented.

4.6 Overview of the Computer Implementation of Approximate Dynamic Programming in this Thesis

Sections 4.1 to 4.5 explain how each element of the approximate DP optimization modeling approach works. In some instances, this is done without being specific about the implementation.

In this section, we describe the implementation of the proposed approximate DP algorithm using the value iteration approach. Equation 4-9 is reproduced here for the purpose of describing the steps in developing and solving it.

$$J_k(i) = \max_{u \epsilon U(i)} \left\{ g(i,u) + \alpha \sum_j p_{ij}(u) \, [\tilde{J}_{k+1}(j,r)] \right\} \qquad (4\text{-}12)$$

The steps described below show how the DP algorithm above is implemented using VB and Excel.

<u>Step 1</u> : Launching the optimization

While launching the optimization, the user sets the scenario to be studied by setting values for all variables shown in the screen shot in figure 4-4.

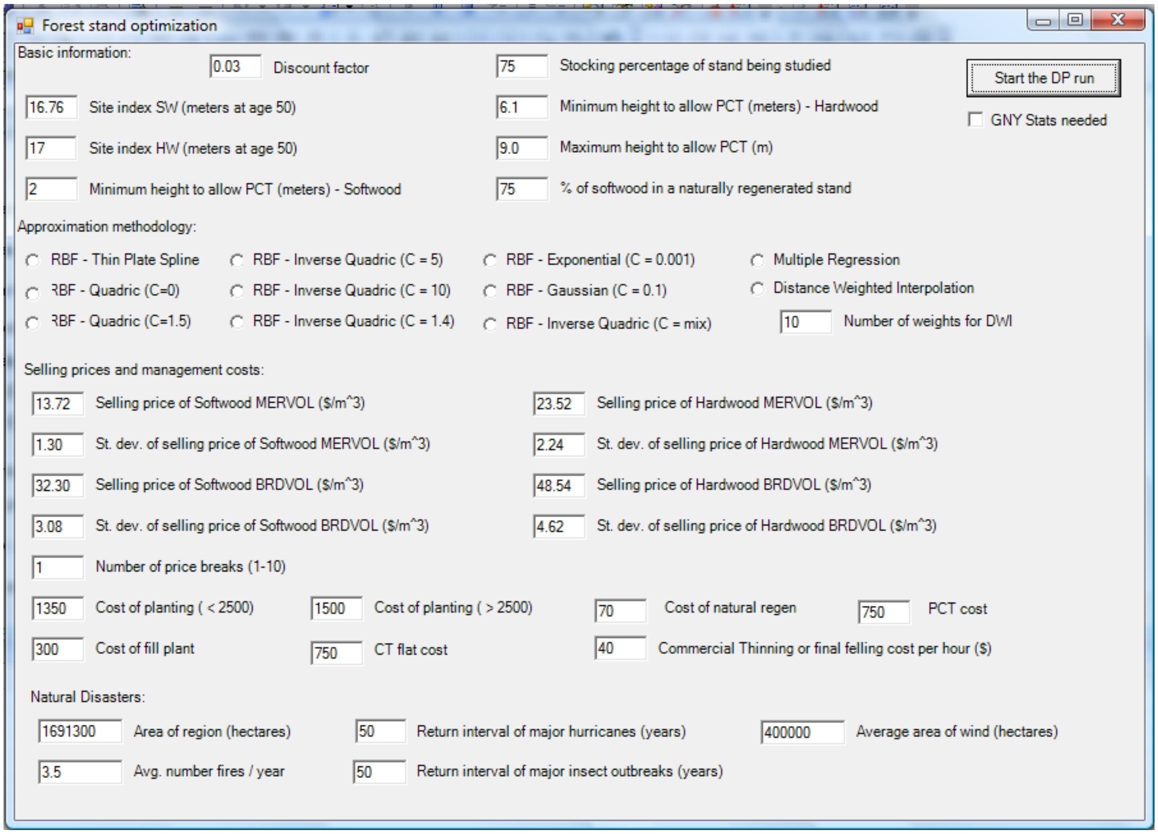


Figure 4-4 – Screen shot of the user interface built using VB

There are four areas in which the user has control over the optimization:

(i) Basic information: The values of these characteristics depend on the stand being studied and the user preferences for managing that stand.

(ii) Approximation architecture: The user has control over which approximation architecture to use for approximating the CTG function.

(iii) Selling prices and management costs: These values depend on the market and on the user's costs for managing its stand. The number of price levels is an indicator of the level of detail preferred by the user.

(iv) Natural disasters: These values depend on long term expected occurances of hurricanes, forest fires and insect infestations.

When the optimization is launched, an Excel spreadsheet is created and is used to store all user data as well as all data and results pertaining to the optimization for the user to see at the end of the optimization.

<u>Step 2</u> : Defining a set of discrete evaluation states and calculating growth and yield

The Nova Scotia Department of Natural Resources publishes guidelines in the form of a Forestry Field Handbook (NSDNR 1993a). Those guidelines recommend how and when each management option may be applied to a forest stand. Therefore, only a specific range of values may exist for any given treatment type. A set of discrete evaluation states has been chosen to give a reasonable representation for each of the 5 treatment types and as a way of covering the state space of the cost-to-go functions. Those states can have the applicable management options applied to them and the results saved for retrieval during the optimization..

In reference to equation 4-12, $g(i, u)$ is the current net profit of taking action u when in evaluation state i which contains forest and market state information. Random disturbances don't have any effect on $g(i, u)$ as all actions u, taken in evaluation state i, result in a specific volume of wood products being removed from the stand as defined by function $g(i, u)$. Furthermore, the decision maker observes the state of the market before making a decision. Therefore profit gained from taking action u when in state i is deterministic. Therefore, that step in the process can be done offline and saved for future retrieval. In addition, $f(i, u, w)$ defines all result states that may occur at the next period subsequent to 5 years of growth after taking action u in state i given disturbances w. This can also be done offline for every $i/u/w$ combination.

<u>Step 3</u> : Offline calculations for function approximation

Many of the calculations for averager and non-averager methods can be done offline before the approximation begins. All preliminary calculations that don't depend on the CTG values are done before the optimization begins and are stored for retrieval at each iteration of the DP algorithm.

For each result state j calculated at step 2, and for each approximation architecture, a set of preliminary calculations, explained below, can be done in order to reduce the time required to do the approximation during the optimization.

MR : We recall the equations presented in section 4.4.1 for calculating the vector of parameters and reproduce one in particular that can be partially computed before the optimization begins: $\beta = (X'X)^{-1}X'y$. The terms in the vector of regressors X are given in section 5.2.1 and an example of how to construct it is given in section 4.4.1. It uses the values of the variables for the evaluation states created in step 2 of this procedure and therefore $(X'X)^{-1}X'$ can be calculated once and stored which will make the calculation of β at each iteration of the optimization more efficient.

RBF : In section 4.4.1, we are working with a linear system of equations $A \times r = f$ where A can be precomputed. The detailed steps are explained in section 4.4.1.
DWI : The weights used to calculate the CTG approximation all depend on the distance between points and not on the CTG values. In this case, the weights are pre-computed using the method described in section 4.4.2.

Step 4 : Calculating CTG values for all discrete evaluation states

We recall that states i and j are combinations of a forest state and a market state. Equation 4-12 is used to calculate the CTG value $J_k(i)$ for all discrete evaluation states i. $J_k(i)$ has two components:

(1) $g(i, u)$: The current net profit of taking action u in state i. The volume of wood removed from the stand, when taking decision u in state i, is transformed to a monetary value according to the procedures described in chapter 2.
(2) $\alpha \sum_j p_{ij}(u) \, [\tilde{J}_{k+1}(j, r)]$: the discounted expected CTG value of taking decision u in state i given transition probabilities $p_{ij}(u)$ and discount factor α where $\tilde{J}_{k+1}(j, r)$ is an approximate value calculated using one of the approximation architectures presented in section 4.4. Step 6 of this algorithm explains how r is updated at each iteration of the algorithm and the steps involved in using the approximation architectures to calculate an approximate value for the CTG function at any state j are explained below.

For TRT = 1 and 3, function approximations don't need to be computed. The state space for those two treatment types are continuous but the nature of the growth models for both species types is such that the discreet states in S_1^{Eval} and S_3^{Eval} can be chosen so that 5 years of growth starting at any $i \epsilon S_1^{Eval}$ or $i \epsilon S_3^{Eval}$ will result in state j being equal to one of the states in S_1^{Eval} or S_3^{Eval} given that the action taken is to let the stand grown. In these cases, $\tilde{J}_{k+1}(j, r)$ is simply a value retrieved from a table which contains values of $J_k(i)$ from the previous iteration.

(i) Multiple Regression – The parameter vector values for the regression equations were updated for TRT = 2, 4 and 5 at step 6 of the last iteration. These regression equation parameters are now used to approximate the CTG function value for state j where the value of the variables in the regression model are equal to the values of the state variables that define discreet state j. Section 5.2.1 describes the order and terms of the regression model used in this thesis.

(ii) Radial Basis Functions – The weights r_m were updated at step 6 of the previous iteration for TRT = 2, 4 and 5. The next step is to calculate, with equation (4-11), Euclidian norms of the distances between each center i^c and the discreet state j for which we need to approximate the CTG function values. With these distances, we can calculate the values of the RBF for each i^c, which when multiplied with the weights r_m and summed, yield the CTG function approximations.

(iii) Distance Weighted Interpolation – The weights were calculated at step 3 of this algorithm. Section 4.4.2 explains how to use DWI to calculate the CTG function approximations.

This process is repeated for all applicable evaluation state/decision combinations and the values of $J_k(i)$ are all stored for future retrieval.

Note: In the first iteration of the DP algorithm, $\tilde{J}_{k+1}(j,r)$ is zero for all result states j because the CTG values are being calculated for the first time so $r = 0$ for all states in the case of TRT = 2, 4 and 5, while the value of $J_{k+1}(j)$ is also zero for all states for TRT = 1 and 3.

Step 5 : Checking if the DP algorithm stopping criteria have been met

Section 4.5 gives a detailed explanation of how the stopping criteria are calculated and, when it`s appropriate, how to stop the DP algorithm. If the stopping criteria have been met, the DP algorithm is deemed to have converged to an acceptable policy π which, along with the CTG value for all evaluation states, is displayed in an Excel workbook. The algorithm is stopped at this point. If the stopping criteria haven't been met, the algorithm proceeds the step 6.

Step 6 : Updating the vector of parameters r

We now have a CTG function value $J_k(i)$ for each evaluation state i. We recall that S_n^{Eval} is a set of discrete states chosen to represent the continuous space S_n. In this step of the algorithm, we use the precomputed values from step 3 and update the values of the fitting parameters r_m for the chosen approximation architecture.

In the case of multiple regression, this means that we now have the values of y which are the $J_k(i)$. We can now complete the calculation of β as described in section 4.4.1 which yields the following multiple regression model:

$$y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \psi$$

where the X_m, $m = 1,\ldots, M$ will be replaced by the values of the state variables that define the discreet state j for which we need to approximate the CTG function at the next iteration of the DP algorithm.

As for RBF approximations, given the values of the vector f which are the $J_k(i)$ values calculated at step 4, the values of the weights r_m can be updated using the procedure in section 4.4.1.

Finally, for DWI, the weights don't depend on the values of $J_k(i)$ therefore they don't change at each iteration of the DP algorithm.

For each of the approximation architectures above, the process of updating the parameter vector needs to be done for TRT = 2, 4 and 5. This thesis uses backward recursive value iteration therefore, at each iteration of the algorithm, we are moving one 5-year period back in time and k from the current iteration becomes $k + 1$ at the next iteration. Return to step 4.

4.7 Policy Simulation

The optimal stationary policy μ obtained from solving the infinite horizon stochastic DP problem must be verified as discussed in section 4.5. This section discusses how μ is simulated in the context of this work.

A policy is a vector of decisions, one for each evaluation state, which results from solving the value iteration algorithm which yielded the vector of CTG function values J_μ. The optimization yielded two vectors that have matching entries where each evaluation state has an associated decision $\mu(i)$ and CTG function value $J_\mu(i)$. Simulating μ for a long time period will yield a CTG function value $J'_\mu(i)$. $J'_\mu(i)$ is used to differentiate it from $J_\mu(i)$ which is the value of the CTG function obtained from solving the DP problem with the value iteration algorithm. If μ is a good approximation to μ^*, $J'_\mu(i)$ should be close to $J_\mu(i)$.

What follows is a step by step description of how policy μ is simulated for state i to yield $J'_\mu(i)$ where x_t is used to represent state i. The simulation advances in 5 year increments and subscript t represents the number of years since the beginning of the simulation replication. Note that many of the states generated in the course of the simulation are not in the set of evaluations states. Thus, the policy corresponding to this state is not available without explicitly carrying out the DP calculation using the calculated approximate CTG function. However, an approximate policy choice is to instead use the policy corresponding to the closest evaluation state.

Step 1 : Choose an initial starting state x_0 along with stand and simulation parameter values for the simulation. Set t = 0. Go to step 2.

Step 2 : Based on policy μ, take action u_t associated with the evaluation state closest to state x_t. Based on this choice of u_t, calculate the resulting state x_{t+5} at time t+5. Calculate the profit gained from applying action u_t to state x_t $(g_t(x_t, u_t))$ and transform to net present value (NPV) at time 0. These profits are all transformed to NPV's according to $\alpha^t g_t(x_t, u_t)$ where $\alpha^t = e^{-\alpha t}$. The distances between states are measured with a Euclidean norm with scaled variable values identical to the one described in section 4.4. Go to step 3.

Step 3 : Generate a random number between 0 and 1. Compare this random number with the natural disaster probabilities calculated in section 4.2. If a natural disaster occurs, state x_{t+5} that would have occurred as calculated in step 2 is modified to reflect the impact of the natural disaster which is for x_{t+5} to become a regeneration state. If x_{t+5} is a regeneration state, either because of the action applied in step 2 or because of a natural disaster, go to step 4. Otherwise, go to step 5.

Step 4 : Apply the optimal action prescribed by policy μ. If the optimal action is to treat the stand in such a way that it either becomes a well stocked natural stand or a plantation, action u_t is applied which results in state x_{t+5}. If the optimal action is to do nothing, generate a random number between 0 and 1, and compare this value with the regeneration probabilities associated with the regeneration state. If, according to the regeneration probabilities, the stand regenerates, x_{t+5} is a 5 year old natural stand. If the stand doesn't regenerate, x_{t+5} is still a regeneration state with a higher probability of regenerating at the next time period. Go to step 5.

Step 5 : Store x_t, u_t, NPV$\big(g_t(x_t, u_t)\big)$ and x_{t+5} for future retrieval.

Step 6 : If $\alpha^t \times (maximum\, harvest\, profit) < 0.10\$$, stop the simulation. The $maximum\ harvest\ profit$ is the maximum profit that could have occurred if a regeneration harvest had been done given the current state of the forest stand. Otherwise, set $t = t + 5$ and go to step 2.

Steps 1 to 6 make up one simulation replication where $J'_\mu(i) = \sum_t \big(g_t(x_t, u_t)\big)$. When enough replications have been done, an average and a standard deviation are calculated and are used to build a confidence interval (CI). As long as the confidence interval contains $J_\mu(i)$ for state i and policy μ, we can say with a confidence level of (1-α) that μ is a good approximation to μ^*.

Let $S^2(n)$ be the sample variance of the CTG value $J'_\mu(i)$ over the n simulation experiments. Then we calculate a confidence interval CI as

$$\mathrm{CI} = \bar{X} \pm t_{n-1,1-\frac{\alpha}{2}} \sqrt{\frac{S^2(n)}{n}}$$ where $t_{n-1,1-\frac{\alpha}{2}}$ is the value from the student-*t* distribution

The number of replications of the simulation has an important impact on the width of the confidence interval. A very large number of replications may create a confidence interval so narrow that it would be difficult for $J_\mu(i)$ to fall within its bounds. If the estimate $\bar{X}$ is such that $|\bar{X} - J_\mu(i)| = \vartheta$ then we say that $\bar{X}$ has an absolute error of ϑ. The confidence interval constructed with the formula above assumes that $S^2(n)$ will not change appreciably as the number of replications increases. The approximate number of replications, $n^*(\vartheta)$, required to obtain an error of ϑ is given by Law and Kelton (2000):

$$n^*(\vartheta) = min\left\{ a \geq n \text{ such that } t_{a-1,1-\frac{\alpha}{2}} \sqrt{\frac{S^2(n)}{a}} \leq \vartheta \right\}$$

We can determine $n^*(\vartheta)$ by iteratively increasing a by 1 until a value of a is obtained for which $t_{a-1,1-\frac{\alpha}{2}}\sqrt{\frac{S^2(n)}{a}} \leq \vartheta$. If $n^*(\vartheta) > n$ and if we make $n^*(\vartheta) - n$ additional replications of the simulation, then the estimate $\bar{X}$ should have an absolute error of approximately ϑ. In practice, the total net present values of individual simulation replications (X) may not be distributed normally therefore the authors recommend the use of the *t*-distribution as it gives better coverage than a confidence interval constructed using the normal distribution. The better coverage is related to the fact that, although the *t*-distribution assumes normal distribution, the value of a *t*-distribution with degrees of freedom less than ∞ is higher for the same level of confidence than that of a normal distribution.

Since each discrete state has its corresponding $\mu(i)$ and $J_\mu(i)$, any state can be chosen as the starting point of the simulation and the corresponding $J_\mu(i)$ compared to the confidence interval above. Section 5.3.3 shows results of confidence intervals constructed for a few discrete states and for different sets of parameters of the DP model.

Chapter 5: Results and Discussion

In chapter 1, we presented a flow chart which gave an overview of the thesis and guides the reader in understanding the links between topics. For convenience, that flow chart is reproduced here and the discussion topics in this chapter are based on the structure presented in this flow chart.

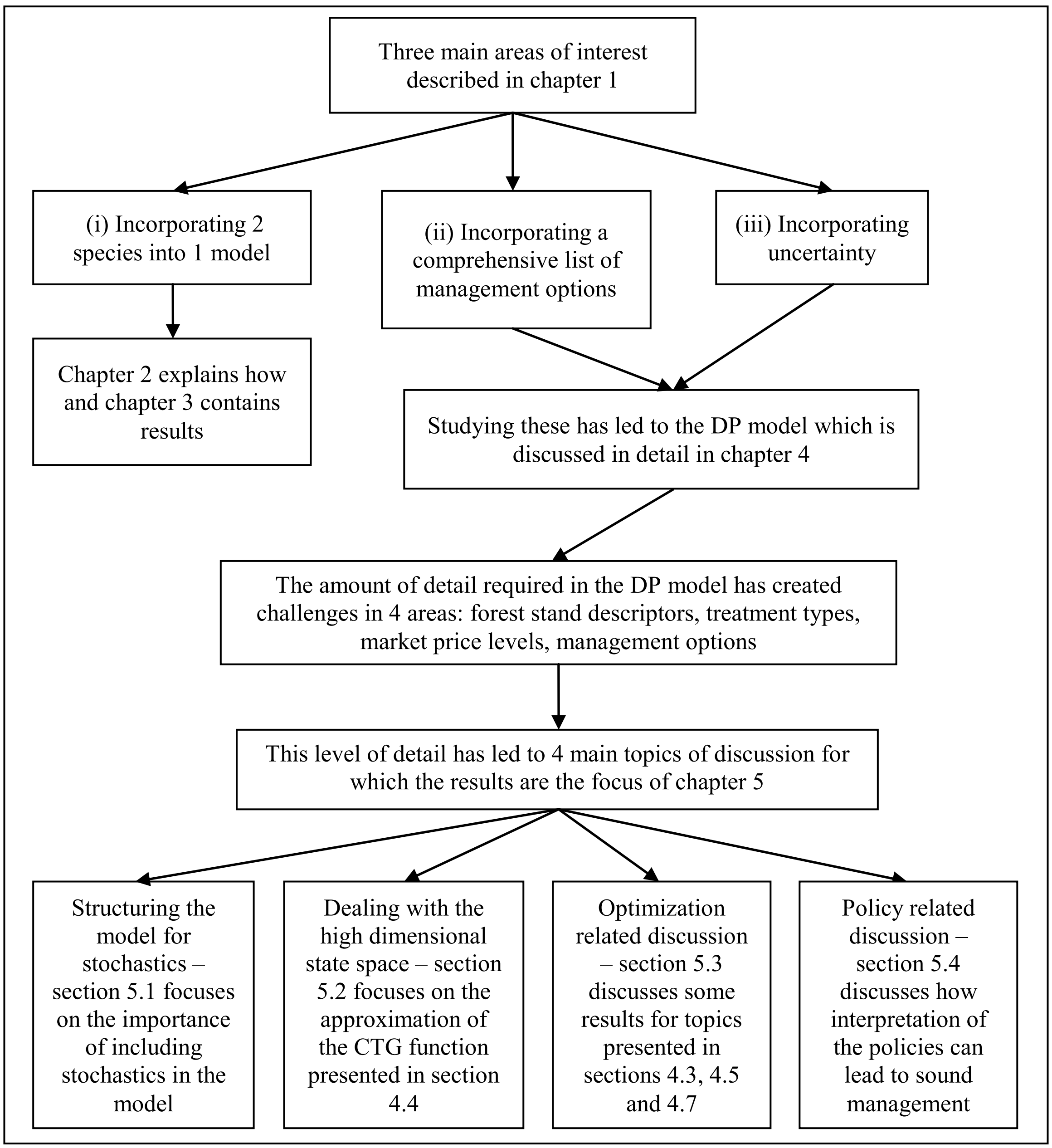


Figure 5-1 – Flow chart of thesis structure

The flow chart in figure 5-1 indicates four main topics of discussion that are the focus of chapter 5. The importance of the first topic, structuring the model for stochastics, is introduced in chapter 1 and developed in detail in section 4.2. Supporting results are presented in section 5.1. The second topic, dealing with the high dimensional state space, is present throughout chapter 4 as it has a major impact on the structure of the DP model. However, the need for appropriate approximation architectures in discussed in detail in section 4.4 and supporting results are presented in section 5.2. The third topic, optimization related discussions, focuses on the items presented in sections 4.3, 4.5 and 4.7, namely setting the value of the discount rate, the rate of convergence and termination criteria of the DP model, and policy simulation. Supporting results are presented in section 5.3. In section 5.4, we present some examples of the type of policy discussion that can occur with the model developed, the importance of which is established in the first page of the thesis.

In the development of the DP model, a basic set of parameters was chosen to represent an average forest stand in Nova Scotia as well as 2010 market and forest management conditions. Table 5-1 shows basic parameters for natural disasters and the explanation of how this information is used in the DP model is described in sections 2.9 and 4.4.

Table 5-1 – Basic parameters for natural disasters in the DP model

Approximate forested area in the west of the province of Nova Scotia (hectares)	1,691,300
Average number of fires per year	3.5
Return interval of major hurricanes (years)	50
Average area of wind for a major hurricane (hectares)	400,000
Return interval of major insect outbreaks (years)	50

Detailed examples of susceptibility calculations are explained in section 2.9. Table 5-2 shows basic parameters for the DP model which will be referred to as the base case scenario in the rest of this chapter. Unless otherwise specified, distance weighted interpolation is always used as the approximation architecture in the DP model. See section 5.2 for results that support this choice. Appendix B lists all possible actions that may apply during the optimization including for what treatment type they are applicable as well as the resulting treatment type after the action has been taken. Appendix B also explains how $BARem$ and $BARem_{split}$ are used to calculate the actual removal for CT for any given stand.

Table 5-2 – Basic parameters of the DP model

Annual discount rate	2%
Site index SW (in meters at age 50)	16.76m
Site index HW (in meters at age 50)	17m
Minimum dominant stand height (hardwood) for doing a pre-commercial thinning	6.1m
Minimum dominant stand height (softwood) for doing a pre-commercial thinning	2m
Maximum average stand height for doing a pre-commercial thinning	9m
Percentage of stand covered with softwood when a stand does naturally regenerate	75%
Natural stocking percentage of the forest	75%
Average selling price of softwood merchantable volume	$13.07/m^3$
Average selling price of softwood board volume	$30.76/m^3$
Average selling price of hardwood merchantable volume	$22.40/m^3$
Average selling price of hardwood board volume	$46.23/m^3$
Number of price level	1
Cost of planting less than 2500 trees on one hectare	$1,350
Cost of planting 2500 or more trees on one hectare	$1,500
Cost of surveying one hectare of newly harvested land	$70
Cost of doing fill planting on one hectare	$300
Cost of doing pre-commercial thinning on one hectare	$750
Cost of one hour of labour for doing commercial thinning or final felling	$40
Flat cost of doing a commercial thinning on one hectare	$750

As stated in chapters 1 and 4, we have an interest in the development of optimal policies for the management of a mixed-species stand. As random events such as changing prices, regeneration and natural disasters occur, policies need to reflect current observed state of the stand and market and the state most likely to occur in the future. Together, they dictate the path taken by the stand and market states and, in reaction to these random changes, the policies that create maximum economic value for the decision maker. In other words, when the state of the forest stand and of the market changes randomly, we need to make decisions that are optimal given the current state and the stochastics of both future states and optimal decisions. This has been achieved and the results presented in sections 5.1 and 5.4 rely heavily on the interpretation of these optimal policies. Therefore, it is useful to present a set of optimal policies and explain the general tendencies before going into specifics in the rest of the chapter. Given the parameters of the stand being studied and, with the observed state of the forest stand and of the market, the decision maker chooses an action based on the optimal policies.

The policies presented in table 5-3 are for the base case scenario with the exception that 6 price levels are used to discuss transitions in policies whereas the base case scenario only has 1 price level.

Table 5-3 – Partial policies for the base case scenario with the exception that there are six price levels instead of one

		Natural unmanaged stands					
Age (years)	Stocking (%)	Price 1	Price 2	Price 3	Price 4	Price 5	Price 6
5	100	1	1	1	1	1	1
10	100	12	12	12	12	12	12
15	100	12	12	12	12	12	12
20	100	1	1	1	1	1	1
25	100	1	1	1	1	1	1
30	100	1	1	1	1	1	1
35	100	1	1	39	39	39	39
40	100	1	38	38	38	38	38
45	100	38	38	38	38	38	38
50	100	19	19	37	38	38	38
55	100	37	37	37	37	38	7
60	100	28	37	37	37	7	7
65	100	37	37	37	7	7	7
70	100	1	37	7	7	7	7
75	100	1	1	7	7	7	7
80	100	1	1	7	7	7	7
85	100	1	1	7	7	7	7
90	100	1	1	7	7	7	7
95	100	19	19	7	7	7	7

Commercially or pre-commercially thinned natural stands											
Stand number	Age (years)	d^S (cm)	d^H (cm)	pct^S (%)	cc (%)	Price 1	Price 2	Price 3	Price 4	Price 5	Price 6
1	15	5.2	0	100	8.4	2	2	2	2	2	2
2	20	6.9	0	100	14.1	2	2	2	2	2	2
3	40	8.7	7.0	92	46.5	2	2	2	2	2	2
4	45	10.6	0	100	44.5	2	2	2	2	2	2
5	50	12.1	0	100	44.7	2	2	2	2	2	2
6	55	13.5	0	100	44.9	2	2	2	2	2	2
7	55	15.0	0	100	51.1	2	2	2	2	2	2
8	55	14.0	0	100	51.6	2	2	2	2	2	2
9	60	15.0	0	100	45.0	2	2	2	2	2	2
10	60	15.4	0	100	51.8	2	2	2	2	2	2
11	65	16.8	0	100	51.9	2	2	2	2	2	7
12	70	18.2	0	100	52.1	2	2	2	2	2	7
13	75	19.4	0	100	52.2	2	2	2	7	7	7
14	65	17.4	0	100	51.7	2	2	2	2	2	7
15	70	22.0	0	100	59	2	2	2	7	7	7
16	100	27.2	0	100	52.0	2	2	7	7	7	7

1 - Let grow	12 - PCT, remove HW	37 - CT, rmv 40% BA, splt 25% (abv)
2 - Let grow	19 - CT, rmv 40% BA, splt 25% (blw)	38 - CT, rmv 40% BA, splt 50% (abv)
7 - ReHar, plt 2500 tr/ha	28 - CT, rmv 40% BA, splt 25% (cros)	39 - CT, rmv 40% BA, splt 75% (abv)

All states in the left side of the table are natural unmanaged stands (TRT=1) and they are discrete elements of S^{Eval}, and all states in the right side of the table are either pre-commercially (TRT=2) or commercially thinned (TRT=5) stands. Each of the colour coded numbers makes reference to an action to be applied to the stand. A pre-commercial thinning action takes the stand from TRT=1 to TRT=2 and a commercial thinning action applied to a natural unmanaged stand takes the stand from TRT=1 to TRT=5. The legend given under table 5-3 and wherever else it is useful in the remainder of this chapter uses the following shortcuts: ReHar = regeneration harvest, plt = plantation, tr/ha = trees per hectare, rmv = remove, splt = $BARem_{Split}$ for SW, blw = remove trees from below, cros = remove trees from across the diameter distribution, abv = remove trees from above. The reader is reminded that the natural stands on the left have 100% crown closure on a 100% stocked stand therefore the stand is supporting as many trees as it possibly can for its age. These natural stands have an average SW content of 75%.

Action 1 is to do nothing and let the stand grow for one 5-year period. In all cases, taking action 1 simply means the stand will be 5 years older at the next decision time. For 5 year old natural stands, it is optimal to do nothing as the trees aren't tall enough to do a pre-commercial thinning and the diameters aren't large enough to have any commercial value. Action 12 is a pre-commercial thinning that eliminates hardwood and spaces softwood to NS DNR recommended spacing between trees and it is optimal to apply this action regardless of price at ages 10 and 15 because NS DNR recommends minimum heights for doing a pre-commercial thinning which correspond approximately to a 10 year old stand. Taking action 12 with 10 and 15 year old natural stands, and letting them grow 5 years, results in transitions to the first two stands respectively (stands 1 and 2) in the right side of table 5-3. We notice that the new stands have had all their hardwood removed (pct^S = 100%) and that the new crown closure percentage is very low. In both cases, it is optimal to do nothing and let the stand grow (action 2) regardless of the observed state of the market. As random disturbances occur and the state of the forest stand and market evolve, the policies are used to continually make optimal decisions based on the state observed at decision time.

Going back to natural unmanaged stands, pre-commercial thinning is permitted beyond 15 years of age but only occurs at ages 10 and 15 because by age 20, the natural stand has self-thinned itself to a point that investing in a PCT to thin out the stand is no longer the optimal action to take. Therefore, between the ages of 20 and 30 inclusively, it is optimal to do nothing and let the stand grow. At those ages, the average diameter of the trees is still too small to have any commercial value.

Decisions 19 to 39 are commercial thinning actions which are optimal for a wide range of ages and prices. Once a stand reaches the age where commercial thinning becomes a viable option, the observed market state becomes an important factor in choosing the optimal action. Generally speaking, CT removes enough wood to create enough revenue to make a profit at the time of harvest and spaces or releases trees so that they may grow larger diameters, and therefore more volume, than they would have grown without being released. The type of commercial thinning that is optimal is a function of the observed market state at decision time where, at higher prices, CT is done from above so maximum volume can be removed from a given amount of BA while prices are high.

All CT actions remove 40% of the total basal area on the stand and, aside from a few exceptions, CT actions are clustered into two groups. The first group, actions 19, 28 and 37, all remove the basal area by taking it as 25% SW and 75% HW. The only difference is the manner in which it is taken where 19 takes trees from the small diameters (from below), 28 takes the trees from across the diameter distribution and 37 takes the trees from the largest diameters (from above). In the second group, actions 38 and 39, CT is done from above where the basal area removed is 50% and 75% SW respectively with the balance in HW. Doing a commercial thinning from above yields slightly higher volumes for the same basal area but, more importantly, it creates a larger proportion of SV which has a much higher market value, and higher market prices encourage the removal of larger trees because there is a high probability that prices will come down at the next period. The majority of the CT actions in the policy from the left hand side of table 5-3 lead to a state where the optimal action is to do nothing for at least 5 years. Stands 3 to 13 from the right side of table 5-3 are a sampling of the resulting forest stands after taking actions 19, 28, 37, 38 or 39 with the natural stands in the left side of table 5-3. Two characteristics are similar for all these stands: pct^S =100% for all stands except stand 3 which still contains a small percentage of HW and cc varies within a narrow range of 44% to 52%. At such low cc, it makes no sense to remove any trees as there isn't enough MV and SV to justify the removal. The policies in the right side of table 5-3 reflect this as it is optimal to do nothing for all stands up to 60 years of age (state 10). Starting at age 65, some regeneration harvests appear at very high prices with more appearing at age 75 (state 14). At this age, the stand diameter is high enough that it is optimal to do a regeneration harvest if the prices are simply above the mean. States 14 and 15 are shown to demonstrate that taking action 2 when in state 14 yields state 15, and that with the rise in cc and diameter, there is a significant change in policy in just 5 years.

The last two commercial thinning actions on the left hand bottom corner are due to the fact that the stand can't grow another period after 95 years of age because the model doesn't include a state for the 100 year old natural stand. Therefore, a CT is done which creates immediate profit and sends the stand to TRT=5 where the age of the stands can go as high as 105 years old. Not surprisingly, the policy for the resulting state at the next time period, represented by state 16 in the right hand side of table 5-3 is identical to the policy for all natural stands between the ages 75 and 90.

Starting at age 55 with natural stands, it is optimal to do a regeneration harvest for very high prices. The older the stand, the higher the probability, based on market prices, that we will do a regeneration harvest but there is a lower limit to the price at which the harvest occurs. At prices 1 and 2 for ages 75 to 90, it is optimal to wait an extra time period before doing a harvest as the volume of wood is quite high at this point and the loss of revenue due to a harvest being done at low prices can't be recovered with future harvests because of discounting.

This introduction serves the purpose of pointing out how the policies can be interpreted and used for optimal decision making. Context specific policy interpretations are done throughout this chapter.

5.1 Structuring the Model for Stochastics

In section 4.2, uncertainty is divided into two independent stochastic processes: market state dynamics and growth and yield dynamics. As discussed in section 4.2.1, market state uncertainty is implemented, in the DP model, as a normal distribution where the prices at period k+1 that are closer to the mean price have a higher probability of occurrence. The mean price for all wood products is stationary as discussed in chapter 4. Section 5.1.1 shows results that support the need to include this market state uncertainty in the DP model. As discussed in section 4.2.2, growth and yield dynamics are represented by two independent Markov chains and, in the implementation of the DP model, one of those Markov chains, stochastic regeneration, is represented by a transition probability matrix and the other, natural disasters, is represented by a transition probability rule. Those Markov chains are discussed in detail in sections 2.5 and 2.9 respectively. Sections 5.1.2 and 5.1.3 discuss the impact of not incorporating growth and yield dynamics into the DP model.

5.1.1 Market State Uncertainty

In individual forest stand management, decisions are made after the state of the market is observed and the information gathered during this observation is used to optimize the decision making process. When the DP model is structured so the decision making can be made in this manner, considerable economic gains are observed over the case where this information is not included in the decision making process.

Prices and probabilities used with the base case scenario are shown in table 5-4.

Table 5-4 – 2010 market prices and their probability of occurrence in any time period

	Price 1	Price 2	Price 3	Price 4	Price 5	Price 6
Probability of occurrence	0.00383	0.087381	0.408789	0.408789	0.087381	0.003799
Z-value	-3.333	-2	-0.666	0.666	2	3.333
Softwood MV	\$9.82	\$11.12	\$12.42	\$13.72	\$15.02	\$16.32
Hardwood MV	\$16.80	\$19.04	\$21.28	\$23.52	\$25.76	\$28.00
Softwood SV	\$23.06	\$26.14	\$29.22	\$32.30	\$35.38	\$38.46
Hardwood SV	\$34.68	\$39.30	\$43.92	\$48.54	\$53.16	\$57.78

Mean prices and variance were chosen such that HW products are worth more than SW products and that there is enough difference between the highest and lowest prices for the policies to be different where price is a contributing factor. There is no good long term market price data available for Nova Scotia, therefore we make the assumption that the prices used in the model are reasonable and that they aren't correlated between discrete time periods, which is a requirement for the infinite horizon approach used in this thesis. The average of prices 3 and 4 is equal to the average of the normal distribution. The probabilities of occurrence are the cumulative probabilities for six equally divided intervals of the Normal distribution between Z = -4 and Z = +4 where the

Z-values given in the table are the mid-points of those six intervals. It is worth noting that the prices for MV and SV for both species are perfectly correlated.

Table 5-5 shows optimal policies and their resulting cost-to-go values for fully stocked natural stands. In order to save space in this section and the rest of the chapter, most policies will be presented as partial policies as the complete policies, in each case, would take up several pages. In this specific case, the complete policy for natural stands has two different stocking percentages: 100% and 75% but only the 100% case is shown. A comparison of the policies for 75% and 100% stocking is discussed in section 5.4.2.

Table 5-5 – Optimal policies and the resulting CTG values for fully stocked natural stands from one optimization that included the 6 price levels given in table 5-4

		Policies – 6 prices						CTG values – 6 prices					
Age	Stocking	1	2	3	4	5	6	1	2	3	4	5	6
5	100%	1	1	1	1	1	1	$8,920	$8,920	$8,920	$8,920	$8,920	$8,920
10	100%	12	12	12	12	12	12	$9,972	$9,972	$9,972	$9,972	$9,972	$9,972
15	100%	12	12	12	12	12	12	$10,870	$10,870	$10,870	$10,870	$10,870	$10,870
20	100%	1	1	1	1	1	1	$11,733	$11,733	$11,733	$11,733	$11,733	$11,733
25	100%	1	1	1	1	1	1	$13,259	$13,259	$13,259	$13,259	$13,259	$13,259
30	100%	1	1	1	1	1	1	$15,046	$15,046	$15,046	$15,046	$15,046	$15,046
35	100%	1	1	39	39	39	39	$16,817	$16,817	$16,938	$17,305	$17,672	$18,039
40	100%	1	38	38	38	38	38	$18,519	$18,707	$19,057	$19,408	$19,758	$20,108
45	100%	38	38	38	38	38	38	$20,137	$20,582	$21,026	$21,470	$21,915	$22,359
50	100%	19	19	37	38	38	38	$21,814	$22,054	$22,356	$22,794	$23,326	$23,859
55	100%	37	37	37	37	38	7	$22,909	$23,285	$23,661	$24,037	$24,434	$25,351
60	100%	28	37	37	37	7	7	$23,825	$24,233	$24,656	$25,078	$26,006	$28,123
65	100%	37	37	37	7	7	7	$24,350	$24,813	$25,277	$25,847	$28,198	$30,549
70	100%	1	37	7	7	7	7	$23,938	$24,383	$24,996	$27,551	$30,106	$32,660
75	100%	1	1	7	7	7	7	$24,833	$24,833	$26,338	$29,079	$31,820	$34,560
80	100%	1	1	7	7	7	7	$25,554	$25,554	$27,384	$30,269	$33,154	$36,038
85	100%	1	1	7	7	7	7	$26,197	$26,197	$28,231	$31,235	$34,240	$37,244
90	100%	1	1	7	7	7	7	$26,753	$26,753	$28,996	$32,109	$35,222	$38,334
95	100%	19	19	7	7	7	7	$26,428	$27,014	$29,690	$32,900	$36,111	$39,322

1 - Let grow	19 - CT, rmv 40% BA, splt 25% (blw)	38 - CT, rmv 40% BA, splt 50% (abv)
7 - ReHar, plt 2500 tr/ha	28 - CT, rmv 40% BA, splt 25% (cros)	39 - CT, rmv 40% BA, splt 75% (abv)
12 - PCT, remove HW	37 - CT, rmv 40% BA, splt 25% (abv)	

Focusing on the policy for age 35 in table 5-5, there are two recommended actions: doing nothing (1) or doing a commercial thinning from above with 40% basal area removal which is split 75/25 between softwood and hardwood (39). If prices 1 or 2 are observed, the optimal policy is to do nothing and let the stand grow another 5 years which results in a CTG value of $16,817. The CTG is the same for prices 1 and 2 because, in both cases, the optimal action is to do nothing and, because prices in period k+1 are independent of prices in period k, the expected value of $\tilde{J}_{k+1}(j)$ is the same for prices 1 and 2 at period k. For prices 3 to 6, the policy is the same and the resulting state is the same but the CTG value rises because revenue is created today from the CT which has different values based on the different currently observed prices.

In contrast, table 5-6 shows policies for 6 individual optimizations each one using 1 of the 6 prices in table 5-4.

Table 5-6 – Optimal policies and the resulting CTG values for fully stocked natural stands from 6 individual optimizations each with one of the prices given in table 5-4

		Policies						CTG values					
Age	Stocking	Price 1	Price 2	Price 3	Price 4	Price 5	Price 6	Price 1	Price 2	Price 3	Price 4	Price 5	Price 6
5	100%	1	1	1	1	1	1	$4,876	$6,370	$7,864	$9,358	$10,872	$12,405
10	100%	12	12	12	12	12	12	$5,451	$7,121	$8,791	$10,461	$12,153	$13,868
15	100%	12	12	12	12	12	12	$5,974	$7,783	$9,591	$11,400	$13,233	$15,089
20	100%	1	1	1	1	1	1	$6,675	$8,539	$10,404	$12,269	$14,151	$16,051
25	100%	1	1	1	1	1	1	$7,551	$9,655	$11,759	$13,863	$15,987	$18,131
30	100%	1	1	1	1	1	1	$8,578	$10,962	$13,346	$15,731	$18,138	$20,567
35	100%	39	39	39	39	39	39	$9,784	$12,497	$15,211	$17,925	$20,664	$23,428
40	100%	38	38	38	38	38	38	$11,155	$14,113	$17,072	$20,030	$23,014	$26,022
45	100%	38	38	38	38	38	38	$12,435	$15,640	$18,845	$22,050	$25,280	$28,536
50	100%	38	38	38	38	38	37	$13,253	$16,620	$19,989	$23,358	$26,753	$30,184
55	100%	37	37	37	37	37	37	$14,025	$17,562	$21,100	$24,638	$28,205	$31,801
60	100%	37	37	37	37	37	37	$14,666	$18,331	$21,996	$25,662	$29,358	$33,085
65	100%	37	37	37	37	37	37	$15,083	$18,833	$22,585	$26,336	$30,120	$33,934
70	100%	6	6	6	6	7	8	$15,926	$19,883	$23,839	$27,796	$31,795	$35,840
75	100%	6	6	6	6	7	8	$16,900	$21,041	$25,183	$29,324	$33,508	$37,738
80	100%	6	6	6	6	7	8	$17,661	$21,946	$26,230	$30,515	$34,841	$39,214
85	100%	6	6	6	6	7	8	$18,271	$22,675	$27,078	$31,481	$35,925	$40,417
90	100%	6	6	6	6	7	8	$18,824	$23,334	$27,845	$32,355	$36,906	$41,505
95	100%	6	6	6	6	7	8	$19,324	$23,932	$28,539	$33,147	$37,795	$42,491

1 - Let grow	8 - ReHar, plt 3250 tr/ha	38 - CT, rmv 40% BA, splt 50% (abv)
6 - ReHar, plt 1750 tr/ha	12 - PCT, remove HW	39 - CT, rmv 40% BA, splt 75% (abv)
7 - ReHar, plt 2500 tr/ha	37 - CT, rmv 40% BA, splt 25% (abv)	

There is a large difference in policy and CTG values between tables 5-5 and 5-6. For prices 1, 2 and 3, all CTG values are lower in table 5-6 than in table 5-5. For prices 4, 5 and 6, the opposite occurs. In table 5-5, if the current observed price is lower than the mean price, there is a higher than 50% probability that the price observed at the next time period will be higher than the current price. If the current observed price is higher than the mean price, there is a higher than 50% probability that the price observed at the next time period will be lower than the current price. The effect of these probabilities is explained below.

Managing a stand using the policies in table 5-6 is the equivalent of making decisions based solely on the current volume of wood on the stand without considering the current state of the market and the possibility of observing a higher or lower price at the next period because only one of those prices can be observed at any given time and it remains unchanged forever. If the current price is as high as the decision maker expects it to get with the prospect of the price going down at the next decision period and the stand being observed is 55 years old, the optimal policy in table 5-5 is to harvest the stand now to take advantage of the current high price. In the case where prices are fixed (table 5-6), the optimal policy is to do a commercial thinning which generates some revenue but not as much as a regeneration harvest. If the decision maker is using the policies in table 5-6 expecting high prices to remain the same in the future but prices go down, a loss of profit may occur because of the use of an inappropriate policy.

Similarly, if observed prices are currently low and the decision maker doesn't consider the possibility that those prices are likely to go up in the next period, an important loss of profit may occur from taking the wrong action. Returning to the policy for the 35 year old stand, the optimal action for price 1 according to table 5-6 is to do the same CT as described earlier for the higher prices. Whereas, in table 5-5 we only did a CT if the current observed price was price 3 or higher, in table 5-6, it is done regardless of the current observed price. For price 1, this decision incurs a lost profit of $7,033 ($16,817 - $9,784). Even when the decision is the same for both policies for any given price, the CTG value is different because the decision maker will be choosing actions at each subsequent time period and, in the case of the policy in table 5-5, decisions will always be made with the knowledge that the prices may go up or down in the future depending on the current observed price.

Table 5-7 – Optimal policies for 75% stocked and 100% stocked natural stands from one optimization that included the 6 price levels given in table 5-4

	100% stocking						75% stocking					
Age	1	2	3	4	5	6	1	2	3	4	5	6
5	1	1	1	1	1	1	1	1	1	1	1	1
10	12	12	12	12	12	12	12	12	12	12	12	12
15	12	12	12	12	12	12	12	12	12	12	12	12
20	1	1	1	1	1	1	12	12	12	12	12	12
25	1	1	1	1	1	1	12	12	12	12	12	12
30	1	1	1	1	1	1	1	1	1	1	1	1
35	1	1	39	39	39	39	1	1	1	1	1	1
40	1	38	38	38	38	38	31	31	31	31	31	31
45	38	38	38	38	38	38	31	31	31	31	31	31
50	19	19	37	38	38	38	31	31	31	31	31	31
55	37	37	37	37	38	7	34	34	34	34	34	34
60	28	37	37	37	7	7	34	34	34	7	7	7
65	37	37	37	7	7	7	1	1	7	7	7	7
70	1	37	7	7	7	7	1	1	7	7	7	7
75	1	1	7	7	7	7	1	1	7	7	7	7
80	1	1	7	7	7	7	1	1	7	7	7	7
85	1	1	7	7	7	7	1	1	7	7	7	7
90	1	1	7	7	7	7	1	1	7	7	7	7
95	19	19	7	7	7	7	16	7	7	7	7	7

1 - Let grow	16 - CT, rmv 30% BA, splt 25% (blw)	34 - CT, rmv 30% BA, splt 25% (abv)
7 - ReHar, plt 2500 tr/ha	19 - CT, rmv 40% BA, splt 25% (blw)	37 - CT, rmv 40% BA, splt 25% (abv)
12 - PCT, remove HW	28 - CT, rmv 40% BA, splt 25% (cros)	38 - CT, rmv 40% BA, splt 50% (abv)
	31 - CT, rmv 20% BA, splt 25% (abv)	39 - CT, rmv 40% BA, splt 75% (abv)

Table 5-7 shows the policies for the base case scenario for 75% stocked and 100% stocked natural stands. The reduction in stocking paired with the random prices yields very different policies that are worth investigating. Note that a stand that starts at 75% stocking will remain so for the duration of the optimization even after a regeneration harvest is applied to the stand. If we observe the policies by chronological age, the first difference is the appearance of PCT actions for 20 and 25 year old stands in the 75% stocked stand, for all prices. PCT is optimal at all ages where it is an acceptable option according to NS DNR. This makes sense as the 75% stocked stand has less volume therefore doing a PCT opens up the stand and accelerates the diameter growth of the trees that are present. CT actions are also very different for the 75% stocked case. The window of opportunity for CT in the 75% stocked case is between the ages of 40-60 whereas it is 35-70 in the fully stocked case. In addition, all CT actions remove 40% of the basal area when the stand is fully stocked and only 20% or 30% when the stand is 75% stocked which makes sense since the stand has less basal area to begin with. The smaller removal percentage will only take a small portion of the basal area on the stand and release the remaining trees to give them extra growing room. The most unexpected difference in CT thinning policy is where prices don't have much of an effect on the policy for the 75% stocked stand. For example, from ages 40 to 50 in the 75% stocked case, it is optimal to do the same CT regardless of the price. The same occurs at age 55 for a different CT action. The policy for the fully stocked stand is very different where the optimal action depends on age and price for all ages except 45 years. The regeneration harvest actions are similar except that, in the fully stocked case, there is a clear relationship between age and price. It is optimal to harvest the stand at lower prices as the stand gets older.

The results in this section indicate that important profit losses may be associated with policies that don't consider the uncertainty of market prices in the modelling of individual forest stand management. Also, these results show that the incorporation of uncertainty to the model in the form of random prices is important because it allows us to capture the dynamics between prices and stand definitions. The next section explains what effects natural disasters can have on policies and CTG values.

5.1.2 Natural Disaster Uncertainty

Three types of natural disasters are of interest: forest fires, hurricanes and insect infestations. They all have different effects on any individual forest stand. In general, forest fires have more effect on small trees that don't have any protection against the heat, hurricanes have more effect on larger trees because they can blow over and insect infestations have the same effect on all trees because they all offer food and shelter. For the basic parameters of the natural disasters discussed in this section, see table 5-1 at the beginning of this chapter.

As expected, the addition of natural disasters to the DP model affects optimal policies and CTG values. Table 5-7 shows the policies for 100% stocked TRT=1 stands, one of which is never subject to natural disasters and three others that are subject to each natural disaster independently of the other two types.

Table 5-7 – Policies for natural disasters added one at a time to the base model

	Age															
Natural disasters	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80
No natural disasters	1	12	12	1	1	1	1	38	38	38	37	37	28	37	3	3
Fire only	1	12	12	1	1	1	1	38	38	37	37	37	37	6	6	6
Hurricane only	1	12	12	1	1	1	1	38	38	37	37	37	37	6	6	6
Insects only	1	12	12	1	1	1	39	38	38	37	37	37	37	7	7	7

1 - Let grow	7 - ReHar, plt 2500 tr/ha	37 - CT, rmv 40% BA, splt 25% (abv)
3 - ReHar, natural regen	12 - PCT, remove HW	38 - CT, rmv 40% BA, splt 50% (abv)
6 - ReHar, plt 1750 tr/ha	28 - CT, rmv 40% BA, splt 25% (cros)	39 - CT, rmv 40% BA, splt 75% (abv)

In these four models, other sources of uncertainty, namely natural regeneration and market price uncertainty, have been eliminated in order to study the effect of each natural disaster added one at a time to the model. Results show that the prospect of losing current or future revenue because of a natural disaster causes the policy to become slightly more conservative. This is indicated by the switch from action 3 at age 75 without natural disasters to action 6 or 7 at age 70 when disasters can potentially occur. The plantations created by actions 6 or 7 grow faster and have, in general, shorter rotation cycles therefore providing protection from the potential future loss due to a natural disaster. In contrast, there is less investment to make in natural stands, with the accompanying slower increase in volume, so if there is no threat of losing revenue due to a natural disaster, letting the stand regenerate naturally gives longer rotation ages and is the optimal decision.

Insects are the most devastating natural disaster as they have the most effect on policy. According to the policy in table 5-7, instead of waiting until the stand reaches age 40 to recommend a CT as is the case when not facing any potential of natural disasters, it is recommended at age 35 when potentially facing natural disasters which shortens the amount of time it takes the stand to get to the regeneration harvest. The shorter the regeneration period, the less probability the stand will be affected by a natural disaster in its lifecycle.

Figure 5-2 shows the change in CTG value associated with each natural disaster when compared to the case without natural disasters.

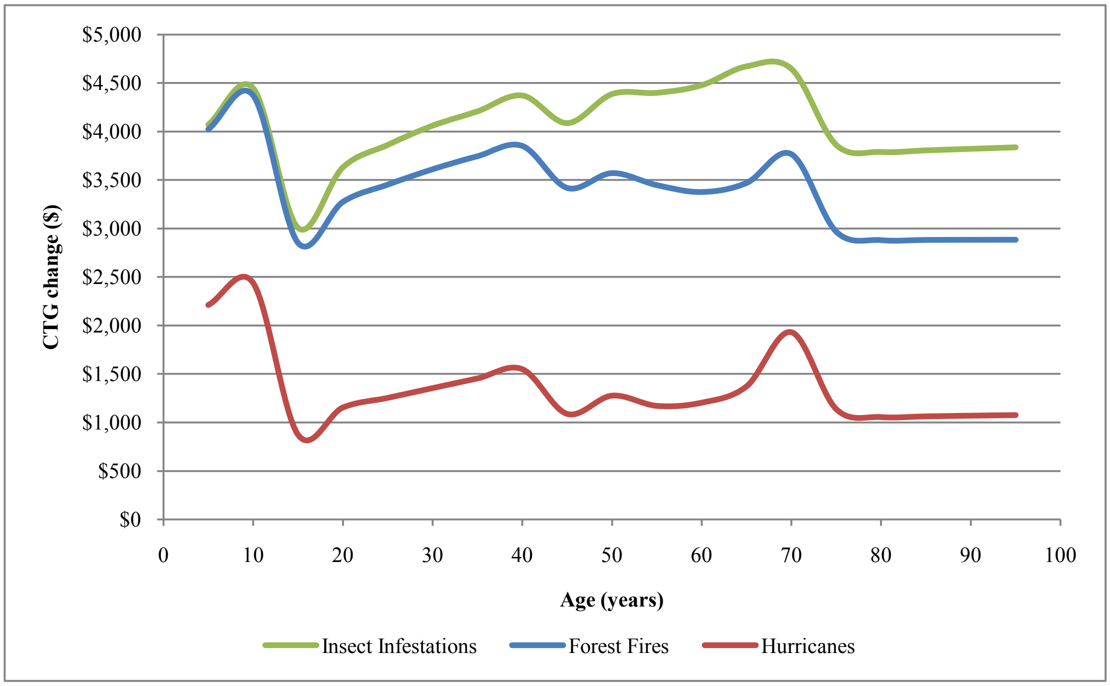


Figure 5-2 – CTG value changes for all three natural disasters added one at a time to the base model

According to table 5-7, for hurricanes and fires, there are policy changes at 10, 20, 40, 50 and 70 years of ages. The curves in figure 5-2 show jumps, in the CTG value change, that coincide with those policy changes. For example, taking decision 38 at age 40 creates revenue immediately by doing a CT and leaves fewer trees to be potentially damaged by a natural disaster which would affect future revenue. Thus figure 5-2 shows the reduction of the effect of natural disasters after CT becomes the optimal decision in the case of forest fires and hurricanes. As the stand gets close to the age where it is optimal to do a regeneration harvest, there is a sharp increase in the effect of natural disasters on the CTG value. Further investigation shows that all stands, where a CT is the optimal decision at ages 60 and 65, will wait at least two periods following CT before a regeneration harvest becomes optimal. Table 5-8 shows, for TRT=1 stands with 100% stocking and ages 60 and 65, the resulting states at the next period immediately following CT actions 28 and 37. Table 5-9 shows, for each result state in table 5-8, the nearby evaluation states with their Euclidian distances and the policies at those states.

Table 5-8 – TRT=1 stands with 100% stocking with result states for actions 28 and 37

Age	Decision
60	37
65	28
65	37

Result state	TRT	Age	d^S	d^H	pct^S	cc
1	5	65	16.8	0	1	52
2	5	70	18.7	0	1	52
3	5	70	18.2	0	1	52

Table 5-9 – The evaluation states that are nearest to the result states from table 5-8

Result state	Euclidian distance	Age	d^S	d^H	pct^S	cc	Policy
1	0.007464	65	19.8	0	55	1	2
1	0.010109	55	17.2	0	55	1	2
1	0.020771	65	19.8	0	40	1	2
1	0.023416	55	17.2	0	40	1	2
2	0.004183	65	19.8	0	55	1	2
2	0.012058	75	22.2	0	55	1	6
2	0.017333	65	19.8	0	40	1	2
2	0.022985	55	17.2	0	55	1	2
3	0.005176	65	19.8	0	55	1	2
3	0.014895	75	22.2	0	55	1	6
3	0.018905	65	19.8	0	40	1	2
3	0.021912	55	17.2	0	55	1	2

As the potential for natural disaster has equal probability of occurrence at each period, there is a high risk of losing revenues in two subsequent periods.

In order to further study the effect of natural disasters on CTG values and policies, we compare average natural disaster cases, those studied so far, to severe natural disasters. In the severe case, hurricanes are larger and return more often, insect infestations return more often and fires burn larger areas. Table 5-10 shows the basic parameters used to calculate the probability that a natural disaster occurs and the stand is affected.

Table 5-10 – Basic parameters for average natural disasters and severe natural disasters

	Average	Severe
Approximate forested area in the west of the province of Nova Scotia (hectares)	1,691,300	1,691,300
Average number of fires per year	3.5	3.5
Average size of a fire (hectares)	10,000	30,000
Return interval of major hurricanes (years)	50	15
Average area of wind for a major hurricane (hectares)	400,000	800,000
Return interval of major insect outbreaks (years)	50	15

The rest of the values presented in section 2.9 are identical. Therefore, susceptibility of the stands doesn't change in the severe case but the higher number and size of fires, the shorter return interval of hurricanes and insect outbreaks and the larger size of hurricanes leads to a higher probability that a stand will end up in a regeneration state.

Figure 5-3 shows the reduction in CTG values associated with severe natural disasters when compared to average natural disasters, for 100% stocked unmanaged natural stands. The graph shows the effect for each natural disaster added to the model one at a time.

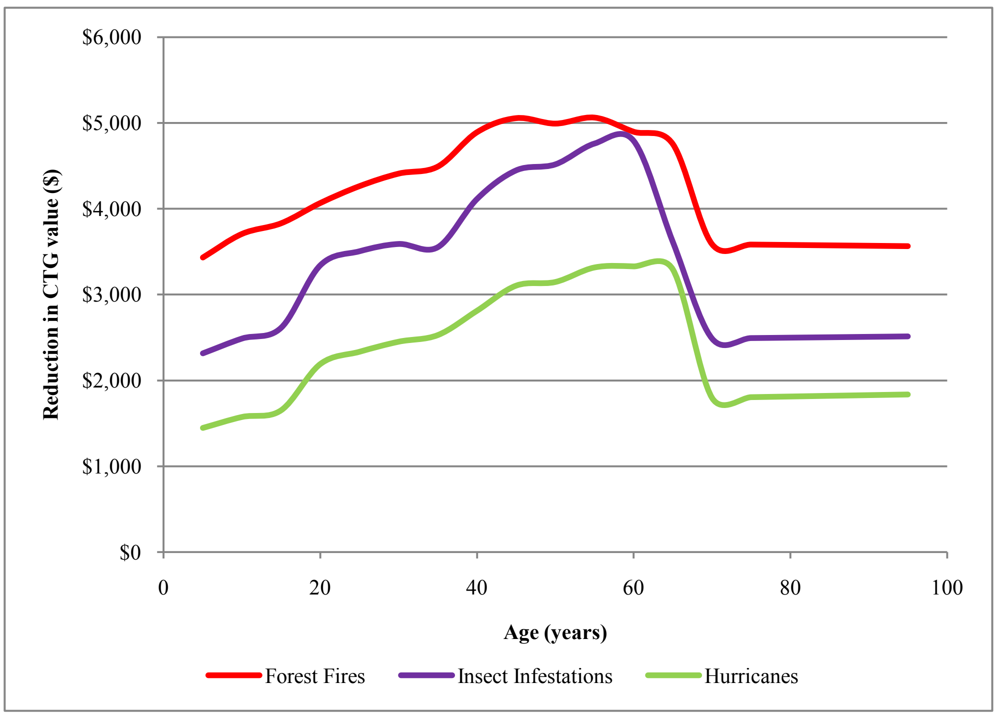


Figure 5-3 – Reduction in optimal CTG value between the average natural disasters and the severe case

The actual values in the graph aren't as important as the shape of the curves because the CTG value changes would be different depending on the values used for the severe cases. However, the shape of the curves shows that after the age 70, when the first regeneration harvest occurs, the effect of making the natural disasters more severe is constant. The optimal policies for the average and severe cases are shown in table 5-11.

Table 5-11 – Policies for 100% stocked unmanaged natural stands with average and severe natural disasters added to the model one at a time in the base case scenario

	Age	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95
Average	Fire	1	12	12	1	1	1	1	38	38	37	37	37	37	6	6	6	6	6	6
	Hurricane	1	12	12	1	1	1	1	38	38	37	37	37	37	6	6	6	6	6	6
	Insects	1	12	12	1	1	1	39	38	38	37	37	37	37	7	7	7	7	7	7
Severe	Fire	1	12	12	1	1	1	39	38	38	38	37	37	37	6	6	6	6	6	6
	Hurricane	1	12	12	1	1	1	39	38	38	37	37	37	37	6	6	6	6	6	6
	Insects	1	12	12	1	1	1	39	38	38	38	37	37	3	3	3	3	3	3	3

1 - Let grow	7 - ReHar, plt 2500 tr/ha	38 - CT, rmv 40% BA, splt 50% (abv)
3 - ReHar, natural regen	12 - PCT, remove HW	39 - CT, rmv 40% BA, splt 75% (abv)
6 - ReHar, plt 1750 tr/ha	37 - CT, rmv 40% BA, splt 25% (abv)	

As the stand gets older and the volume of wood, and therefore the monetary value of the stand, grows, severe natural disasters have a larger and larger effect on the CTG values. The regeneration harvests at age 65-70 create large revenues after which a stand doesn't provide any revenue for a lengthy period. Therefore, after the initial regeneration harvest, severe natural disasters cause a constant reduction in CTG values. In terms of policy, the largest change shown in table 5-11 is for severe insect outbreaks where the optimal policy for older stands changes from creating plantations to letting the stands grow naturally. This change is due to a long term reduction of the value of the stand which leads to earlier harvest and the elimination of investment in plantations in favour of a natural regeneration option that costs much less in terms of initial set-up.

Results show that managing a forest stand without considering the effects of natural disasters when the forest stand being managed is susceptible to natural disasters may lead to suboptimal decision making.

5.1.3 Regeneration Uncertainty

Regeneration uncertainty was presented in detail in section 2.5 and the result of incorporating it into the DP model is discussed here. Table 5-12 shows the optimal policies and CTG values for untreated natural stands in the case where the stand may take several periods to regenerate, hereafter referred to as uncertain regeneration, and the case where regeneration is guaranteed in the first period following a regeneration harvest, hereafter referred to as certain regeneration. In reference to the results discussed in section 2.5, the average number of periods of regeneration is 2.155 which means that, on average, the stands take 10.77 years to regenerate which leads to a reduction of CTG value for all stands and to a change in optimal policy.

Table 5-12 – Policies and CTG values for the cases where natural stands have an uncertain regeneration period and the case where there isn't uncertainty

		Policies		CTG values	
Age	Stocking	No regeneration uncertainty	Uncertain regeneration period	No regeneration uncertainty	Uncertain regeneration period
5	1	1	1	$14,603	$12,725
10	1	12	12	$16,139	$14,064
15	1	1	12	$15,625	$15,133
20	1	1	1	$17,268	$16,657
25	1	1	1	$19,084	$18,410
30	1	1	1	$21,091	$20,346
35	1	1	1	$23,309	$22,487
40	1	38	38	$25,761	$24,853
45	1	38	38	$27,498	$27,121
50	1	38	37	$28,984	$28,427
55	1	37	37	$30,253	$29,845
60	1	37	37	$31,200	$30,761
65	1	28	37	$31,940	$31,325
70	1	37	6	$32,384	$30,884
75	1	3	6	$33,032	$32,327
80	1	3	6	$34,075	$33,454
85	1	3	6	$34,995	$34,368
90	1	3	6	$35,828	$35,197
95	1	3	6	$36,583	$35,947

1 - Let grow	12 - PCT, remove HW	38 - CT, rmv 40% BA, splt 50% (abv)
3 - ReHar, natural regen	28 - CT, rmv 40% BA, splt 25% (cros)	39 - CT, rmv 40% BA, splt 75% (abv)
6 - ReHar, plt 1750 tr/ha	37 - CT, rmv 40% BA, splt 25% (abv)	

In the certain regeneration case, if we start with a 5 year old natural unmanaged stand, table 5-12 tells us that it is optimal to do a pre-commercial thinning at age 10 at which point the stand becomes a TRT=2 stand. Table 5-13 shows partial policies for pre-commercially thinned stands (TRT=2) for the base case scenario with certain regeneration. This policy shows that it is optimal to do nothing until age 40 and, regardless of crown closure, it's optimal to do a regeneration harvest at age 40 and let the stand naturally regeneration, at which point the policy in table 5-12 applies and the cycle restarts. The life cycle is very different for a 5 year old natural unmanaged stand in the case with uncertain regeneration. In this case, the policy in table 5-12 tells us that it's optimal to do a pre-commercial thinning at age 10 at which point the stand becomes a TRT=2 stand. Table 5-14 shows the optimal policy for a TRT=2 stand in the uncertain regeneration case and, regardless of crown closure, it is optimal to do a regeneration harvest between ages 35 and 45, and plant 1750 trees/hectare at which point the stand becomes a TRT=3 stand. Table 5-15 shows the policy for a TRT=3 stand with an initial planting density of 1750 trees/hectare in the case of uncertain regeneration. This policy shows that it is optimal to let the stand grow up to age 50 at which point a regeneration harvest is done and the cycle restarts.

Table 5-13 – Partial optimal policy for TRT=2 (pre-commercially thinned) for the base case scenario with certain regeneration

Age	d^S	d^H	cc	pct^S	Optimal decision
30	13.1	0	33.3	1	2
30	15.1	0	33.3	1	2
40	15.5	0	40	1	3
40	16.9	0	40	1	3
30	13.1	0	58.3	1	2
30	15.1	0	58.3	1	2
40	15.5	0	70	1	3
40	16.9	0	70	1	3
30	13.1	0	83.3	1	2
30	15.1	0	83.3	1	2
40	15.5	0	100	1	3
40	16.9	0	100	1	3

2 - Let grow	3 - ReHar, natural regen	6 - ReHar, plt 1750 tr/ha

Table 5-14 – Partial optimal policy for TRT=2 (pre-commercially thinned) for the base case scenario with uncertain regeneration

Age	d^S	d^H	cc	pct^S	Optimal decision
35	15.0	0	33.3	1	2
35	16.7	0	33.3	1	2
45	17.0	0	40	1	6
45	18.2	0	40	1	6
35	15.0	0	58.3	1	2
35	16.7	0	58.3	1	2
45	17.0	0	70	1	6
45	18.2	0	70	1	6
35	15.0	0	83.3	1	2
35	16.7	0	83.3	1	2
45	17.0	0	100	1	6
45	18.2	0	100	1	6

Table 5-15 – Partial optimal policy for TRT=3 (plantation) for the base case scenario with uncertain regeneration

Age	d^S	Initial planting density	Optimal policy
5	0	1750	2
10	4.7	1750	2
15	8.9	1750	2
20	12.1	1750	2
25	14.7	1750	2
30	16.8	1750	2
35	18.6	1750	2
40	20.1	1750	2
45	21.5	1750	2
50	22.6	1750	6

The life cycle of the 5 year old natural unmanaged stand is very different when natural regeneration isn't guaranteed after a regeneration harvest if no planting is done. The results discussed in sections 5.1.1 to 5.1.3 support the claim that modelling individual forest stand management without incorporating uncertainty into the model can lead to poor decision making which may have economic impacts.

5.2 Dealing with the High Dimensional State Space

The biggest challenge created by the high dimensional state space, and the one which is discussed in this section, is the approximation of the CTG function values in the DP algorithm. Section 4.4 discusses two classes of approximation architectures: averager and non-averager methods. As discussed in section 4.4, averager methods are generally better at approximating the CTG function in high dimensional state spaces. Results in this section support that claim.

Section 4.6 presents details of the DP algorithm and identifies when the approximation architecture is used to approximate the CTG function. Calculating $J_k(i)$ requires the approximation $\tilde{J}_{k+1}(j,r)$ for each admissible state/decision combination. There are two key components in this approximation: proper scaling of the discrete states and the need to bound the CTG function approximation.

Scaling has a significant effect on the implementation of radial basis functions (RBF) and distance weighted interpolation (DWI) while bounding the CTG function is critical when using multiple regression (MR) for function approximation. The rest of this section focuses on these three CTG function approximations and the critical elements that ensure accurate approximation. Section 5.2.1 discusses the implementation of multiple regression as the first non-averager approximation architecture in the DP model and includes a discussion on the difficulty of choosing the appropriate terms for the regression model. Results also show the importance of CTG bounds on its accuracy as an approximation architecture for DP. Section 5.2.2 shows implementation results for radial basis functions and discusses the choice of the shape parameter ω. The impact of scaling is discussed at length. In section 5.2.3, results show that distance weighted interpolation, the only averager method studied, does a good job of approximating the CTG function without the difficulties associated with non-averager methods.

5.2.1 Multiple Regression

As discussed in chapter 2, there are five treatment types but in the implementation of the DP model, we only need to develop CTG function approximations for three of those 5 because the other two describe stands that follow a pre-defined path through the state space as they grow and discrete states exist for every state of the forest stand that can occur through its life cycle. As a reminder, the 5 treatment types are restated in table 5-16.

Table 5-16 – Treatment types

Treatment	Description
1	Unmanaged natural stand
2	Pre-commercially thinned natural stand
3	Unthinned plantation
4	Commercially thinned plantation
5	Commercially thinned natural stand

CTG function values are calculated for each of those discrete states at each iteration of the DP model, saved in an array and retrieved at a subsequent iteration. Therefore, no approximation is required for two of the 5 treatment types. For the other three, TRT=2, TRT=4 and TRT=5, the terms of the regression equation are defined based on the variables representing its discrete states. Table 5-17 gives the terms in the regression equation.

Table 5-17 – Terms in the multiple regression equation for CTG function approximations (TRT=2, TRT=4 and TRT=5)

TRT	Terms
4	age, d^S, cc, age^2, ${d^S}^2$, cc^2, $age \times d^S$, $age \times cc$, $d^S \times cc$
2, 5	age, d^S, d^H, pct^S, cc, age^2, ${d^S}^2$, ${d^H}^2$, pct^2 , cc^2, $age \times d^S$, $age \times d^H$, $age \times pct^S$, $age \times cc$, $d^S \times d^H$, $d^S \times pct^S$, $d^S \times cc$, $d^H \times pct^S$, $d^H \times cc$, $pct^S \times cc$

A second order polynomial was arbitrarily chosen for all treatment types with the terms being limited to two way interactions between variables. The fitting is done on known deterministic CTG function values at the data points therefore the results shown here aren't for a model fit on random data. These results are used simply for discussion and for analysing which terms should remain in the model.

For TRT=4, only the three state variables and all two way interactions are used as terms in the regression equation. ANOVA results in table 5-18 support the use of this set of terms with very high values for R^2 and F-statistic. The F-statistic with 9 and 140 degrees of freedom and a 95% level of confidence is 1.95. These results were obtained by doing a regression analysis using the 9 terms in table 5-17 and the CTG function values for TRT=4 from the first iteration of the DP optimization with the base case scenario.

Table 5-18 – ANOVA results for TRT=4 optimal CTG function values for the base case scenario (R^2 = 0.9901)

	df	*SS*	*MS*	*F*	*Significance F*
Regression	9	17,190,166,540	1.91E+09	1554.043	1.4E-135
Residual	140	172,068,952	1229064		
Total	149	17,362,235,493			

Table 5-19 shows the t-statistic values for the terms in the regression equation where the first column indicates the terms used in the regression equation and the three columns with t-statistics indicate which terms were used in three different regressions using the same data set. This information is used only for purposes of deciding which terms should remain in the model and not for doing any statistical testing. The t-statistic for 9 degrees of freedom and 95% confidence level is 2.2622 so this value is used for deciding which terms explain variation. Therefore, based on the results from table 5-19, the following terms should be removed from the model: d^S, age^2, ${d^S}^2$ and $age \times d^S$. The t-statistics for the 5 term regression show that one additional term should be removed: $age \times cc$.

Table 5-19 – T-statistics for the terms given in table 4 for TRT=4 and fitted on the CTG function values from the first iteration of the DP optimization of the base case model

Term	T-statistics with 9 terms	T-statistics with 5 terms	T-statistics with 4 terms
Intercept	-9.80512	-3.1574	-4.16911
age	7.273237	3.458408	7.686034
d^S	0.21985	---	---
cc	9.189767	4.51568	4.814916
age^2	-2.0691	---	---
${d^S}^2$	-0.19153	---	---
cc^2	-10.7209	-5.24492	-5.17448
$age \times d^S$	-0.75562	---	---
$age \times cc$	2.757003	2.230362	---
$d^S \times cc$	5.575867	10.75619	15.23759

However, in regression analysis, it isn't sufficient to use ANOVA information to determine which terms should be included in the regression model. Figure 5-4 shows residual plots for 5-term and 4-term regressions. Both of these clearly indicate that the terms used in the model are not sufficient to explain all of the variability in the response variable. The quadratic form of the residuals versus age plots below indicates that a term that includes age^2 needs to be added to the model to improve the fit of the relation, which is clearly not linear. The process of adding variables to the model and verifying the residuals graphs is done iteratively until we get a model that yields residual plots such as those in figure 5-5. Alternatively, we could have searched for a model which yields random errors as those in figure 5-5 and which minimizes the maximum error.

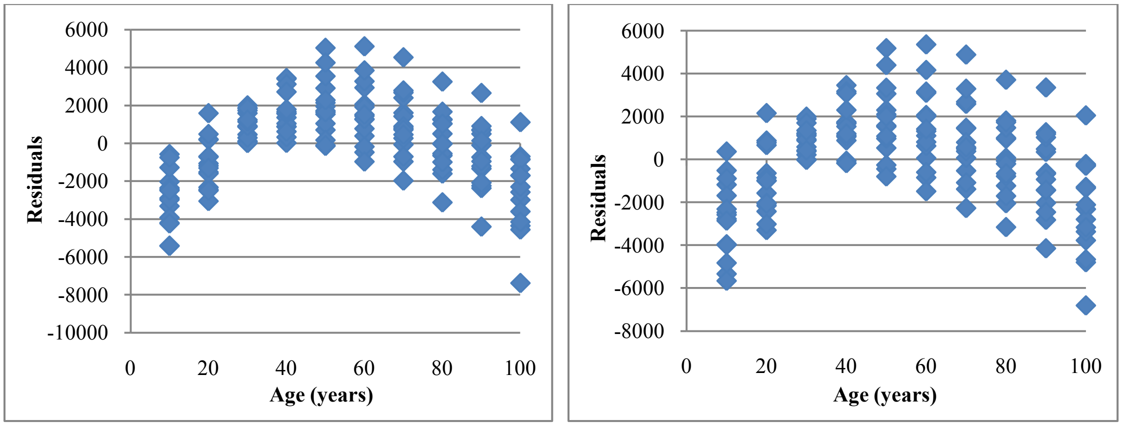


Figure 5-4 – Residual plots for the 5 term and 4 term models shown in table 5-19

The graphs in figure 5-5 show residual plots for the same regression model that created the plots in figure 5-4 but with the 9 terms shown in table 5-17. The inclusion of those 9 terms gives the ANOVA results in table 5-18 and the residual plots in figure 5-5 which don't show any discernable tendencies.

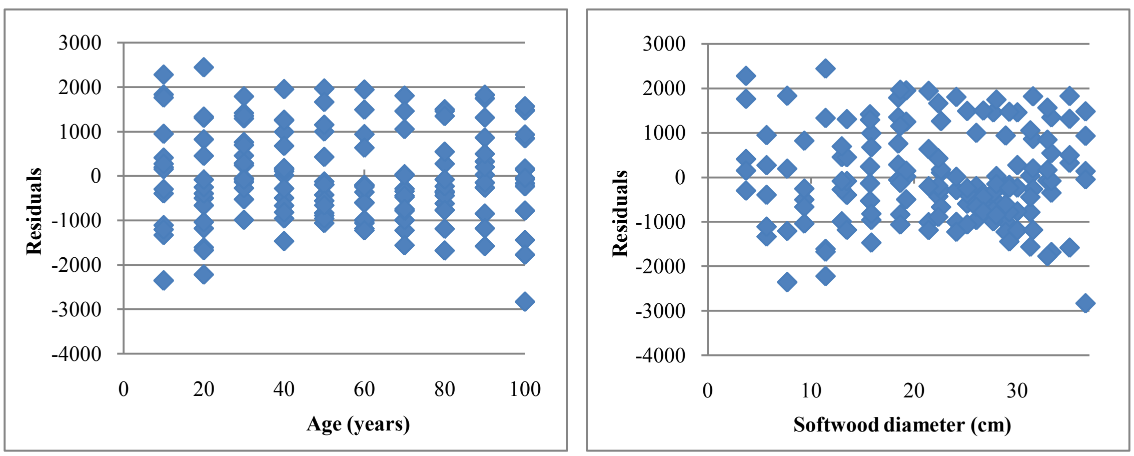


Figure 5-5 – Residual plots for the multiple regression results shown in table 5-18

The randomness of the residuals and the very high F-statistic for the regression seem to indicate that the multiple regression approximation architecture closely models the CTG function values yet the value of the residuals in figure 5-4 tell a different story. The maximum and minimum values for the residuals are respectively $2,444.84 and $-2,831.60. DP is sensitive to the values of the CTG function approximations as it will always take the highest value when choosing between more than one option. If the multiple regression shown above can overestimate the CTG function value by almost $2,500 for any given state, the DP optimization is affected. In addition, it is inconsistent in its function approximation. For example, for a stand defined by age = 80 years, d^S = 30.0cm and cc = 40%, the CTG function approximation is $20,245.50 while for a stand defined by age = 80 years, d^S = 33.3cm and cc = 40%, the CTG function approximation is $19,974.52. The actual CTG value for these two stands is identical at $20,519.45. The analysis discussed above for TRT=4 was repeated for TRT=2 and TRT=5 and that analysis led to the development of regression models with the terms listed in table 5-17.

Figure 5-6 shows the multiple regression CTG function approximation for iteration 1 of the deterministic base case scenario (TRT=2). This graph was created by selecting a series of 5 evaluation states and 17 result states that closely represent the growth, using NS DNR GNY models, of a randomly chosen initial state and plotting its actual CTG function values, in the case of the evaluation states, and its approximate CTG function values, in the case of the result states.   Those states are given in table 5-20.

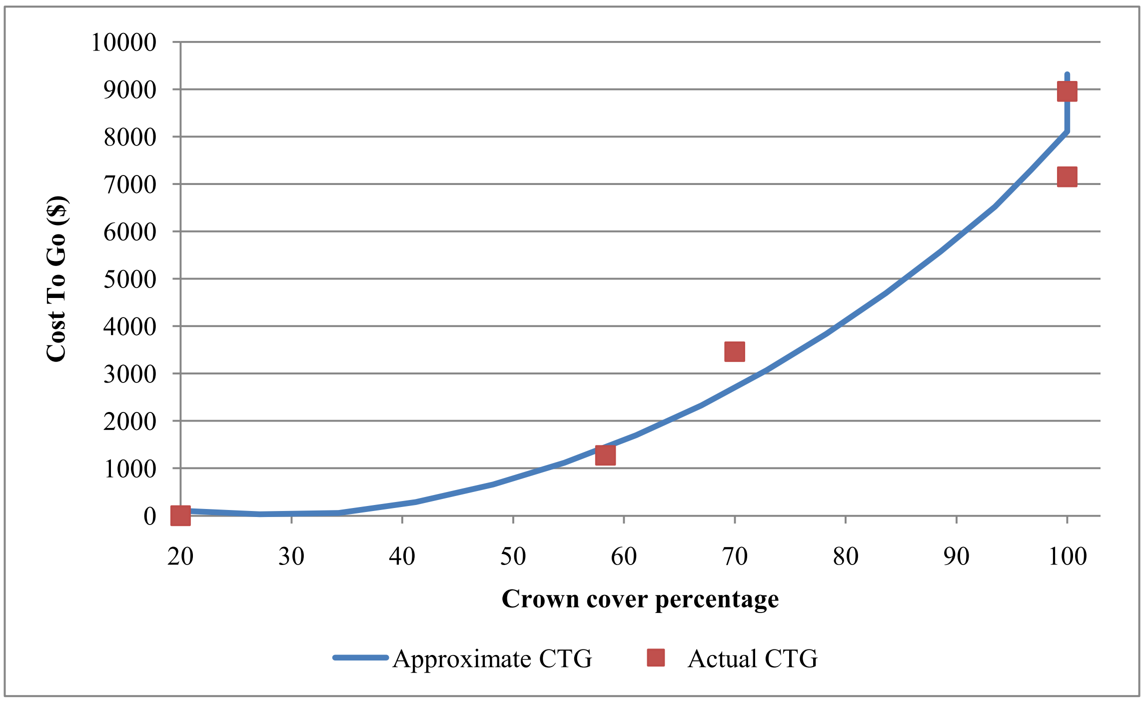


Figure 5-6 – Actual and approximate CTG values for the same optimization as above but for TRT=2 (multiple regression CTG function approximation)

The irregular placement of the CTG function values for evaluation states in figure 5-6 is caused by the fact that we are only plotting the CTG values for 1 of 5 dimensions. The graph is different when plotted against the other 4 variables but plotting them would lead us to the same conclusions so they are omitted. The graph shows portions of the set of result states where it is overestimating the CTG function and others where the opposite occurs. For the set of result states and CTG function values plotted in figure 5-6, multiple regression does a decent job of approximating the CTG function but as shown on the residual plots of figure 5-5, there may be large discrepancies in CTG function approximations for many states.

Table 5-20 – Evaluation and result states used to create the graph in figure 5-6

	Age	d^S (cm)	d^H (cm)	cc (%)	pct^S (%)
Evaluation states	20	6.5	5.5	33	20
	30	11.1	9.7	33	58.3
	40	15.5	12.1	33	70
	50	18.5	13.9	33	100
	60	21.3	15.3	33	100
Result states	20	6.5	5.5	33	20
	22.5	8.0	6.7	30	27
	25	9.5	8.0	26	34
	27.5	10.7	8.8	27	41
	30	11.9	9.7	28	48
	32.5	13.0	10.3	28	55
	35	14.0	11.0	29	61
	37.5	15.0	11.6	29	67
	40	15.9	12.1	30	73
	42.5	16.7	12.6	30	78
	45	17.5	13.1	30	84
	47.5	18.2	13.5	31	89
	50	18.9	13.9	31	93
	52.5	19.5	14.2	31	97
	55	20.1	14.6	31	100
	57.5	20.7	14.9	32	100
	60	21.3	15.3	33	100

Large discrepancies in CTG function approximations have led to the implementation of the CTG bounds explained in section 4.4.1. Upper and lower bounds are calculated for each result state where a CTG approximation is required. This is done at every iteration of the DP algorithm as the CTG values are changing thus the bounds must follow. If an approximation falls outside those bounds, it's adjusted to be within the calculated bounds by lowering or raising its value but if it is within the bounds, no adjustment is made.

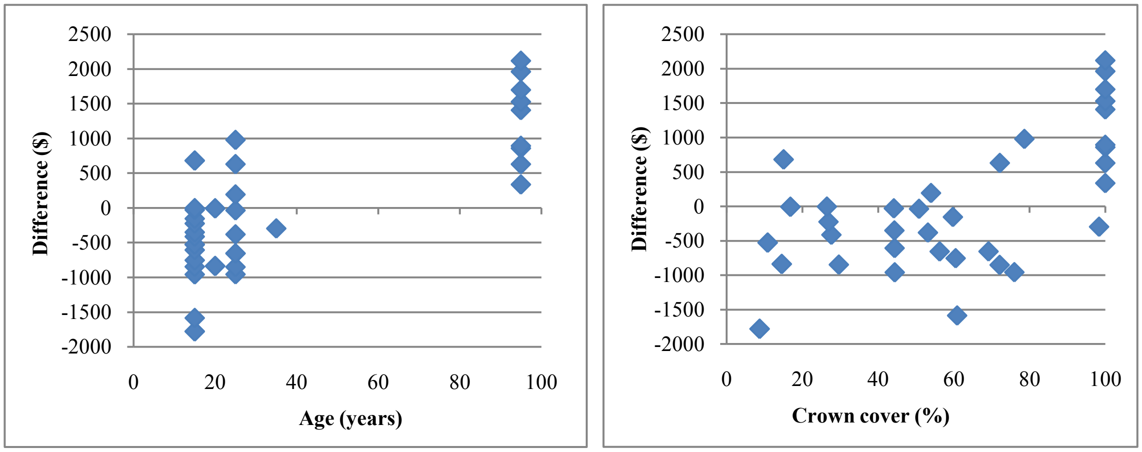


Figure 5-7 – States for which the approximate CTG value is outside the CTG bounds and the difference indicates by how much they have to be adjusted

For iteration 1 of the DP optimization of the base case scenario with all uncertainty removed using the multiple regression approximation architecture for TRT=2, figure 5-7 shows graphs of the states for which the approximate CTG value needs to be adjusted. The graph on the left indicates that young stands are particularly difficult to approximate and generally give CTG values which are below the lower limit. At young ages, stands grow quickly and the value of wood on the stand grows rapidly therefore, the quadratic regression equation can't react to this sudden change in CTG values. At old ages, once the stand has reached maximum MAI and it starts to decline, the rate of growth of value of a stand slows. The regression equation doesn't react well to this change. Putting in place CTG bounds ensures that the CTG value of young and old stands aren't under or over estimated.

In table 5-21, a set of result states was chosen for TRT=2 to illustrate the importance of using upper and lower approximation levels with the multiple regression approximation architectures. The table shows 9 result states that need the largest adjustment. For example, the CTG function approximation for result state 9, using MR, is $38,400.90. But the highest and lowest values observed for the CTG function in the neighbourhood of result state 9 at the previous DP iteration were respectively $36,281.23 and $15,422.10. The lack of use of an upper approximation bound would lead to an over-evaluation of $2,119.67. The lack of use of approximation bounds for these 9 result states would lead to an over-evaluation of a forest stand by as much as 18.2% in one case and an under-evaluation of $1,779.73 in another. Setting these bounds and adjusting the value of the CTG function approximation accordingly ensures consistency in CTG function approximations.

Table 5-21 – 9 result states, from TRT=2 in the base case scenario with all uncertainty removed, that need the largest adjustments of CTG value approximations

State	Lower bound	CTG approximation	Upper bound	Difference
1	$0	$-1,779.73	$0	$-1,779.73
2	$0	$-1,779.73	$0	$-1,779.73
3	$0	$-1,586.25	$5,385.63	$-1,586.25
4	$0	$-958.98	$0	$-958.98
5	$0	$6,364.65	$5,385.63	$979.02
6	$15,422.10	$37,810.31	$36,281.23	$1,529.07
7	$7,709.61	$14,539.82	$12,839.93	$1,699.89
8	$7,709.61	$14,802.79	$12,839.93	$1,962.86
9	$15,422.10	$38,400.90	$36,281.23	$2,119.67

5.2.2 Radial Basis Functions

We recall from section 4.4.1 that, in order to ensure exactness of the method, we will force $\tilde{J} = J(i_n)$ at all centers i_n where $J(i_n)$ are the CTG function values at those centers. We also recall the set of equations $J(i_n) = \sum_{m=1}^{M} r_m \varphi(|i_n - i_m|_2)$ for $n = 1 \ldots m$ used to calculate the weights r_m. We assume that φ and p are given and fixed thus we have *m* unknowns, the r_m, and *m* equations giving a linear system A × r = *f*, where the elements of A are given by $\varphi(|i_n - i_m|_2)$ and *f* = $J(i_n)$. If **A** is invertible, we have $r = f \times \mathrm{A}^{-1}$ and by substituting in $\tilde{J}(j, r)$, we have an interpolation equation. The results in this section will focus on two of the four forms of RBF mentioned in section 4.4.1, namely the thin plate spline $(\varphi = \delta^2 \ln \delta)$ and the multiquadric $\left(\varphi = (\delta^2 + \omega^2)^{1/2}\right)$ where ω is a tunable shape parameter.

Matrix inversion is impacted by the condition of the matrix to be inverted. In the context of this research, the condition of the matrix is a function of the number of centers used for the RBF and the scaling of the distances between those centers. Scaling, in all cases, improves the condition of the matrix. However, there is a trade-off between the improved condition of the matrix due to the reduction in the number of centers, and the quality of the CTG function approximation due to a higher number of centers. A lower number of centers yields a more accurate inverted **A** matrix ($\mathbf{A}^{-1}$) for the calculation of the

$\boldsymbol{r}$ parameter vector. However, when approximating the CTG function using $\tilde{J}(j,r)$ with a small number of centers, the shape of the RBF may cause the CTG approximation to be much higher or lower than the actual CTG value. Figure 5.8 shows an example of the use of RBF's for CTG function approximation for TRT=2. On this graph, the CTG function is shown for 8 evaluation states (in red) as a function of stand age. The full basis RBF and reduced basis RBF are plotted on the same graph by varying the value of the 5 variables that define a TRT=2 stand as follows: if the first 2 evaluation states on the graph are represented by i_1 and i_2, the states used for plotting the two RBF curves are given by $i_{plot} = \lambda i_1 + (1-\lambda)i_2$ where λ varies from 0 to 1. Both RBF curves fit exactly at the evaluation states which were used as basis points. There are only slight differences between the full basis and reduced basis RBF's for the majority of the range being shown in figure 5.8. This might indicate that we can take advantage of the reduced computation associated with the reduced basis RBF. However, there is an exception at low ages where the CTG function approximation is negative which is infeasible. With fewer basis points through which it is forced to pass, the reduced basis RBF can have some difficulty fitting on portions of the range of variable values. The use of CTG approximation bounds may eliminate these errors. A note is made about their use with RBF's at the end of this section.

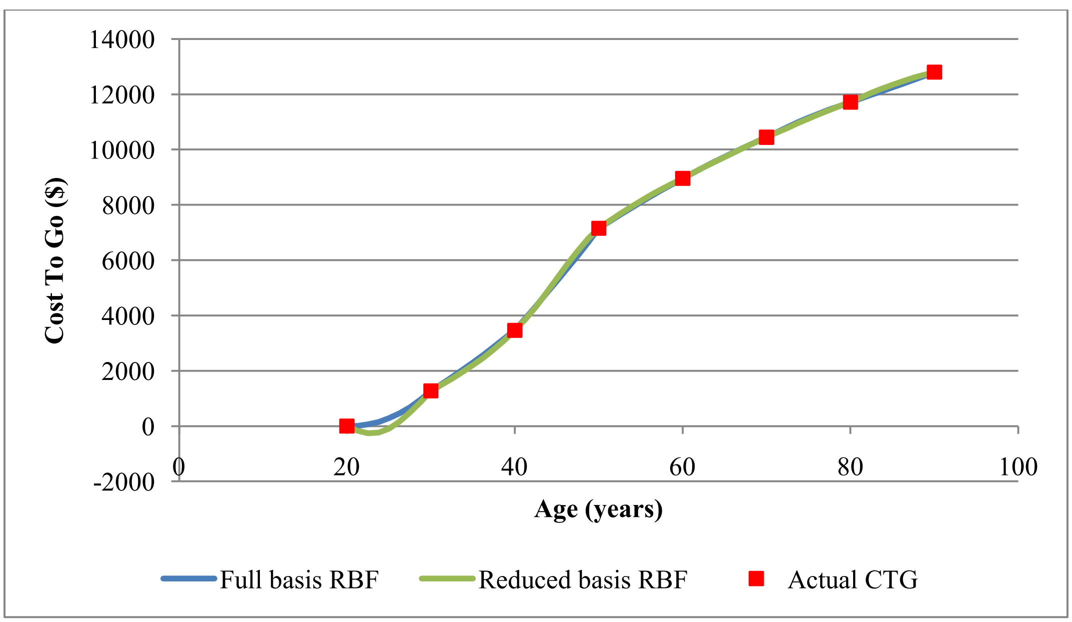


Figure 5-8 – RBF fits for TRT=2

In the DP model, the **A** matrix is formed by the application of the chosen RBF to the distance between the basis points chosen out of the set of evaluation states that represent the continuous state space of individual forest stand descriptors. This distance is calculated using the Euclidian norm on 3 or 5 dimensional data sets. These data sets are a combination of forest stand descriptors with a wide range of values such as 0 to 105 for the age and 0 to 1 for the softwood percentage of the stand. Before calculating the Euclidian norm, each dimension of all discrete states is transformed to a value on a scale between 0 and 1. For example, treatment type 3 stands are defined by three variables: age, softwood diameter and initial planting density. The maximum discrete value for age in the model is 95 years, the maximum softwood diameter is 29.1cm and the maximum initial planting density is 4000. Given 1 and 2 below:

State 1: age = 50 years, d^S= 20cm, initial planting density = 2000
State 2: age = 60 years, d^S= 22cm, initial planting density = 2400.

The scaled distance or scaled Euclidian norm for these two states is calculated as:

$$Distance = \left(\frac{60\text{-}50}{95}\right)^2 + \left(\frac{22\text{-}20}{29.1}\right)^2 + \left(\frac{2400\text{-}2000}{4000}\right)^2 = 0.0258$$

Without scaling, the distance would be 160,104 which is largely dominated by the initial planting density. All elements of the **A** matrix for treatment types 2, 4 and 5 are calculated using this transformation of the Euclidian norm. In order to avoid the problem of fitting between basis points that is depicted in figure 5-8, all evaluation states are used as basis points for the RBF in the DP model and scaling is done for all of them. The use of such large basis, 294 in the case of TRT=2 and 1080 in the case of TRT=5, leads to a very large **A** matrix that needs to be inverted. Without scaling of distances, the condition of the **A** matrix for TRT=2 is 1.9502×10^{18} versus 6.9496×10^{7} for the **A** matrix with scaled distances. Both these condition numbers are nowhere near the scale of the condition number for the matrix inverted for the example in figure 5-8. The larger the condition number, the more difficult it is to invert the matrix with any level of accuracy even with the best inversion algorithms. Therefore, it is critical to properly scale the distances.

GNU Octave (Eaton 2009), an open source software similar to Matlab, was used to do the matrix inversion. It was also used to calculate the condition number of the matrix to invert. Figure 5-9 shows the difference between the approximate CTG values and actual CTG values for the 294 basis points chosen as the centers for fitting the RBF. The 294 centers represent 100% of the discrete states for TRT=2 in the DP model. The distances in this case are scaled. The largest absolute difference between the approximate CTG values and actual CTG values for the 294 basis points is 0.00809. If the matrix inversion was done without error, the difference would be zero.

Without scaling of the distances, the results are much different. Figure 5-10 shows those differences. In this second example, the ill-conditioning of the **A** matrix yields poor approximations at the basis points. These differences should all be zero if the inversion could be done without loss of precision. The matrix inversion was done using the same inversion algorithm for the results in figure 5-9 and 5-10.

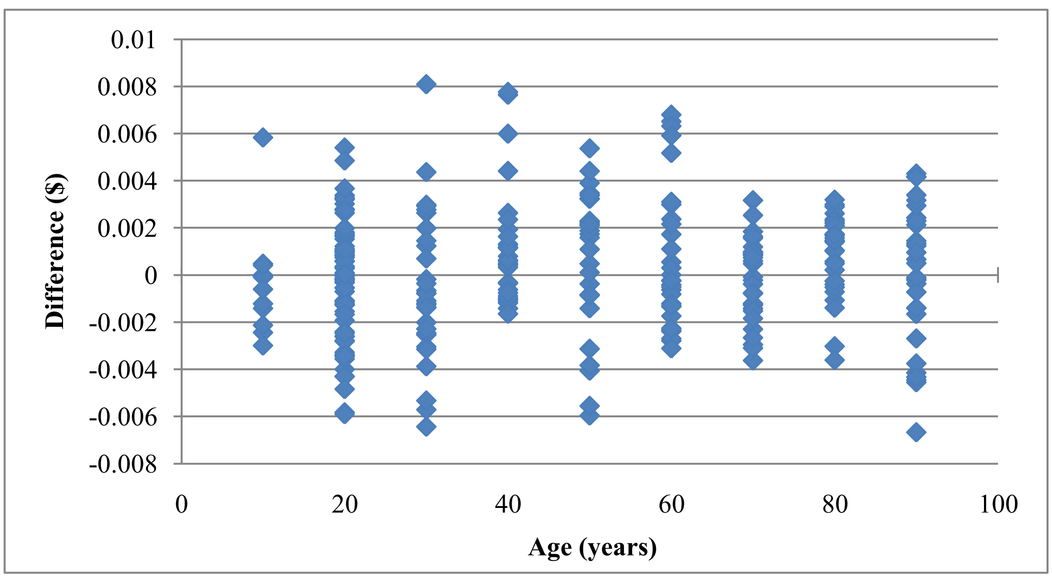


Figure 5-9 – Difference between approximate CTG value and actual CTG for the 294 discrete states for TRT=2 with those 294 states being used as centers in the RBF with distances scaled to 0-1

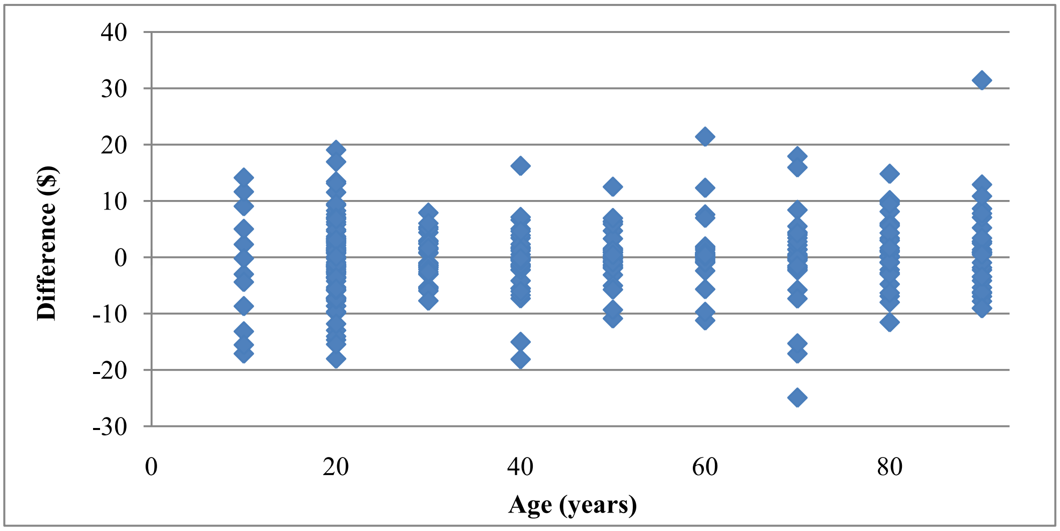


Figure 5-10 – Difference between approximate CTG value and actual CTG for the 294 discrete states for TRT=2 with those 294 states being used as centers in the RBF without scaled distances

The beginning of this section discusses the trade-off between the additional accuracy gained by adding centers to the basis used for the RBF versus the added difficulty in doing the matrix inversion with additional centers. One eighth (37) of the 294 evaluation states for TRT=2 were randomly chosen to cover the entire range of state variable values and, with proper scaling of the distances between these centers, the condition number of the **A** matrix is 5.8042×10^4 which is three orders of magnitude better than the condition number for the matrix with all evaluation states being used as centers in the RBF. In the case of the smaller number of centers, the largest absolute difference between the actual CTG value and the approximate CTG value at the 37 centers is 1.3424×10^{-10}. In contrast with differences in the case where all discrete states are used as centers, this is 7 orders of magnitude better. However, as discussed earlier, there is a trade-off for this additional precision. Figure 5-11 shows the distribution of the differences at the 294 discrete states when 37 centers are used for the RBF. Although it's difficult to show the behaviour of the CTG approximation architecture between the centers in a meaningful way, it clearly doesn't perform as well as the full basis.

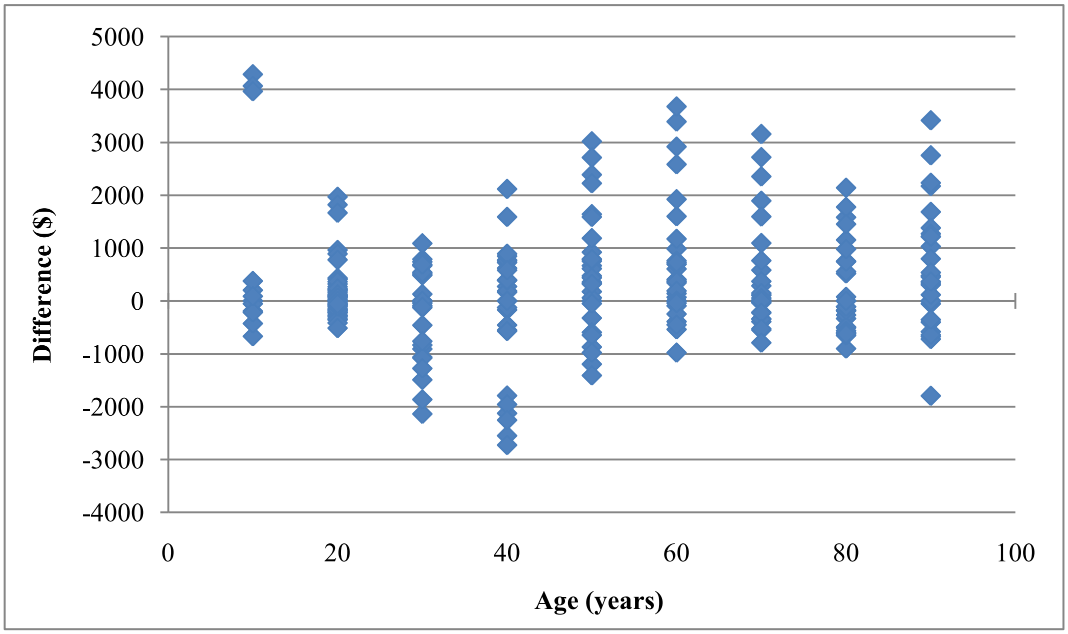


Figure 5-11 – Difference between approximate CTG value and actual CTG for the 294 discrete states for TRT=2 with 37 states being used as centers in the RBF with scaled distances

Function approximation using RBF's requires a high degree of precision and is very sensitive to a small change in the **A** matrix. For example, multiplying $\boldsymbol{f} \times \mathbf{A}^{-1}$ to obtain $\boldsymbol{r}$ using single precision variables in Visual Basic yields a maximum absolute error of 2582.03 in the CTG value at the centers whereas, using double precision, the maximum absolute error is 0.00809. Adding precision, when doing a calculation as simple as a matrix multiplication, yields more accurate results.

All of the RBF results discussed so far use thin plate splines. In the case of the inverse multiquadric, the shape parameter ω has a big impact on the capacity of the RBF to fit the CTG function values with accuracy. Figure 5-12 shows the differences between the actual CTG and approximate CTG at the 294 centers using the multiquadric with ω = 1 and ω = 0.1. The condition number of the **A** matrix are, in the case of ω = 1, 2.8051 x 10^{12}, and in the case of ω = 0.1, 4.2821 x 10^{9}. The latter gives better precision. As discussed in section 4.4.1, there are no algorithms to find the optimal value of the shape parameter ω, only good values. Therefore, given the precision of the thin plate spline, no further effort has been put into finding a better value for ω.

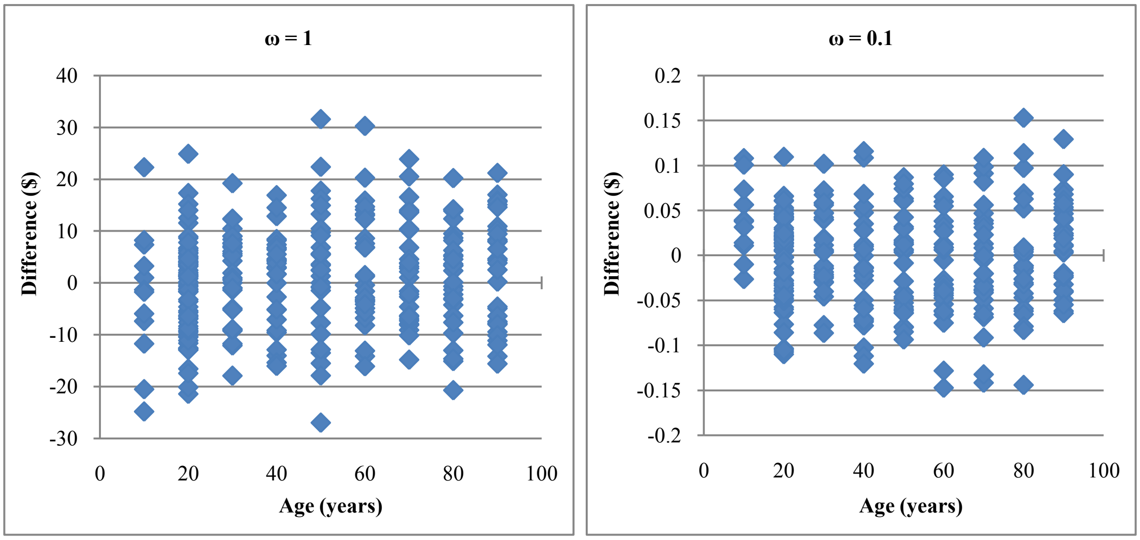


Figure 5-12 - Differences between the actual CTG and approximate CTG at the 294 centers using the multiquadric with $\omega = 0.1$ and $\omega = 1$

Given that RBF's give such good approximations when using a large number of centers, the addition of CTG bounds to the process of approximating the CTG function with the RBF doesn't seem necessary. A DP optimization of the base case scenario described at the beginning of this chapter was done using the thin plate spline RBF as the approximation architecture. During iteration 10 of the optimization, statistics were collected on the number of states with approximate CTG values which fell outside the CTG bounds. None were observed which is in stark contrast with the results discussed in the previous section. Although RBF's are more complicated to calculate than multiple regression, the additional accuracy combined with the processing capability of present day computers, which is adequate and provides solutions to the DP model in a matter of minutes, are reason enough to use RBF's over MR for DP.

5.2.3 Distance Weighted Interpolation

In section 4.4.2, we discussed the importance of properly scaling the distances between points when implementing DWI as an approximation architecture in DP. Results in this section show the impact that a slight modification in the weighting scheme can have on CTG value approximation.

From section 4.4.2, we recall that

$$weight_{mj} = \left(\frac{\left(\sum_m d_{mj}\right) - d_{mj}}{\left(\sum_m d_{mj}\right) * d_{mj}}\right)^2 \text{ where } m = 1,\ldots,v$$

where v is the number of evaluation states used for approximation and d_{ij} is the Euclidian norm between evaluation states i and result states j. Finally, the weight r_{ij} is calculated and used for the approximation $\tilde{J}_{k+1}(j,r)$.

$$r_{ij} = \frac{weight_{mj}}{\sum_m weight_{mj}} \qquad \tilde{J}_{k+1}(j,r) = \sum_{i \epsilon M_j} r_{ij}\left(J_k(i)\right) \qquad \sum_{i \epsilon M_j} r_{ij} = 1$$

When using DWI for function approximation where there is a large set of evaluation states representing the space where the function is being approximated, it makes sense to use a small portion of this set (M_j) to approximate CTG function at each result state j. M_j should be sufficiently large for the approximation to closely follow the changes in the function it is fitting. Table 5-22 shows the CTG function values for a set of evaluation states for TRT=2 from the same DP optimization as table 5-23. Table 5-23 shows a list of result states that create a path through time for a TRT=2 stand that starts with state variable values shown in the first line of the table and grows for 70 years. The CTG function values shown in the last three columns are approximations of the CTG function at the result states during the first iteration of the DP optimization of the base case scenario. The approximations are done using DWI with v = 2, 10 and 40.

Figure 5-13 shows a plot of the values from tables 5-22 and 5-23. With v = 2, there are large approximation errors as the approximation moves through the state space from one result state to the next. The approximations are exact at the evaluation states but don't do a good job of interpolating between the evaluation states because there aren't enough states being used to do the interpolation. The approximations with v = 10 and v = 40 are almost undistinguishable from one another except for the range of 40 to 50 years. The CTG approximation shown in figure 5-13 is typically how DWI function fitting behaves in the DP model and is a reasonable method of approximating a function which, in this case, has 5 continuous variables and uses only 294 evaluation states to cover its entire range.

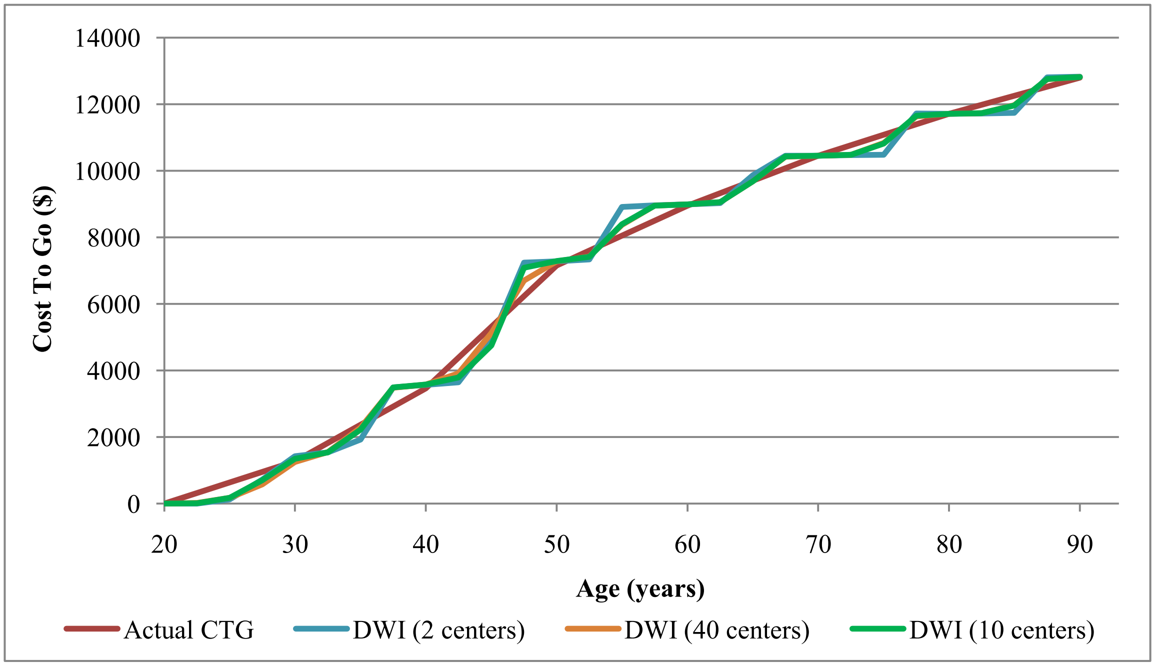


Figure 5-13 – Plot as a function of age for the CTG function values and DWI approximations given in tables 5-22 and 5-23

Table 5-22 – CTG function values for a set of evaluation states during the first iteration of the same optimization used to obtain the values in table 5-23

Evaluation states					CTG function values
Age	d^S (cm)	d^H (cm)	cc (%)	pct^S (%)	
20	6.5	5.5	33%	20	$0
30	11.1	9.7	33%	58	$1,273
40	15.5	12.1	33%	70	$3,461
50	18.5	13.9	33%	100	$7,152
60	21.3	15.3	33%	100	$8,955
70	24.0	16.8	33%	100	$10,446
80	26.6	18.2	33%	100	$11,713
90	28.5	19.7	33%	100	$12,798

Table 5-23 – CTG function value approximations for a set of result states during the first iteration of the base case scenario DP optimization using DWI with 2, 10 and 40 centers

Result states					CTG function approximations using DWI		
Age	d^S (cm)	d^H (cm)	cc (%)	pct^S (%)	2 centers	10 centers	40 centers
20	6.5	5.5	33	20	$0	$0	$0
22.5	8.1	6.8	30	27	$7	$2	$21
25	9.8	8.2	27	34	$178	$125	$178
27.5	11.0	9.1	28	41	$703	$722	$580
30	12.3	9.9	28	48	$1,352	$1,419	$1,248
32.5	13.3	10.6	29	55	$1,539	$1,537	$1,552
35	14.4	11.3	30	61	$2,225	$1,923	$2,279
37.5	15.4	11.9	30	67	$3,494	$3,495	$3,486
40	16.3	12.4	31	73	$3,573	$3,567	$3,576
42.5	17.1	12.9	31	78	$3,790	$3,642	$3,913
45	17.9	13.4	31	84	$4,751	$4,936	$5,121
47.5	18.7	13.8	32	89	$7,082	$7,238	$6,701
50	19.4	14.2	32	93	$7,289	$7,276	$7,280
52.5	20.0	14.6	32	97	$7,413	$7,336	$7,425
55	20.7	14.9	32	100	$8,396	$8,909	$8,378
57.5	21.3	15.3	32	100	$8,955	$8,963	$8,953
60	21.8	15.6	32	100	$8,992	$8,992	$8,992
62.5	22.4	15.9	32	100	$9,056	$9,030	$9,060
65	23.0	16.3	32	100	$9,683	$9,865	$9,679
67.5	23.6	16.6	32	100	$10,428	$10,448	$10,422
70	24.1	16.9	32	100	$10,447	$10,447	$10,447
72.5	24.7	17.2	32	100	$10,481	$10,469	$10,484
75	25.2	17.6	32	100	$10,819	$10,483	$10,835
77.5	25.8	17.9	32	100	$11,651	$11,715	$11,637
80	26.3	18.2	31	100	$11,713	$11,714	$11,713
82.5	26.8	18.5	31	100	$11,730	$11,723	$11,731
85	27.4	18.8	31	100	$11,962	$11,734	$11,965
87.5	27.9	19.2	32	100	$12,754	$12,801	$12,746
90	28.5	19.7	33	100	$12,818	$12,820	$12,817

The denominator in the $weight_{mj}$ function puts a large emphasis on any increase in distance between evaluation states i and result states j. This means that the value of the weights $r_{m,j}$ for any evaluation state beyond the first few closest states in the neighbourhood, regardless of v, decrease quickly. For example, table 5-24 gives ten distances and their corresponding weights for a randomly chosen result state j from TRT = 2, and its ten closest neighbouring evaluation states i. Although the distances for m = 1 and m = 2 are relatively close to each other, the weights heavily favour m = 1. Starting at m = 3, the weights are negligible. This is typically the case, therefore for the implementation of DWI in the DP model, we use v = 4.

Table 5-24 – Distances and weights for result state j and its 10 closest neighbouring evaluation states i

m	Distance d_{ij}	$r_{m,j}$
1	0.000696	0.941383
2	0.002785	0.05621
3	0.01763	0.00098
4	0.018854	0.00083
5	0.02151	0.000593
6	0.075952	2.76E-06
7	0.083091	7.95E-07
8	0.09	6.9E-08
9	0.090696	4.19E-08
10	0.092785	1.27E-09

Table 5-25 – Example CTG function approximation for the first four evaluation states of TRT = 2

Evaluation state i	Decision	m	States i that are closest to result state j	Distance	$r_{m,j}$	CTG value state j	Approximate CTG
1	2	1	2	0.008026	0.675757	$948.16	
1	2	2	100	0.011188	0.169413	$1,671.47	
1	2	3	3	0.011737	0.131690	$1,022.30	
1	2	4	101	0.014899	0.023140	$1,624.88	$1,096.12
2	2	1	102	0.003859	0.825683	$1,700.96	
2	2	2	101	0.007476	0.148587	$1,624.88	
2	2	3	100	0.014141	0.014986	$1,671.47	
2	2	4	5	0.015102	0.010743	$1,347.17	$1,685.42
3	2	1	102	0.008071	0.489278	$1,700.96	
3	2	2	5	0.008253	0.457503	$1,347.17	
3	2	3	101	0.014668	0.052231	$1,624.88	
3	2	4	4	0.022296	0.000989	$1,085.22	$1,534.52
4	2	2	102	0.015592	0.954662	$1,347.17	
4	2	3	101	0.025176	0.038047	$1,700.96	
4	2	4	4	0.025196	0.003656	$1,624.88	
4	2	5	3	0.03478	0.003635	$1,085.22	$1,360.70

The impact of the weighting scheme is shown in the examples given in table 5-25. Evaluation states 1 through 4 in the first column are the first 4 states out of 294 states representing TRT = 2. The second column indicates that the results shown in this table result from taking the same decision for all four states which is to do nothing and let the stand grow an additional 5 years. The fourth column gives the 4 evaluation states that are the closest to the result state, at time t+5 after taking action 2 in each evaluation state i at time t. Tables 5-26 and 5-27 show the value of the descriptor variables for the states in table 5-25.

The distances and weights for the four discrete states in table 5-25 indicate the large bias towards increasing distance. The table shows that many of the same states are being used for CTG function approximation but the large bias towards increasing distance results in large changes in CTG value approximation for these four young stands. Nevertheless, this bias towards increasing distance is necessary and has proved to work well because there are portions of the state space where the CTG value changes rapidly so it's important that the closest states be given a substantial portion of the total weight.

Table 5-26 – Four evaluation states for TRT = 2 shown in table 5-25 at period k with result states at period $k+1$ after 5 years of growth

	Evaluation states					Result states				
State	Age_k	d_k^S	d_k^H	cc_k	PCT_k^S	Age_{k+1}	d_{k+1}^S	d_{k+1}^H	cc_{k+1}	PCT_{k+1}^S
1	10	0	2.59	8.9	0	15	0	4.94	26.5	0
2	20	0	5.49	20	0	25	0	7.55	34.3	0
3	20	0	6.26	20	0	25	0	8.30	32.2	0
4	20	0	7.02	20	0	25	0	9.05	30.7	0

Table 5-27 – Values of the state variables for the evaluation states shown in column four of table 5-25

Evaluation states i	Age_k	d_k^S	d_k^H	cc_k	PCT_k^S
2	20	0	5.5	20	0
3	20	0	6.3	20	0
4	20	0	7.0	20	0
5	30	0	9.7	33.3	0
100	20	0	5.5	35	0
101	20	0	6.3	35	0
102	20	0	7.0	35	0

5.2.4 Choosing an Approximation Architecture

For the implementation of the DP model and for the results presented in this chapter, it is useful to choose one of the approximation architectures and keep it for all results in order to compare policies and CTG values across a broad spectrum of scenarios. Evidence is given through sections 5.2.1 to 5.2.3 that support the use of all three approximation architectures. The performance of DWI and RBF's is clearly superior to MR, therefore the final choice is made between the latter two architectures.

Figure 5-14 shows DWI and two versions of RBF used to fit the same path through time as that shown in figure 5-13.

Figure 5-14 – Actual and approximate CTG values for a typical individual forest stand (TRT=2) as it grows through time, plotted against the age of the stand

The reduced basis RBF, although easier to implement than the full basis, clearly suffers from a lack of performance in the lower portion of the graph. The full basis RBF doesn't suffer from the same difficulties because it covers the state space with a denser mesh of basis points which allows it to more closely approximate the CTG function. This is a reflection of the fact that, with RBF's, the approximations are forced to be equal to the CTG function values at the basis points. Therefore, more basis points means more contact points between the CTG function and the RBF function. For the set of states plotted in figure 5-14, the full basis RBF does a better job than the reduced basis RBF of approximating the CTG function.

Figure 5-15 shows the same approximations as those in figure 5-14 but plotted against crown closure instead of age. The last 5 evaluation states used for plotting all have cc = 100% and all three approximation architectures fit the CTG function values well between those points.

When comparing the approximation results of the RBF and DWI approximation architectures, we observe that DWI may make small sudden changes in CTG function approximations but its average error along the path in figures 5-14 and 5-15 is small. Because of the challenges encountered when implementing RBF's with Visual Basic, and because of the effectiveness of DWI in fitting CTG functions, all results discussed in this chapter are obtained using DWI as the approximation architecture in the DP model.

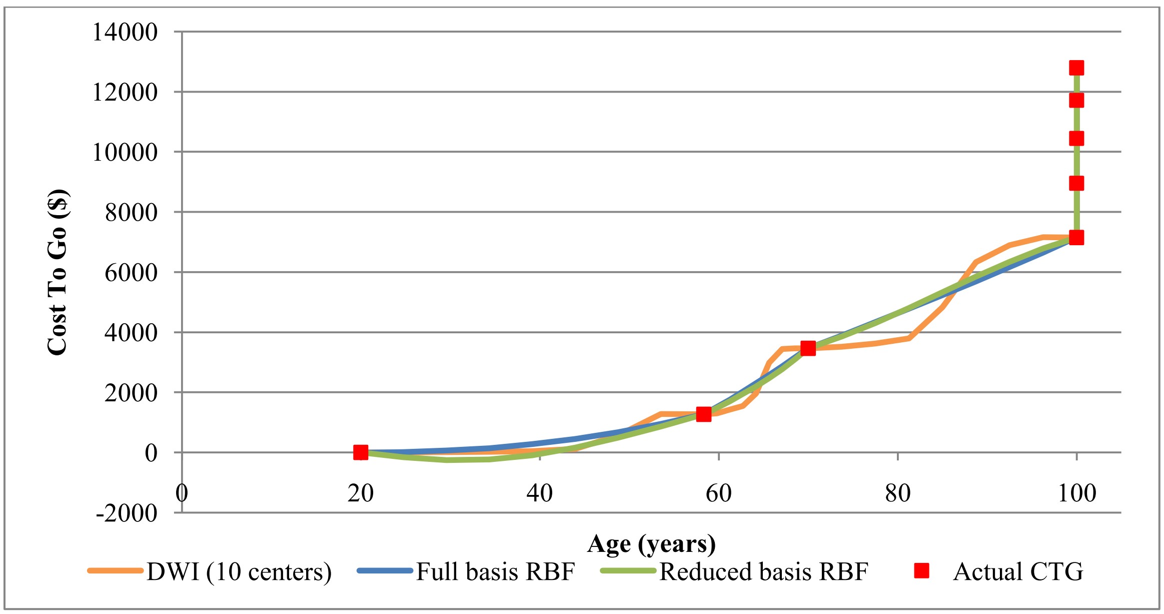


Figure 5-15 – Actual and approximate CTG values for a typical individual forest stand (TRT=2) as it grows through time, plotted against crown closure percentage

5.3 Optimization

This section focuses on the DP optimization and the factors that have an effect on its convergence. In infinite horizon DP, discounting and value iteration combine to create stable policies and CTG values. Section 5.3.1 discusses the importance of properly choosing the discount rate and its effect on the optimal policy. Directly related to discounting is the convergence rate of the DP. Higher discounting will ensure quicker convergence to the optimal policy and the optimal CTG value. The stopping criteria for value iteration DP is an important factor in determining when the optimal CTG value has been reached and the results in section 5.3.2 will discuss the impact of that stopping criteria. As discussed in section 4.7, policy simulation is used to verify that the optimal policy and the optimal CTG value obtained from the DP optimization match. Section 5.3.3 discusses the results of that simulation.

5.3.1 Effect of the Discount Rate

In this section, the results focus on the effect of the discount rate on CTG values and optimal policies. As can be seen, the discount rate has a significant effect on policies. In practical applications, it would be up to the decision maker to choose the policy that matches their guiding interest rate.

Table 5-28 shows how policies change for natural stands as the interest rate rises.

Table 5-28 – Policy for fully stocked natural stands for the base case scenario with variable yearly interest rate

		Yearly interest rate										
Age	Stocking	0.1%	1%	2%	3%	4%	5%	6%	8%	10%	12%	15%
5	100%	1	1	1	1	1	1	1	1	1	1	1
10	100%	12	12	12	12	12	12	1	1	1	1	1
15	100%	1	12	12	12	12	12	12	1	1	1	1
20	100%	1	1	1	1	1	1	1	1	1	1	1
25	100%	1	1	1	1	1	1	1	1	1	1	1
30	100%	1	1	1	1	1	1	1	1	1	1	1
35	100%	1	1	39	39	39	39	39	39	39	39	3
40	100%	38	38	38	38	38	38	38	38	3	3	3
45	100%	38	38	38	38	38	38	38	3	3	3	3
50	100%	38	37	38	38	38	38	4	3	3	3	3
55	100%	37	37	37	37	4	4	4	3	3	3	3
60	100%	37	37	37	4	4	4	4	3	3	3	3
65	100%	37	37	37	4	4	4	4	3	3	3	3
70	100%	7	7	6	4	4	4	4	3	3	3	3
75	100%	7	7	6	4	4	4	4	3	3	3	3
80	100%	7	7	6	4	4	4	4	3	3	3	3
85	100%	7	7	6	4	4	4	4	3	3	3	3
90	100%	7	7	6	4	4	4	4	3	3	3	3
95	100%	7	7	6	4	4	4	4	3	3	3	3

1 - Let grow	6 - ReHar, plt 1750 tr/ha	37 - CT, rmv 40% BA, splt 25% (abv)
3 - ReHar, natural regen	7 - ReHar, plt 2500 tr/ha	38 - CT, rmv 40% BA, splt 50% (abv)
4 - ReHar, fill plant, no ECC	12 - PCT, remove HW	39 - CT, rmv 40% BA, splt 75% (abv)

Very low interest rates put emphasis on volume MAI and try to maximize it as there is very little loss in present value of future profits. We don't have MAI charts for mixed species stand in Nova Scotia but volume maximum MAI for a typical TRT=1 pure softwood stand is between 60 and 70 years for a stand with SI^S = 17m at 50 years of age. This is reflected in the policy in table 5-28. At a very low interest rate of 0.1%, a regeneration harvest would only be optimal for a stand that is 70 years or more but there are many CT opportunities from age 40 to age 65. CT creates profit at the time of the harvest but also leaves trees for future harvesting and revenue creation. Because of the slow rate of decline in present value due to the low interest rate, it is optimal to wait for maximum volume MAI before doing a harvest. In contrast, with a guiding rate of 15%, the stand should be harvested at age 35 which is the first age at which the stand has any harvestable volume. Future profits are discounted so much that as soon as the stand can create some profit, it is harvested and no future investment is made by planting or doing a PCT or CT. Table 5-28 shows a clear transition in policy from very low to very high interest rates for fully stocked natural stands.

Decision 12 is a PCT where hardwood trees are removed and softwood trees are spaced accordingly and, at low interest rates, it makes sense to invest in doing a PCT to release the trees on the stand as that is an investment in future growth of the stand. In terms of the regeneration harvest, the age at which to do the regeneration harvest and what to do with the stand after the regeneration harvest shifts as the interest rate goes up. At low interest rates, it is optimal to plant more trees per hectare which cost more to do. At a higher planting density, the diameter of the trees will grow more slowly but there will be more of them therefore, in the long term it is worth the investment. At 2% interest, fewer trees are planted (1750 vs. 2500) and starting at 3%, the optimal policy is not to grow a plantation but rather to fill plant which is much cheaper than planting and the result is a fully stocked natural stand. And at very high interest rate, the optimal decision is to do the regeneration harvest and let the stand regenerate on its own without any monetary investment other than surveying the land. And all of these regeneration harvests are occurring at younger and younger ages as the interest rate goes up.

However, the case with 75% stocked natural stands shows different policies that warrant discussion. These policies are found in table 5-29. Focusing on the policy for

4% interest rate, the change from optimal policy 1 to optimal policy 12 at age 25 requires further investigation. We would expect that once the policy has changed from 1 to 12 and back to 1, it would remain 1 until there is enough wood volume to warrant either a CT or a regeneration harvest.

Table 5-29 – Policy for 75% stocked natural stands for the base case scenario with variable yearly interest rate

		Yearly interest rate										
Age	Stocking	0.1%	1%	2%	3%	4%	5%	6%	8%	10%	12%	15%
5	75%	1	1	1	1	1	1	1	1	1	1	1
10	75%	12	12	12	12	12	12	1	1	1	1	1
15	75%	12	12	12	12	12	12	12	12	1	1	1
20	75%	1	1	12	1	1	1	1	1	1	1	1
25	75%	1	1	12	12	12	12	12	12	1	1	1
30	75%	1	1	1	1	12	12	12	12	12	1	1
35	75%	1	1	1	1	1	1	1	1	3	3	3
40	75%	31	31	31	31	31	31	31	3	3	3	3
45	75%	31	31	31	31	31	31	31	3	3	3	3
50	75%	31	31	31	31	31	4	4	3	3	3	3
55	75%	34	34	34	34	34	4	4	3	3	3	3
60	75%	34	34	6	4	4	4	4	3	3	3	3
65	75%	7	7	6	4	4	4	4	3	3	3	3
70	75%	7	7	6	4	4	4	4	3	3	3	3
75	75%	7	7	6	4	4	4	4	3	3	3	3
80	75%	7	7	6	4	4	4	4	3	3	3	3
85	75%	7	7	6	4	4	4	4	3	3	3	3
90	75%	7	7	6	4	4	4	4	3	3	3	3
95	75%	7	7	6	4	4	4	4	3	3	3	3

1 - Let grow	6 - ReHar, plt 1750 tr/ha	31 - CT, rmv 20% BA, splt 25% (abv)
3 - ReHar, natural regen	7 - ReHar, plt 2500 tr/ha	34 - CT, rmv 30% BA, splt 50% (abv)
4 - ReHar, fill plant, no ECC	12 - PCT, remove HW	

Table 5-30 shows CTG function values for a discount rate of 4%, for decisions 1 and 12, and for ages 5 to 40. $Currentnet$ is the profit in the current period from taking decision 1 or 12 with the current stand, $futurenet$ is the approximate CTG function value of ending up in state j after having taken decision u in state i, and $totalnet$ is the $currentnet$ plus the discounted $futurenet$. From these results, we can see that there isn't much difference between the optimal CTG value when we do nothing and when we do a PCT at ages 10, 20 and 30. Both decisions 1 and 12 have monotonically increasing $totalnet$ values as the age increases but they increase at different rates. At age 20, the difference in totalnet value between decisions 1 and 12 is only $72 and a slight difference in approximations of $futurenet$ values for decisions 1 and 12 could have led to a different optimal policy at age 20. Section 5.2.3 discussed some of the effects of the weighting scheme in DWI and the results shown in table 5-30 were obtained using DWI as the approximation architecture in the DP. Therefore, it isn't surprising that the policy is slightly different than expected.

Table 5-30 – CTG values of natural stands (TRT=1) for a 4% discount rate

Age	Decision	Currentnet	Futurenet	Totalnet
5	1	0	$3,516.44	$2,879.01
10	1	0	$4,220.04	$3,455.07
10	12	-750	$5,273.72	$3,567.76
15	1	0	$4,431.35	$3,628.08
15	12	-750	$6,176.41	$4,306.82
20	1	0	$5,536.43	$4,532.84
20	12	-750	$6,363.50	$4,459.99
25	1	0	$5,643.63	$4,620.61
25	12	-750	$7,875.46	$5,697.88
30	1	0	$6,989.88	$5,722.83
30	12	-750	$8,022.00	$5,817.86
35	1	0	$8,847.54	$7,243.76
40	1	0	$9,858.77	$8,071.68

Section 4.3 puts emphasis on properly choosing the guiding rate that represents the decision maker's preference. Results in this section show that changing the discount rate has a big impact on the optimal policy and the CTG function value. Therefore, choosing that discount rate is indeed very important. The next section discusses the rate of convergence of the DP optimization where the discount rate also has an impact.

5.3.2 Rate of Convergence and Termination Criterion

The most important factors in the convergence of the value iteration algorithm implemented in this thesis are the value of the discount rate, the value of the stopping criteria and the consistency of the CTG function value approximations. Results in section 5.1 show that CTG function approximations consistently represent the known CTG function values they are approximating. Therefore, results in this section focus on the other two important factors.

The equations used to determine when the DP algorithm should be stopped are given in section 4.5. They are presented here for convenience:

$$c_k^L = \frac{\alpha}{1-\alpha} \min_{i=1,\dots,n} [T^k J(i) - T^{k-1} J(i)]$$
$$c_k^U = \frac{\alpha}{1-\alpha} \max_{i=1,\dots,n} [T^k J(i) - T^{k-1} J(i)]$$

$$c_k^U - c_k^L \le \varepsilon$$

where ε is the stopping criterion and α is the discount factor. Table 5-31 shows results for the base case scenario with the value of the stopping criterion set to 0.2 and DWI used for CTG function approximations. These optimizations were done using compiled VB, running in the Vista Business Operating System (OS) on a Toshiba Tecra M8 laptop with a 2.0 gigahertz dual core Intel processor with 2MB L2 cache and 800 megahertz front side bus, and 3GB of DDR2 SDRAM running at 667 megahertz.

Table 5-31 – Results for the base case scenario with stopping criterion = 0.2 and DWI used for CTG function approximation

α (%)	# iterations for convergence	Iteration # of last policy change	Largest CTG value change at last policy change	CPU time (seconds)
Set-up	---	---	---	905
0.1	835	258	$244.70	125
1.0	199	65	$81.25	47
2.0	83	50	$13.40	17
3.0	57	30	$28.80	56
4.0	43	23	$22.81	12
5.0	34	16	$54.47	111
6.0	28	14	$52.24	12
8.0	20	9	$52.38	96
10.0	17	10	$6.84	4
12.0	14	9	$9.54	9
15.0	11	3	$1,174.14	53

There is a clear relationship between the discount rate and the number of iterations to convergence. The first line shows a CPU time of 905 seconds for set-up and is the time needed for the VB code to set-up the Excel application and to do the preliminary calculations described in steps 1, 2 and 3 in section 4.6. 905 seconds is the shortest time observed during a test of 10 set-up runs discussed below. The rest of the values of CPU time are the time to convergence of the DP algorithm, starting with the first iteration, assuming the set-up time of 905 seconds was always the same for each optimization which may not be the case. Clearly, there is some inconsistency in CPU time for the total optimization and set-up times. Set-up time variability was tested by running the set-up portion of the DP optimization 10 times, on the same laptop, while performing simple web surfing and word processing operations, which are the same conditions under which the optimizations in table 5-31 were done. The average set-up time for those 10 test runs was 934 seconds with a standard deviation of 22.02. The minimum observed set-up time was 905 seconds and the maximum was 981 seconds. This leads to us to believe that simple tasks such as word processing and web browsing, which were done during the DP optimizations, may have affected the CPU times. Table 5-31 also shows the number of iterations to convergence versus the iteration number of the last policy change. The number of policy changes is recorded during each iteration of the DP algorithm. If the policy for an evaluation state changes from one iteration to the next, it is counted as one policy change for that iteration. This verification is done for each evaluation state at each iteration and the number of policy changes is recorded for each iteration and shown in table 5-31. The iteration number shown in column three is the last iteration during which a policy change was recorded. The fourth column of the same table shows the largest CTG function value change for the last iteration where there is a policy change. Clearly, regardless of the discount rate, the CTG function values have not converged when the policy stops changing. Table 5-32 shows results for the base case scenario with discount rate = 3%.

We recall that the value iteration algorithm is terminated when the maximum CTG value change $(c_k^U - c_k^L)$ is smaller than the stopping criteria (ε). Therefore, smaller values of ε lead to more iterations. The example shown in table 5-32 supports this claim. The optimization would have been terminated approximately at iteration 50 with $\varepsilon = 1$ whereas it would have gone approximately to iteration 70 with $\varepsilon = 0.5$.

The results in this section support the claim that the termination criterion value and the discount rate are important contributing factors to the length of each optimization. The next section shows results that support the claim that the cost-to-go values and policies are consistent.

Table 5-32 – Number of policy changes and the maximum CTG value change for iterations 15 to 90 for the optimization of the base case scenario with 3% discount rate

Iteration	Number of policy changes	Maximum CTG value change	% of maximum CTG value
15	994	\$913.94	1.98%
20	7	\$333.59	0.72%
25	0	\$115.51	0.25%
30	1	\$28.80	0.062%
35	0	\$12.74	0.028%
40	0	\$4.91	0.011%
45	0	\$1.68	0.0036%
50	0	\$0.65	Negligible
55	0	\$0.27	Negligible
60	0	\$0.11	Negligible
65	0	\$0.054	Negligible
70	0	\$0.048	Negligible
75	0	\$0.024	Negligible
80	0	\$0.024	Negligible
85	0	\$0.024	Negligible
90	0	\$0.006	Negligible

5.3.3 Policy Simulation

Section 4.7 establishes the need and method for simulating policies as a means to verify that the DP optimization yields policies and CTG values that are close to optimal. Using this method, results in this section show that the DP model does yield near optimal policies. The methods in section 4.7 are applied to calculate confidence intervals (CI) for these simulations and the number of replications required to obtain a pre-determined error. As discussed in section 4.7, the number of replications needed during a simulation is dependent on the value of ϑ. During a simulation, the number of replications required to achieve an absolute error ϑ is recalculated at each replication and the simulation is stopped when $a \geq n$, where a is the replication number and n is the number of replications that yield an error of ϑ. As a general rule of thumb, Law and Kelton (2000) suggest using a value of ϑ that is equivalent to 15% of the optimal CTG function value of the DP algorithm for the starting state i being simulated. This value is very conservative for our purposes and would lead to pre-mature termination of the policy simulation and lead to very wide confidence intervals that aren't very useful in determining if $J'_\mu(i)$ is a good approximation to $J_\mu(i)$. Much tighter confidence intervals can be calculated and this is discussed later.

We state, in section 4.7, that μ is simulated until future revenues created from harvests no longer have any significant discounted present value. Therefore, the length of each replication should reflect the value of the discount rate which explains why simulation 5 is longer than the others. Simulations 1 and 2 could have been shortened but there isn't any significant additional cost to letting those replications go to 295 years. The states chosen for the simulation were chosen either because they were recurring states and thus are important to study, or they were transient states that offer the possibility to simulate a larger part of the policy.

Table 5-33– Details of five policy simulations

Number	Discount rate	# of price breaks	Length of each replication	Approximation architecture	Starting state
1	0.04	3	295 years	DWI	TRT = 3 Age = 25 Initial density = 2500
2	0.04	3	295 years	DWI	TRT = 3 Age = 35 Initial density = 3250
3	0.03	1	295 years	Multiple regression	TRT = 3 Age = 30 Initial density = 2500
4	0.03	1	295 years	Multiple regression	TRT = 3 Age = 45 Initial density = 3250
5	0.02	1	620 years	DWI	TRT = 1 Age = 15 Stocking = 100%

Simulations 1 and 2 have three price levels so they require the simulation of random market prices. The simulation average given in table 5-34 is the average $\bar{X}$ of the

CTG values for all the replications X^m for a given simulated optimal policy. The value of the relative error ϑ given in table 5-34 is for the number of replications $n^*(\vartheta)$ shown in the next column. The number of replications shown in the last column is the number it would have taken had we used ϑ = 15% to decide when to stop the simulation. In all cases, the confidence interval contains $J_\mu(i)$ for starting state i and policy μ with ϑ smaller than what is recommended by Law and Kelton (2000). Therefore, according to the discussion in section 4.5, we can say with 95% confidence and a relatively small absolute error, that μ is a good approximation to μ^* in those 5 cases.

Table 5-34 – Confidence intervals for the 5 simulations described above

Simul.	Lower CI	Simulation average	Upper CI	DP CTG function value	ϑ	$n^*(\vartheta)$	# replications with ϑ = 15%
1	$3,061.85	$3,390.17	$3,718.48	$3,403.40	9.7%	50	7
2	$6784.36	$6,913.09	$7,041.82	$6,945.36	1.9%	50	4
3	$4,803.22	$5,163.84	$5,524.45	$4,890.85	7.0%	50	5
4	$7,813.62	$8,003.00	$8,192.37	$7,999.86	2.4%	50	3
5	$11,032.46	$11,565.85	$12,099.24	$11,399.81	4.6%	150	6

What follows is a discussion of the distribution of the simulated CTG values and how they relate to the policies they are simulating.

Figure 5-16 shows a histogram of the distribution of simulated CTG values for the first 4 simulations. Simulations 1 and 2 have three price levels therefore simulation is done with random market values. The distribution of CTG values for these simulations reflects the random nature of market prices where the CTG values for a majority of the replications are situated around the mean with values above and below. Simulation 3 has almost 80% of its replications produce CTG values within a narrow range whereas the values in simulation 4 are distributed more evenly. The distribution of CTG values for simulation 5 is very similar to that of simulation 4 so the chart is omitted to save space.

In simulation 3, the average number of time periods per replication where a stand is affected by a natural disaster is 2.62 and few of them occur before the first regeneration harvest which occurs 15 years, or three time periods, after the start of the replication. The policy simulated in simulation 3 states that, given the starting forest state, it is optimal to do nothing until age 40 at which point, it is optimal to do a regeneration harvest and replant 2500 trees/hectare. Of course, if a natural disaster occurs before the regeneration harvest can occur at age 40, no salvage is possible and the plant/grow cycle restarts. The starting state is 15 years away from its first harvest therefore the majority of the revenue created by harvests in simulation 3 occur early in the replication and are highly discounted to the present.

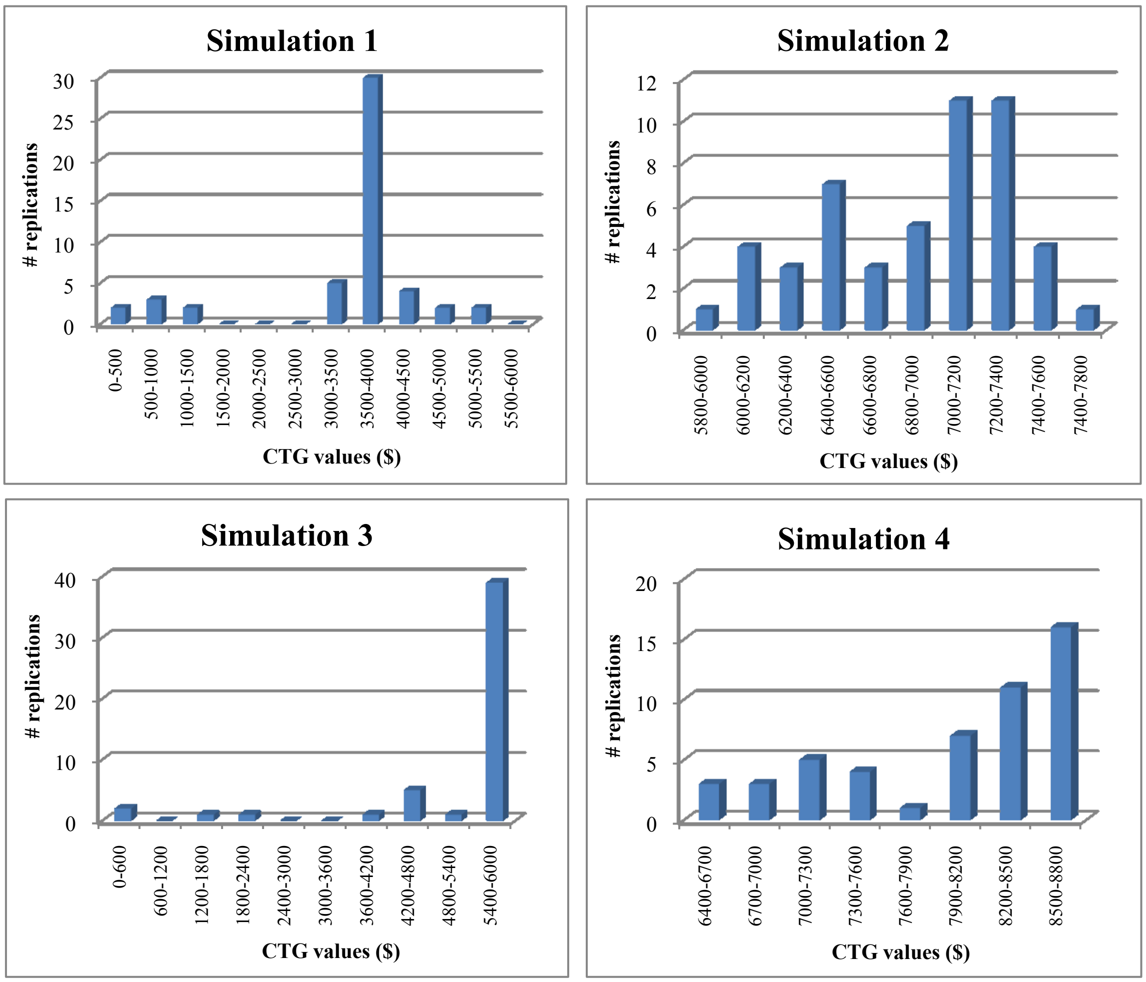


Figure 5-16 – Histograms for simulations 1-4

In simulation 4, the average number of stands affected by natural disasters per replication is 3.38 which means that, on average, potential profits are cancelled more often than in simulation 3 and this creates more variability in profits because they occur randomly over time and discounting affects the monetary impact of each natural disaster. By contrast, the optimal policy for the starting state in simulation 4 is to do an immediate regeneration harvest which creates profit that isn't discounted and can't be affected by a natural disaster. Yet, because this revenue occurs so early in the replication and discounting is low, potential future revenues that could contribute a substantial quantity to the CTG value are subject to being cancelled by a natural disaster. This creates variability between replications.

Simulation 5 has a lower discount rate therefore it is run much longer. The starting state is such that the optimal action at the first time step of the replication is to do a PCT and remove all hardwood which takes the stand to TRT=2. The optimal action for a pre-commercially thinned stand is to do nothing until age 40 where it is optimal to do a regeneration harvest and plant 1750 trees/hectare. The optimal policy for the resulting plantation is to let it grow until age 45 and restart the plant/grow/harvest cycle. Low discounting yields higher present values for future profits. The simulated CTG values for this policy are highly variable because the first opportunity for creating profit is at the 6th time step of the simulation which gives a high probability, relative to the first 4 simulations, that a natural disaster will occur and destroy the stand. Combine that with the fact that future profits still have a high present value, and you get a high level of variability in simulated CTG values. Therefore, it takes a larger number of replications to achieve the desired precision.

The randomness of natural disasters has an important impact on the present net value of the stand. In the five simulations presented here, when a natural disaster occurs and a stand ends up in a regeneration state, it takes at least 40 years before revenue can be created from a harvest. If a natural disaster occurs early in the replication before a harvest can occur, the next earliest harvest after the natural disaster is 40 years hence. The four graphs in figure 5-16 reflect this phenomenon. In all simulations, there is a small number of replications with very small CTG values. In these cases, natural disasters occur very early in the replication before a harvest can be done to create revenue. Discounting limits the ability of future harvests to add to the total CTG value of the simulated policy. The starting state in simulations 2 and 4 are older stands that have a better chance, statistically, of being harvested before a natural disaster destroys the stand. Consequently, there aren't any very low CTG values in those two simulations whereas there are a few in simulations 1 and 3.

Results in this section show that the policies created with the DP are near optimal and the simulation results are consistent with the stochastic processes that are incorporated into the DP model. The next section discusses how proper interpretation and implementation of the policies can lead to sound management.

5.4 Policy Discussion

The main goal stated in section 1.1 is to develop decision policies for individual forest stand management. In the first three sections of this chapter, we use policies and changes in policy to explain and demonstrate the implementation of the DP model. In this section, we focus on the policies themselves and what they mean for individual forest stand management. The policies developed with the DP model are complex and the discussion in this section demonstrates their real world application. Section 5.4.1 demonstrates that the optimal policies developed with the DP model reflect the underlying growth and yield models used to construct the DP model. Section 5.4.2 discusses the implication of managing individual forest stands according to these optimal policies.

5.4.1 Policies Reflect Underlying GNY and Stochastic Models

The results, for most optimizations, are a reflection of the softwood percentage and crown closure of the stand. Softwood trees grow faster than hardwood in Nova Scotia and diameter growth is directly related to crown closure. When crown closure is at or near 100%, the growth rate of trees is at its lowest. Therefore, when stands have large trees and high crown closure, it is often optimal to do a CT to create revenue and release the trees, or to do a regeneration harvest. Figure 5-17 shows the same optimal policy for commercially thinned natural stands (TRT = 5) on two different graphs. The top graph shows softwood diameter vs. age, and the bottom graph shows crown closure percentage vs. age. There are 1080 evaluation states in subset S_5^{Eval} and the optimal policy for the majority (807) of these states is to do a regeneration harvest. Another 219 states have an optimal policy of doing nothing and letting the stand grow an additional 5 year period. The 807 states where it is optimal to do a regeneration harvest have been removed from the graph in order to concentrate on a relatively small number of cases. The remaining 54 states (5% of all states) combine together to show that a second CT is only done when crown closure is above 80% and when the softwood portion of the stand has an average diameter above 22cm but these optimal CT actions will not be applied often in the course of managing any given stand. The stands that lead to a second CT are mostly transient states in the infinite horizon.

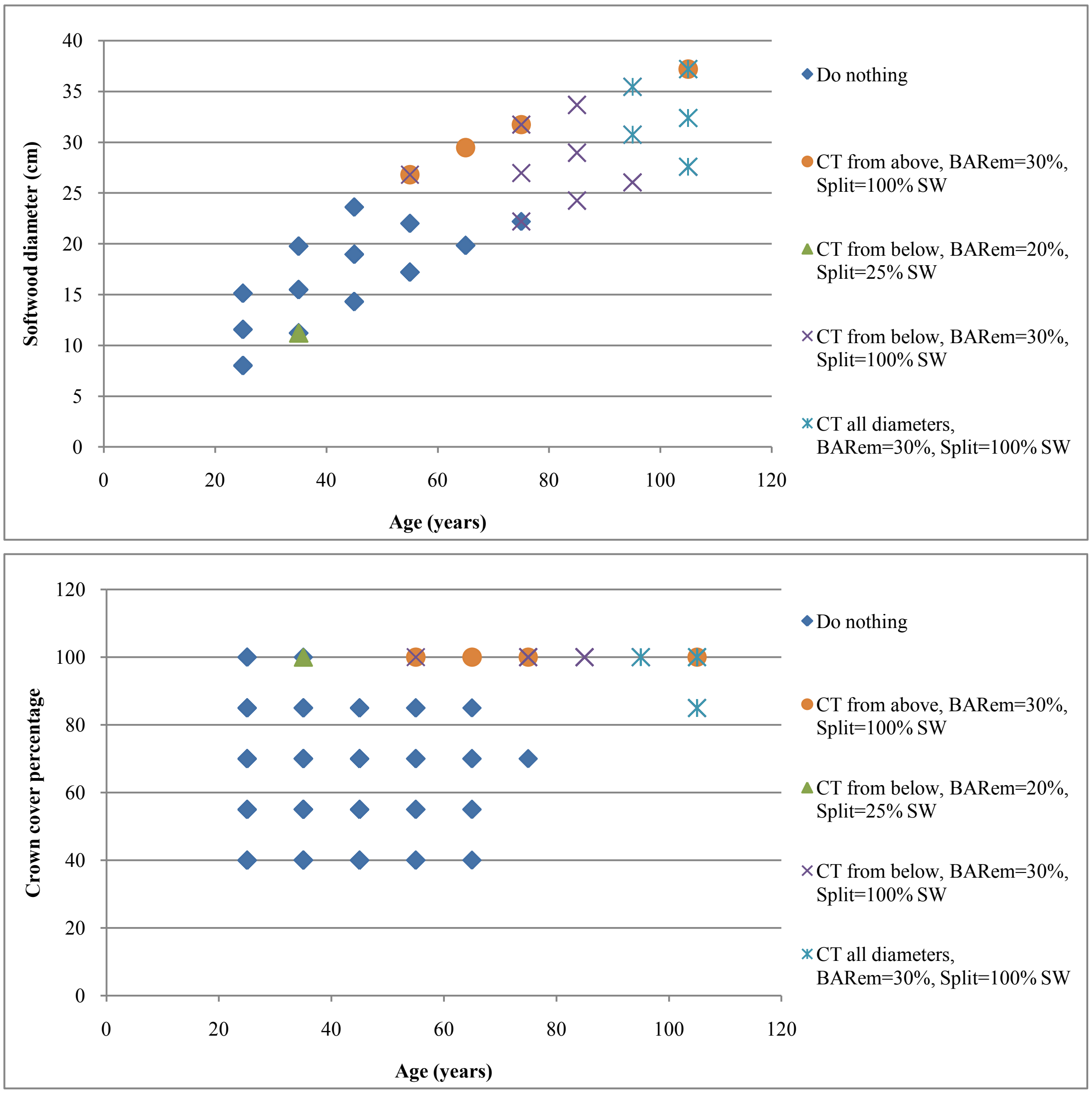


Figure 5-17 – Optimal policy for commercially thinned natural stands for the base case scenario described at the beginning of chapter 5

In one isolated case, the optimal policy is to remove 20% of the BA on the stand and to make that removal 75% hardwood. This policy is a reflection of growth and yield models because the stand average diameter is small and removing HW leaves a stand that is mostly softwood which grows faster than hardwood. All other CT are done at larger diameters and only SW trees are removed in those cases (split = 100% SW). Regardless of the type of CT, if the proportion of SW on the stand is small before the CT, the result is a stand with very little SW after CT, and at the next decision period, a regeneration harvest is done and the stand becomes a SW plantation. This sequence is a reflection of the growth rate of SW being higher than that of the HW. Wherever possible, the removal of commercially valuable SW trees is preferred.

As indicated above, management of individual forest stands puts emphasis on growing SW rather than HW stands. In Nova Scotia, modelling mixed-species stands is difficult because the hardwood growth and yield models don't have a good way of quantifying the quality of hardwood trees. The selling price per cubic meter of hardwood SV is higher for larger trees because the products that are made with large hardwood trees command a premium price on the market. Bigger higher quality trees are worth more money per m^3 but the hardwood growth and yield model doesn't reflect this.

Hardwood plantations don't exist in Nova Scotia because twigs and buds on very young hardwood trees are a source of food for many herbivores (Doucet and Thompson 2008) so young hardwood trees have to be protected from deer and rabbits where there are enough of them to have a serious effect on the trees. Such an investment is not made in Nova Scotia. Therefore, the growth and yield models used in the DP model don't include the option of creating hardwood plantations.

The reality in Nova Scotia is that softwood trees grow faster than hardwood trees in similar conditions. The policies reflect this reality. Regardless of the discount rate and market prices used in the DP optimization, anytime a pre-commercial thinning is the optimal policy, it's always optimal to remove all hardwood and space softwood accordingly.

The structure of stochastics is also reflected in the policies. Several examples were given in section 5.1 for the three sources of uncertainty. Further observations are given here to support the argument that the policies reflect the dynamics of price uncertainty. As discussed in section 5.1.1, very high and very low prices have less probability of occurrence than average prices. This probability distribution is reflected in optimizations that have more than one price level. Table 5-35 shows the policies for three states.

Table 5-35 – Policies for three states and 6 different price levels. The numbers in color are the optimal decision in each of these three states

	Age (yrs)	d^S (cm)	d^H (cm)	cc (%)	PCT^S (%)	Initial planting density	Price 1	Price 2	Price 3	Price 4	Price 5	Price 6
State 1	85	27.3	19.4	100	33	N/A	21	21	21	21	7	7
State 2	90	28.1	19.9	60	5	N/A	2	7	7	7	7	7
State 3	5	0	0	N/A	100	2500	2	2	2	2	2	2

2 - Let grow	6 - ReHar, plt 2500 tr/ha	21 - CT, rmv 40% BA, splt 75% (blw)

Prices 1 to 6 are low to high prices. Decision 2 is to do nothing, decision 21 is a CT with 40% $BARem$ from below with $BARem_{Split}$ = 75% SW, and 7 is a regeneration harvest followed by planting 2500 SW trees per hectare. If decision 21 is taken when in state 1, state 2 occurs at the next time interval (5 years). If decision 7 is taken in either state 1 or 2, the resulting state at the next time interval is state 3.

Interestingly, with low to average prices in state 1, the optimal decision is to remove most of the softwood (decision 21) and let the hardwood grow another 5 years at which point the entire stand is harvested unless a very low price is observed. With a young plantation, the optimal decision is always to do nothing because PCT is not an option with plantations and there isn't any growing stock (d^S = 0) at this young age. There is no correlation between prices from one period to the next and this is reflected in the policy. It takes advantage of a high price in the current time interval by doing a regeneration harvest because there is a high probability that the price in the next period will be lower. Seemingly in contrast, state 1 has enough HW that if the price is low, it's optimal to remove almost all SW from the stand and leave the HW to grow an extra 5 years before the entire stand is harvested. But since there is very little SW on the stand to begin with, it makes sense to remove it and give the HW more room to grow. HW will gain 1.5m^3 more volume by growing 5 years after it's been released following the CT.

Another set of policies is given in table 5-36. It shows the optimal policy and optimal CTG value for 3 states of TRT = 4 for the base case scenario without uncertainty.

Decision 2 is to do nothing and let the stand grow and decision 3 is to do a regeneration harvest and let the stand grow naturally without any intervention.

Table 5-36 – Optimal policy and CTG values for three states in the base case scenario without uncertainty

Age (yrs)	d^S (cm)	cc (%)	# trees	Optimal decision	Optimal CTG value
50	18.6	55	1100	Let grow	\$27,940.70
50	22.3	55	825	ReHar, natural regen	\$26,590.20
50	26	55	650	ReHar, natural regen	\$27,061.00

At first glance, we would expect the CTG value to be higher for a stand of the same age, same crown closure and with a larger diameter. Using the maximum stocking line for softwood plantations, similar to figure 2-3, we determine, based on crown closure which is a percentage of maximum stocking for any given diameter, the number of trees on each of the stands in table 5-36. The equal crown closure for the three stands indicates approximately equal basal area for each stand. The growth models indicate that the stand with the smallest diameter which has the largest number of trees will gain the most basal area in the next 5 years (7.8 m^2) whereas the other two stands will gain much less basal area (2.3 m^2 and 1.95 m^2). Therefore, it is optimal to do a regeneration harvest at the current time period for the two stands with the largest diameter and keep the stand with the smallest diameter for an additional 5 years at which point it is optimal to harvest it. The policies given by the DP model are as expected given the growth and yield models used to build it.

5.4.2 Policy Interpretation

We start this section by further discussing the policies given in table 5-35. CT productivity, the number of cubic meters of wood products harvested per hour, is higher when the average stand diameter is larger. In the case of state 1 from table 5-35, the hardwood is left on the stand because it can be harvested at the next period with a larger diameter and thus a higher productivity which will reduce the cost per unit of volume harvested. This is an example of real life policy implementation and interpretation that can be done with the structure proposed in this thesis.

The policies developed with the proposed DP model allow us to do what-if and cause-effect analysis of the resulting policies. Enough precision is built into the DP model to give answers to typical questions forest managers would ask. The rest of this section is structured in a question and answer format.

1. Do very small values of the discount factor cause regeneration harvests to appear at a later age?

We need to differentiate between regeneration harvests and rotation ages. A regeneration harvest removes all trees from the stand and any wood products that have commercial value are sold to create revenue. The rotation age is the age at which a stand will receive its regeneration harvest over an infinite horizon. According to the policies in table 5-29, starting with stands that are already 25 years old or more, the answer is yes. However, looking at the entire optimal policy for the base case scenario, we observe that starting with a 5 year old natural stand, the optimal policy is to do a PCT at age 10 in which case the stand becomes a TRT = 2 stand. The optimal policy for the resulting stand is to wait until the stand age is 45 years to do a regeneration harvest and plant 1750 trees/hectare. Scanning the rest of the policy, regardless of the discount rate used for the base case scenario, the optimal infinite horizon policy is always for the stand to get to a 45 year rotation age. But the management policy is vastly different. A very high discount rate (6%) creates a rotation of natural stands that are pre-commercially thinned at 15 years and harvested at 45 years. The policy for 6% discount is given in table 5-37. Decision 12 at age 15 for TRT = 1 yields the first TRT = 2 stand 5 years later. The right side of the table shows the succession of states up to age 45 where it is optimal to take decision 4 which creates a natural stand and the cycle restarts. A very low discount rate (0.1%) creates a rotation age of 45 years on a softwood plantation with initial density of 2500 trees/ha. The policy for 0.1% is given in table 5.38. Decision 12 at age 10 creates a 15 years old TRT = 2 stand which is shown in the right table. It is optimal to let the stand grow to age 45 and do a regeneration harvest and plant 2500 trees/hectare. Once the stand is a plantation, it is optimal to let the stand grown to age 45 and restart the cycle of harvesting, planting and letting grow. The transition between these policies occurs between 1 and 3%. The policies in tables 5-37 and 5-38 are portions of the entire policy only for the states through which the stand will transition given the decisions made using the optimal policies for 6% and 0.1% discounting.

Table 5-37 – Partial optimal policy for the base case scenario with discount rate = 6%

TRT = 1

Age (yrs)	Stocking	Decision
5	75%	1
10	75%	1
15	75%	12

1 – Do nothing
4 – ReHar, fill plant, no ECC
12 – PCT, remove HW

TRT = 2

Age (yrs)	d^S (cm)	d^H (cm)	cc (%)	PCT^S (%)	Decision
20	6.9	0	14	100%	2
25	10.1	0	29	100%	2
30	12.7	0	44	100%	2
35	14.8	0	59	100%	2
40	16.6	0	73	100%	2
45	18.0	0	86	100%	4

Table 5-38 – Partial optimal policy for the base case scenario with discount rate = 0.1%

TRT = 1

Age (yrs)	Stocking	Decision
5	75%	1
10	75%	12

TRT = 3

Age (yrs)	d^S (cm)	Initial Planting Density	Decision
5	0	2500	2
10	4.2	2500	2
15	7.9	2500	2
20	10.7	2500	2
25	13.0	2500	2
30	14.9	2500	2
35	16.5	2500	2
40	17.9	2500	2
45	19.1	2500	7

TRT = 2

Age (yrs)	d^S (cm)	d^H (cm)	cc (%)	PCT^S (%)	Decision
15	5.2	0	8.4	100%	2
20	8.8	0	23	100%	2
25	11.7	0	38	100%	2
30	14.0	0	53	100%	2
35	15.9	0	68	100%	2
40	17.4	0	81	100%	2
45	18.8	0	93	100%	7

1 – Do nothing
2 – Do nothing
7 – ReHar, plt 2500 tr/ha
12 – PCT, remove HW

In Markov chain terminology, there are very few recurrent states in individual forest stand management. Most states are transient and will never be visited again.

2. Is it feasible to manage a stand as a HW only stand?

As discussed on several occasions, hardwood plantations don't exist in Nova Scotia. However, hardwood does grow naturally in mixed wood stands and hardwood growth models do exist for Nova Scotia. The DP model can be used to answer the question above. The entire focus of this work has been on economic objectives with certain assumed values for hardwood and softwood. Strictly economically speaking, given the choice between starting with a young softwood stand and a young hardwood stand, assuming a hardwood stand could grow without being killed by predators at no extra cost for protection, the softwood stand would have 16 times the present net value of the hardwood stand with an annual discount rate of 2%. A 5 year old natural pure softwood stand has an optimal CTG value of 12,947$ and a 5 year old natural pure hardwood stand has an optimal CTG value of 789$. In the softwood only model, the recurrent state is a 45 year old softwood plantation with an initial planting density of 1750 trees/hectare and, in the hardwood only model, the recurrent state is a 110 year old natural stand. Table 5-39 shows the optimal policy for a pure softwood stand which has a rotation age of 45 years old and the partial policy for the hardwood stand where the policy from age 25 to 90 doesn't change.

Table 5-39 – Policies for pure SW and HW stands with parameters from base case scenario

Softwood

Age (yrs)	d^S (cm)	Initial Planting Density	Decision
5	0	1750	2
10	4.8	1750	2
15	9.0	1750	2
20	12.2	1750	2
25	14.8	1750	2
30	16.9	1750	2
35	18.7	1750	2
40	20.3	1750	2
45	21.6	1750	6

Hardwood

Age (yrs)	Stocking	Decision
5	0.75	1
10	0.75	1
15	0.75	1
20	0.75	1
⋮	⋮	⋮
95	0.75	1
100	0.75	1
105	0.75	1
110	0.75	3

1 – Do nothing	2 – Do nothing	3 – ReHar, Nat Regen	6 – ReHar, plt 1750 tr/ha

With a stand that can't contain any softwood on an infinite horizon, it is never optimal to do a commercial thinning or pre-commercial thinning. In certain scenarios, it is optimal for a small number of transient states to keep hardwood for future harvests but none of the scenarios studied in this research had a recurrent state that was 100% hardwood unless the DP model was programmed to not include softwood. With the assumptions made in this thesis, hardwood management is unlikely to be optimal in an even aged management regime but with different numbers, anything is possible.

3. If planning to do mixed management (HW and SW) in even-aged stands, does the species mix change during the life of the stand?

The species mix is measured by the proportion of the stand basal area represented by each species. When the crown closure is below 100%, each species on the stand grows to fill in the open spaces in the stand until it reaches 100% crown closure. As this growth occurs, the SW and HW portions of the stand are filling in that space at different rates. Once the stand has reached 100% crown closure, the SW and HW portions of the stand continue to grow according to their respective fully stocked growth models or maximum stocking lines. At this point, the species mix stabilizes as there are no longer any open spaces to be filled in by the growing trees. Therefore, the answer to the question above is that the species mix changes if a stand has open spaces for the trees to fill. Interestingly, a stand with two very different SW and HW diameters, which has open spaces for the trees to fill, will have large changes in species mix because the total stand BA for one of the species changes very quickly. Table 5-40 shows one example of this change.

Table 5-40 – Example of species mix change for even-aged stand with initial crown closure = 50%

Year	Softwood		Hardwood	
	d^S (cm)	pct^S	d^H (cm)	pct^H
0	12.00	80	2.00	20
5	13.85	69	3.58	31
10	15.12	67	4.63	33
15	16.18	66	5.48	34

The stand in table 5-40 has an initial crown closure = 50%. Crown closure reaches 100% in 15 years after which there are only small changes in species mix, where pct^S drops to 60% by year 85. Notice the changes in diameter where the SW gains 35% growth and the HW diameter gains 174% growth. The difference in diameter growth percentage accounts for the swing in pct^S.

4. Can we force the model to prefer a species over the other with the right parameters?

There are many user definable parameters in the DP model and they can be divided into two major categories: growth and yield, and economic factors. These two will be discussed separately.

In the case of growth and yield parameters, they are not controlled by the user but are dependent on the stand being modelled. There are many examples in this chapter of the policies given by the DP model for typical stands found in Nova Scotia. Here are a few scenarios to demonstrate how the DP model handles atypical stands.

For example, with a site index that is much higher for HW than for SW, 17m at 50 years for HW and 12m for SW, the optimal policy eventually transforms all stands into plantations which is the same as the base case model. However, starting with a 5 year old natural stand with 100% stocking, the optimal policy is not to do a PCT at age 10 as is the case with the base case scenario, but rather to let the stand grow as a natural stand up to age 50 at which point a CT from above is done with 40% removal and split 50/50 between SW and HW. After the CT, the stand grows for another 10 years before a regeneration harvest occurs and it's transformed to a plantation.

In another case, with a stand that has 25% SW in a naturally regenerated stand instead of the common value of 75%, the change in policy is significant. The policies are given in table 5-41. With 6% annual discounting, there is a change in policy for a small number of transient states, mostly for natural stands. For the case of 100% stocking with 25% SW, pre-commercial thinning is optimal at ages 25 and 30, and we wait an extra 5 years before doing a commercial thinning. These changes are not surprising as the stand with 25% SW has very few trees of commercial value at those young ages, and doing a PCT will eliminate HW, space the remaining SW trees accordingly, and give them room to grow until it is optimal to do a regeneration harvest. In the case of 75% stocking, the changes are in the type of commercial thinning that is optimal. With a stand that contains 75% softwood, it is optimal to do 20% BA removal with a split of 25% SW / 75% HW. When the stand contains 25% SW, is it optimal to remove 40% of the BA and split it 50/50 or 75% SW / 25% HW. Thinning is done from above in all these cases. In the first case, when there is a large proportion of SW, it is optimal to remove the HW and leave the softwood so that it gains as much diameter growth as possible. In the latter case, it is optimal to remove at least half of the SW on the stand and leave the HW to grow at least another 5 years.

Table 5-41 – Optimal policies for natural stands with a 6% discount rate and 25% vs 75% SW in a natural regenerated stand

Age	Stocking	75% SW	25% SW	Stocking	75% SW	25% SW
5	100%	1	1	75%	1	1
10	100%	1	1	75%	1	1
15	100%	12	12	75%	12	12
20	100%	1	1	75%	1	1
25	100%	1	12	75%	12	12
30	100%	1	12	75%	12	12
35	100%	39	1	75%	1	38
40	100%	38	39	75%	31	38
45	100%	38	39	75%	31	39
50	100%	4	4	75%	4	39
55	100%	4	4	75%	4	4

These policies make sense because SW is more valuable and should be kept on the stand if there is a significant portion of the stand covered with it. However, if a stand is left with mostly hardwood but with softwood in the mix and less than 100% crown closure, the faster growth rate of SW will allow it to fill the gaps in the stand faster and slow the rate of growth of the HW. The policies reflect the dynamics of growth of each species.

Economic factors are a combination of user preferences mostly in the form of the discount rate, with observed market prices and common silvicultural practices and their costs. Average market prices and their effects on policy are presented in detail in this chapter and their effect and importance is supported by the results discussed. In Nova Scotia, silvicultural practices and costs are mainly governed by government policy in the form of a credit system which is part of the sustainable forest management policy (NSDNR 2009). The credit system assigns a number of credits per hectare for silviculture operations The costs used in the DP model are a reflection of that system and it is not in the scope of this work to debate that system. The discount rate is the focus of section 5.3.1 and its impact is supported by results. Therefore, policy discussion in the last part of this section will focus on atypical market states, and on user preferences for the discount rate as discussed in section 4.3.

In an atypical scenario where prices are higher than usual for both SW products (42% higher), we get a change in policy as expected. In the base case scenario with average market prices, there are more transient states, meaning that some stands take longer to get to the plantation state which has a rotation age of 40 years and a plantation density of 1750 trees per hectare. In the atypical case described above, within a maximum of 45 years starting with any stand, all stands are transformed into SW plantations with a planting density of 3250 trees per hectare and a rotation age of 40 years. The higher market prices support a higher planting density which has a higher planting cost per hectare. In a different scenario with HW prices higher than usual, there is no change in the optimal policy. HW prices have to be extremely high relative to SW in order to have any impact on policy. Results show that if the prices of HW are high enough (360% higher than normal), the optimal policy changes from one of converting all stands to SW plantations, to one of fill planting and changing the stand to a fully stocked natural stand which has a 25% HW proportion. But the case being described here is one where the prices of HW are 7.5 times higher than those of SW which is very unlikely to occur in practice in the current economy. Therefore, there is no reason to believe that softwood plantations will be replaced by stands containing a majority of hardwood in any commercial operations in the foreseeable future in Nova Scotia, strictly based on typical market prices and the dynamics of growth and yield of typical Nova Scotia forest stands.

Chapter 6: Conclusion

We focus on issues related to the optimal management of an individual forest stand. More specifically, we are interested in developing policies that allow forest managers to make decisions while considering information such as the state of the market and of the forest, and some knowledge of the uncertainty in regards to market and forest growth dynamics. There are three main areas of interest related to the objective of developing management policies:

i. Incorporating a two-species growth and yield model into a single stand management model
ii. Incorporating a comprehensive list of management options into a single stand management model
iii. Incorporating uncertainty into a single stand management model.

The forest stand management problem, as modelled in this thesis, has a large dimensional state space with a mix of discrete and continuous state variables. Because the cost and profit functions and the dynamics are complex, no analytical solution is possible. Thus we resort to numerical methods and value iteration to solve the DP problem. However, since some of our state variables are continuous, we cannot examine all states in attempting to create the cost-to-go function. Therefore, we calculate the cost to go function at a given stage of the algorithm at a finite set of state points and then approximate these cost-to-go values with a continuous function on the continuous portion of the state space. These cost-to-go function estimates are computed as expected values in the context of the random processes impacting state transitions.

The approach used created challenges, which are documented throughout chapters 2 to 5. These challenges include the need to deal with a mix of discrete and continuous state variables, random disturbances and a large number of stand management options. The main challenge with DP in a context such as the one here is finding a way to reduce the need to use a very large number of discrete values of the continuous variables in order to solve the DP algorithm. We accomplish this by using a relatively small number of evaluation states and approximating the cost-to-go function on the continuous portion of the state space. The small number of evaluation states allows us to consider a large number of management options and to study the effect of several sources of uncertainty, individually or collectively. The ability to consider this level of detail is a focus of this work.

In these concluding remarks, we focus on the contributions of this thesis and give opportunities for future research.

6.1 Contributions

In section 1.3, a brief overview of the contributions was given. In this section, we recall the main contributions and discuss why they are important.

Foresters need to understand uncertainty and, most importantly, the impact of not incorporating it into their models. The three forms of uncertainty studied in this thesis, prices, regeneration and natural disasters, weren't meant to be an exhaustive study of the uncertainty inherent in forest stand management. Rather, the goal was to properly describe a framework for incorporating uncertainty into a model for the development of optimal forest stand management policies. The key is to focus on the decision problem and to exploit uncertainty. For example, decisions shouldn't be made based on average prices because, if current market conditions aren't good, the potential exists for doing better than average if a decision is delayed based on the knowledge that better market conditions are likely to occur at the next decision period. Also, as discussed in chapter 5, knowing that a natural disaster may potentially destroy a stand when it is almost ready to be harvested could lead to an accelerated decision to remove a portion or all of the wood on the stand in order to avoid the potential loss of profit. Therefore, understanding and exploiting uncertainty makes sense from a decision problem perspective.

In order to study mixed-species individual forest stands in detail, a methodology had to be developed to combine individual species growth models for SW and HW together into a single growth model. The concept presented in this thesis relies on the fact that crown closure is directly related to spacing between the trees on a stand and, in the growth and yield models published by NS DNR (NSDNR (1993b), O'Keefe and

McGrath (2006)), spacing is the major driving force of individual SW and HW growth models. The proposed model capitalizes on this fact and calculates the fraction of the area of a stand covered by each species, given their respective basal areas, which leads to a different spacing for softwood and hardwood. The respective spacing is then used to calculate diameter growth before combining the two species and calculating the crown closure for the new stand. The proposed method allows us to study the interaction between species growing together on the same stand with a great deal of flexibility.

Once we can combine two species together and study how they grow and interact, we can start modelling commercial thinning of these stands in more detail. NS DNR only has diameter change models for commercial thinning (CT) from below for both softwood and hardwood as individual species. In order to incorporate a comprehensive list of management options into one single stand management model, a model had to be developed for CT from above for softwood and hardwood. We also had to develop an approach to modeling CT on a mixed-species forest stand. Section 3.2 describes the development of the method and the resulting diameter change equations for SW and HW, with thinning from above and below with user definable removal percentages. The method used to develop the new diameter change models is based on work by Gunn et al. (2000) and uses simulation to model a range of tree removal on a stand. This allows us to expand the list of silvicultural options considered while developing policies for individual forest stand management. The expansion of the list of silvicultural options leads to the development of treatment types, which reflect the history of the past decisions that have been made about the management of the stand. The majority of the reviewed papers optimize models for one treatment type at a time (Arthaud and Klemperer (1988), Peltola and Knapp (2001), Brodie et al. (1978)) or don't differentiate between treatment types. Most studies use variables that can describe the state of the stand regardless of treatment history. The development of the mixed-species model has lead to the development of the list of management options used in this work which wouldn't have been otherwise possible.

With the development of the mixed-species growth and CT model, detailed lists of forest stand silvicultural options can be created. The list of 48 management options used in the DP model includes letting the stand grow, doing regeneration harvests followed by plantations or naturally regenerated stands, doing three types of pre-commercial thinning on natural stands to encourage better growth of the trees on the stand, and doing commercial thinning with a wide range of combinations of removal types, removal percentages and removal split between SW and HW. All these management options can be applied to a stand with any proportions of HW and SW. The ability to deal with this range of management options provides the ability to generate policies that cannot be examined otherwise. Two observations warrant discussion. To clarify, a regeneration harvest leaves a stand with no standing trees and is often referred to as a clear cut. The first is that transitory policies developed in this work are complex and, in some cases, the inclusion of CT as a decision option allows the decision maker to delay clear cutting the stand and may lead to higher profits than if the only choices of decision were clear cut / no clear cut which is the case in many studies. The second observation is one of the possible social impact of having the choice between clear cut and commercial thinning. In the cases where the best decision, on a purely economic level, is to do a clear cut, the policies developed in this work could allow us to use commercial thinning instead of clear cutting if the loss of profit incurred due to the use of a less economical policy is acceptable to society. At least, the comparison can be made using these policies. On a landscape level, or multi-stand level, the use of commercial thinning could reduce the need for doing large amounts of clear cutting.

In DP, the approximation architecture used for CTG function approximations at each step of the value iteration algorithm and for each state/decision combination must ensure that cost-to-go function approximations on the continuous portion of the state space are consistent with the cost-to-go function values at the evaluation states. Because approximate value iteration additively builds approximations to the CTG function values at the evaluation states, inconsistent cost-to-go function approximations on the continuous portion of the state space could prevent the DP algorithm from converging in a finite number of iterations (Bertsekas 2001). In the case of multiple regression (MR), the creation of approximation bounds ensures this condition is met and allows us to use a simple approximation technique that is easy to implement but would not otherwise be suitable for use in this context. Therefore, when using MR as an approximation architecture to solve the DP model in a high dimensional state space where the exact form of the cost-to-go function is not known, the proposed method allows us to calculate more consistent cost-to-go function approximations. In the case of RBFs and DWI, much emphasis was put on properly scaling the distances between basis points. This isn't a contribution as much as a reiteration that DP is not trivial and requires careful consideration of the state space in order for the optimization to converge and for the resulting policies to be good approximations of the optimal policies.

Past research has properly demonstrated the development of optimal policies for individual forest stand management in a stochastic setting (Haight and Holmes (1991), Plantinga (1998)). The main challenge here stems from the fact that we have an infinite number of states, which makes it impossible to exactly solve the DP model. Thus, we resort to approximating the CTG function, and consequently the policies, at 1600 evaluation states, some of which are described by 5 variables, divided into 5 distinct treatment types, with the addition of three sources of random disturbances all combined together with 48 silvicultural options. All of these elements are considered simultaneously and the resulting policies are good approximations to the optimal policies in the continuous context. The policies are highly detailed and span a wide range of management options and treatment types, which include mixed-species stands. Chapter 5 gives a sample of the type of policy analysis that is achievable when working with management policies as detailed as the ones developed in this work. The use of approximate DP in the development of optimal policies for the individual forest stand management problem is worth the effort because these types of policies can't be obtained any other way.

6.2 Limitations

Additional avenues could have been investigated and they are discussed in this section. They revolve around additional details or methods that could have been added to the DP model and include defining susceptibility to natural disasters by including crown closure, finding actual market price data from Nova Scotia and using it in the DP model, using policy iteration to solve the DP algorithm, adding more approximation architecture possibilities and modeling using a finite horizon framework.

Whereas the height, diameter and crown closure of a stand affect its susceptibility to a hurricane (Duryea and Kampf 2008), Gilman et al. (2008)), susceptibility was modelled as depending only on age or diameter. Crown closure information is included in stand definitions therefore it could have been incorporated into the DP model. However, the challenge lies in finding models that appropriately describe the relationship between diameter, crown closure and hurricane variables. The approach used demonstrates how natural disaster information can be incorporated into the DP model. If better natural disaster information were available, it could easily be substituted into the DP model.

As discussed in section 5.1.1, there is no good long-term price information for forest products in Nova Scotia. The intent of this work is to show the structure that can be used with any stationary price information. There is price information available for the New England states in the United States of America which has a mix of tree species similar to Nova Scotia but research would need to be conducted to determine if such information is transferrable to Nova Scotia's economic reality and wood products industry.

Section 4.1 mentions a second approach to solving the DP algorithm: policy iteration. Policy iteration generates a sequence of stationary policies, each with improved profit over the preceding one. With a finite number of states and controls, policy iteration converges in a finite number of iterations but when the number of states and controls is large, solving the linear system in the policy evaluation step of the policy iteration approach can be time consuming. The main advantage of policy iteration over value iteration is that, in general, it converges in a smaller number of iterations (Bertsekas 2001). However, finding exact values of the CTG function for a given policy at the policy evaluation step of the algorithm requires solving a complex system of linear equations where the dimension of this system is equal to the number of evaluation states. With over 1600 states in the DP model, the dimension of the system of linear equations renders the method unattractive thus value iteration is used.

A considerable amount of time was spent on developing and refining three approximation architectures to be used in the value iteration algorithm. The chosen architectures represent both averager and non-averager methods, and represent very different ways of approaching function approximation. However, together, they only represent a portion of available function approximation methods. Methods such as splines and artificial neural networks (ANN) could be used for CTG function value approximations and they would present their own challenges. For example, neural networks need to be retrained at each iteration of the DP algorithm when new values of the CTG function are calculated. Furthermore, ANN's in general don't force the output of the network to be equal to the known function values at any evaluation state. Splines might suffer the same issues as multiple regression because they are simply piecewise polynomials used to fit the CTG function on subsets of the state space. In both these cases, it may be necessary to incorporate approximation bounds similar to those used with multiple regression.

The finite horizon framework is very closely related to its infinite horizon counterpart with only a few modifications required to make the transfer. The finite horizon framework can be used, and in some cases is required, when any of the components of the DP model aren't stationary. These could include but not be limited to market price dynamics, natural disaster susceptibility probabilities and growth and yield models. The main objective of this thesis was to develop a flexible modeling approach and demonstrate its use. However, finite horizon modeling opens up possibilities that will be discussed in the next section.

6.3 Opportunities for future research

The structure is now in place to study other objectives. Whereas the objective of this study was strictly to maximize profit for maximum economic gain, most stakeholders in forestry have specific objectives when managing forest stands. As long as those objectives can be described in economic terms, they can be studied with the model developed in this work. Objectives such as protection of habitat, carbon sequestration and public use of forests are all worthy objectives but they require the transformation of those objectives into economic terms. For example, Kline et al. (2000) develop an empirical model describing owners' willingness to accept an economic incentive to adopt a 200-foot harvest buffer along streams as a function of their forest ownership objectives and socioeconomic characteristics. Slaney et al. (2009) employ a spruce budworm (*Choristoneura fumiferana Clem*.) decision support system to examine costs and benefits of sequestering, or protecting, carbon in forests through pest management. Results provide forest managers with important information needed to justify such carbon sequestration programs on economic grounds. Bestard and Font (2010) develop a model where the goal is to improve current applications of inferring the recreational value of forests in the region of Mallorca, Spain. The stated objective is to better understand and implement policies and regulations on a wider geographical area. In these three cases, the studies have found ways to describe their objectives in economic terms to be included into their models. The nature of the model developed is well suited for these types of approaches and could be adapted to include these types of objectives.

Finite horizon DP is another possible avenue of research. As discussed earlier, the finite horizon framework can be used, and in some cases is required, when any of the components of the DP model aren't stationary. These could include but not be limited to market price dynamics, natural disaster susceptibility probabilities and growth and yield models. In these three cases, there is a vast literature on non-stationary models and incorporating them into the DP model requires modifications of the transition probabilities used in the value iteration algorithm. The structure used in this thesis allows for those modifications and is a natural extension of the DP model. Finite horizon DP could also allow us to study the effect, on the optimal policies, of adding a termination state and stopping the DP model after a finite number of periods. The use of a termination state allows the study of scenarios where a specified use of the stand is pre-planned at a given termination date but there is a need to develop a policy to get to that termination date. Examples of termination states include turning the stand into a public use property, or forcing a final harvest of the stand at a specific date for economic reasons of the forest stand owner. In all these cases, the structure of the DP model wouldn't be very different and would allow for the study of a host of new scenarios.

The policies developed in this thesis are more detailed than what has been seen before because the DP model uses function approximations that allow us to considerably reduce the number of evaluation states and thus add a comprehensive set of management options to the model. Even with a very large number of iterations, the VB code written for this DP model converges to an optimal policy in less than 15 minutes of computer processing time which is short. We assume that much more complexity could be added to the model without causing the optimization time to become unmanageable. This additional complexity could come in the form of a multi-stand analysis. Studying a new set of objectives as described above with a multi-stand forest could lead to useful policy development and interpretation. The complexity of the DP model would grow quickly but the structure can handle that additional complexity. The only real limit to the size of the model, and therefore its level of detail, is the limit of the computer on which it is being modelled and the programming language in which it is being coded.

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Appendix A: Growth Functions

The functions in this appendix were initially based on Fortran code received from NS DNR (McGrath 2005). They were adjusted, as needed, according to growth and yield models published by the Nova Scotia Department of Natural Resources (NSDNR (1993b), O'Keefe and McGrath (2006)) and are used to calculate growth and yield in this thesis. The notation f_i was used in chapter 2. The i refers to the function numbers in this appendix.

Function 1 – Dominant height:

This function calculates dominant height of natural unmanaged stands based on age of the stand since last regeneration harvest (ha_t). The height of the trees on a stand depends on site index (SI^k). Softwood and hardwood have different equations but the stand can only have one dominant height. The dominant height of the stand is the average of the dominant heights of softwood and hardwood.

$dht_t = f_1(\mathrm{ha_t}, \mathrm{SI^k})$

Softwood :

dht_t^S measured in meters

SI^S measured in meters at age 50 years

A = $3.2808 \times SI^S$

B = − 0.019070142

C = 3.063581805

D = − 0.228589318

$$dht_t^S = \left(4.5 + (\mathrm{A} - 4.5) \times (1 - \mathrm{e}^{\mathrm{B}\times 50})^{-(\mathrm{C}\times\mathrm{A}^{\mathrm{D}})} \times (1 - \mathrm{e}^{\mathrm{B}\times ha_t})^{(\mathrm{C}\times\mathrm{A}^{\mathrm{D}})}\right) \div 3.2808$$

Hardwood :

dht_t^H measured in meters

SI^H measured in meters at age 50 years

A = SI^H

B = 10.5513

C = - 0.0192

D = 0.0339

E = 1.7826

F = 0.7565

G = 1.076

$$dht_t^H = \mathrm{A} \times \left(\left(1 - \mathrm{e}^{\left(\mathrm{C}*50+\left(\mathrm{B}-\mathrm{F}\times\mathrm{A}+\mathrm{D}\times\mathrm{A}^{\mathrm{E}}+1\right)\right)}\right)^{-\mathrm{G}}\right) \times \left(\left(1 - \mathrm{e}^{\left(\mathrm{C}*ha_t+\left(\mathrm{B}-\mathrm{F}\times\mathrm{A}+\mathrm{D}\times\mathrm{A}^{\mathrm{E}}+1\right)\right)}\right)^{\mathrm{G}}\right)$$

$$dht_t = (dht_t^S + dht_t^H) \div 2$$

Function 2 – Average height:

This function gives a simple relationship between dominant height and average height of trees in a single species softwood or hardwood stand. In the case of softwood, the function is applicable to all TRT types. As for hardwood, the appropriate functions are given below.

$aht_t^k = f_2(dht_t)$

Softwood :

dht_t and aht_t^S measured in meters

$$aht_t^S = (-1.466091 + 0.861959 \times (3.2808 \times dht_t) + 0.00108537 \times (3.2808 \times dht_t)^2) \div 3.2808$$

Hardwood :

dht_t and aht_t^H measured in meters

$aht_t^H = 0.9654 \times dht_t - 0.752$ for TRT=1

$aht_t^H = 0.962 \times dht_t - 0.3979$ for TRT types 2, 3, 4 and 5

Function 3 – Diameter for natural unmanaged stands :

This function calculates the diameter for softwood and hardwood with different formulas that depend on average height for each species and on site index in the case of hardwood. It is only applicable to natural unmanaged stands.

$d_t^k = f_3(aht_t^k, SI^H)$

Softwood :

aht_t^S measured in meters

d_t^S measured in centimeters

A = 0.286385

B = 0.134989

C = 0.00137156

D = 0.0000546409

E = 0.000000820707

F = 0.00000000724082

G = 0.0000000000372075

H = 0.000000000000102997

J = 0.000000000000000118654

M = $aht_t^S \times 3.2808$

$d_t^S = (A + B \times (M - 4.5) - C \times (M - 4.5)^2 + D \times (M - 4.5)^3 - E \times (M - 4.5)^4 + F \times (M - 4.5)^5 - G \times (M - 4.5)^6 + H \times (M - 4.5)^7 - J \times (M - 4.5)^8) \times 2.54$

Hardwood :

SI^H measured in meters at age 50 years
aht_t^H measured in meters
d_t^H measured in centimeters
$d_t^H = (0.3068 + 0.0004 \times SI^H) \times (aht_t^H)^{(1.6713 - 0.0171 \times SI^H)}$

Function 4 – Diameter growth for softwood :

This function uses information about the current state of the stand to calculate the quadratic mean diameter of the softwood trees on a stand with an additional 5 years of growth. This function applies to softwood stands of TRT types 2, 3, 4 and 5 that have $SP_t^S \geq 3.1m$.

$d_{t+5}^S = f_4(d_t^S, SI^S, maxtrees_t^k, cc_t)$

$maxtrees_t^S$ measured in trees per hectare
cc_t measured in percentage
d_t^S measured in centimeters
d_{t+5}^S measured in centimeters
SI^S measured in meters at age 50 years
B1 = 0.02785202
B2 = -0.367548143
B3 = -0.005540854
DBHIB = (d_t^S/ 2.54 - 0.016384) / 1.057711
TMPBA = (DBHIB)^2 × 0.005454 × ($maxtrees_t^S$ × cc_t ÷ 100) / 2.47105
D5IB = (B1 × SI^S × 3.28084 + B2) × e$^{(B3 \times TMPBA)}$
d_{t+5}^S = ((DBHIB + D5IB) × 1.057711 + 0.016384) × 2.54

Function 5 – Diameter and spacing dependent age :

This function calculates a diameter dependent age and is used in conjunction with function 6 to calculate the diameter of the stand after 5 years of growth. It is only applicable to softwood stands of TRT types 2, 4 and 5 that have $SP_t^S < 3.1m$.

$fa_t^S = f_5(SP_t^S, d_t^S, SI^S)$

d_t^S measured in centimeters
SP_t^S measured in meters
SI^S measured in meters at age 50 years
A = 0.963860501
B = 0.063249499
C = -0.179264128
D = 0.789203213
E = -0.029174191
DBHIB = (d_t^S ÷ 2.54 - 0.016384) ÷ 1.057711

$fa_t^S = (\ln(1 - (\text{DBHIB} \div (\text{A} \times SP_t^S \times 3.2808 + \text{B} \times SI^S \times 3.2808 + \text{C}))^{(1 \div \text{D})})) \div \text{E}$

Function 6 – Age and spacing dependent diameter :

In conjunction with function 5, this function calculates the diameter of a softwood stand after i years of growth. It is only applicable to softwood stands of TRT types 2, 4 and 5 that have $SP_t^S < 3.1m$.

$d_{t+i}^S = f_6(fa_{t+i}^S, SP_{t+i}^S, SI^S)$

SP_{t+5}^S measured in meters
SI^S measured in meters at age 50 years
fa_{t+i}^S in i years
A = 0.016384
B = 1.057711
C = 0.963860501
D = 0.063249499
E = 0.179264128
F = 0.029174191
G = 0.789203213
H = $SP_{t+5}^S \times 3.2808$
J = $SI^S \times 3.2808$

$$d_{t+i}^S = \left(\text{A} + \text{B} \times \left((\text{C} \times \text{H} + \text{D} \times \text{J} - \text{E}) \times \left(1 - \text{e}^{(-\text{F} \times fa_{t+i}^S)}\right)^{\text{G}}\right)\right) \times 2.54$$

Function 7 – Hardwood diameter :

This function is used to calculate the diameter i years in the future or past based on basal area and diameter at time t. This function is only applicable to hardwood stands of TRT type 2 and 5.

$d_{t+i}^H = f_7(d_t^H, BA_t^H, SI^H)$

d_{t+i}^H and d_t^H measured in centimeters
SI^H measured in meters at age 50 years
BA_t^H measured in m^2
A = 0.6636
B = 0.0971
C = 0.0625
D = 0.0005
E = 0.3062

$$d_{t+i}^H = d_t^H + (\text{A} + \text{B} \times SI^H) \times \text{e}^{((-\text{C} + \text{D} * SI^H) \times BA_t^H)} + \text{E}$$

Function 8 :

To calculate crown closure for any stand of TRT types 2, 3, 4 or 5, we need the number of trees on the stand ($TFREQ_{t+5}^{k}$) for each species.

$$TFREQ_{t+5}^{k} = f_8(d_{t+5}^{k})$$

d_{t+5}^{k} measured in centimeters

$$TFREQ_{t+5}^{S} = 2.47105 \times 10^{\left(4.62277 - 1.87401 \times Log_{10}\left(d_{t+5}^{k} \div 2.54\right)\right)}$$

$$TFREQ_{t+5}^{H} = 10^{\left(5.1557 - 1.688 \times Log_{10}\left(d_{t+5}^{H}\right) + 0.049\right)}$$

Function 9 :

Calculating MV and SV requires the use of a series of equations, which we`ll call procedures, that determine the portion of the total stand volume that can be harvested. There are four procedures categorized by their species and TRT types.

- Procedure 1 – Softwood, TRT = 1
- Procedure 2 – Hardwood, TRT = 1
- Procedure 3 – Softwood, TRT types 2, 3, 4, 5
- Procedure 4 – Hardwood, TRT types 2, 3, 4, 5

Procedure 1

Step 1: Calculate merchantable basal area as a fraction of stand total basal area

d^S measured in centimeters
$MERBA^S$ measured in m^2
BA^S measured in m^2
If ($d^S \geq 20.32$) Then
    $RATIO_1 = 1$
ElseIf ($d^S \leq 7.1$) Then
    $RATIO_1 = 0$
Else
    A = 0.82932659
    B = 0.7183961
    C = 0.10184704
    E = 0.0000066210904
    F = 0.0054868242
    G = d^S / 2.54
    $RATIO_1 = -A + B \times G - C \times G^2 + F \times G^3 - E \times G^5$
    If ($RATIO_1 > 1$) Then
        $RATIO_1 = 1$
    ElseIf ($RATIO_1 < 0.05$) Then
        $RATIO_1 = 0$
    End If

End If
$MERBA^S = RATIO_1 \times BA^S$

If ($MERBA^S < 0.1$) Then
$MERBA^S = 0$
End If

Step 2 : Calculate ratio of $MERBA^S$ to BA^S

$\text{RATIO}^S = MERBA^S \div BA^S$

Step 3 : Calculate merchantable diameter

$MERDIAM^S$ measured in centimeters
If ($RATIO^S \geq 1$) Then
$MERDIAM^S = d^S$
ElseIf ($RATIO^S \leq 0$) Then
$MERDIAM^S = 0$
ElseIf ($d^S \geq 20.32$) Then
$MERDIAM^S = d^S$
ElseIf ($d^S \leq 2.54$) Then
$MERDIAM^S = 0$
Else
A = 3.0746534
B = 0.30615578
C = 0.040103103
$MERDIAM^S = (A + B \times d^S / 2.54 + C \times (d^S / 2.54)^2) \times 2.54$
End If

Step 4 : Calculate merchantable frequency

$MERFREQ^S$ measured in number of trees per hectare
If ($MERDIAM^S \neq 0$) Then
$MERFREQ^S = MERBA^S \div (3.141592654 \times (MERDIAM^S \div 200)^2)$
Else
$MERFREQ^S = 0$
End If

Step 5 : Calculate board basal area

$BRDBA^S$ measured in m^2
If ($MERBA^S \leq 0$) Then
$RATIO_5 = 0$
ElseIf ($MERFREQ^S \leq 0$) Then
$RATIO_5 = 0$

Else

$X1 = 1 \div ((MERDIAM^S \div 2.54)^6)$

$X2 = (MERDIAM^S \div 2.54)^{10}$

$X3 = 1 \div ((MERDIAM^S \div 2.54)^5)$

$X4 = (MERDIAM^S \div 2.54)^9$

$X5 = (MERDIAM^S \div 2.54)^2$

$X6 = 1 \div ((MERDIAM^S \div 2.54)^{10})$

A = 1.0794209

B = 21281.691

C = 0.0000000000088421416

D = 6064.0629

E = 0.000000000011016046

F = 0.00050860475

G = 394808.99

$RATIO_5 = A + B \times X1 - C \times X2 - D \times X3 + E \times X4 - F \times X5 - G \times X6$

End If

$BRDBA^S = RATIO_5 \times MERBA^S$

If ($BRDBA^S > MERBA^S$) Then

$BRDBA^S = MERBA^S$

ElseIf ($BRDBA^S \leq 0$) Then

$BRDBA^S = 0$

End If

Step 6 : Calculate board diameter

$BRDDIAM^S$ measured in centimetres

If ($MERDIAM^S > 25.4$) Then

$BRDDIAM^S = MERDIAM^S$

ElseIf ($MERDIAM^S = 0$) Then

$BRDDIAM^S = 0$

Else

$X1 = 1 \div (MERDIAM^S \div 2.54)$

$X2 = (MERDIAM^S \div 2.54)^{10}$

$X3 = (MERDIAM^S \div 2.54)^{0.1}$

$X4 = (MERDIAM^S \div 2.54)^{0.5}$

$X5 = 1 \div (MERDIAM^S \div 2.54)^6$

$X6 = 1 \div (MERDIAM^S \div 2.54)^{10}$

A = -8.5204673

B = 7.0903534

$C = 1.2249943 \times 10^{-20}$

D = 7.6542127

E = 0.2563372

F = 493.03614

G = 55803.496

$RATIO_6 = A + B \times X1 - C \times X2 + D \times X3 - E \times X4 - F \times X5 + G \times X6$

$BRDDIAM^S = RATIO_6 \times MERDIAM^S$

End If
If ($BRDBA^S$ < 0 or $BRDDIAM^S$ < 14.22) Then
    $BRDDIAM^S$ = 0
End If
If ($BRDDIAM^S$ < $MERDIAM^S$) Then
    $BRDDIAM^S$ = $MERDIAM^S$
End If

Step 7 : Calculate board frequency

$BRDFREQ^S$ measured in number of trees per hectare
If ($MERDIAM^S$ > 25.4) Then
    $BRDFREQ^S$ = $MERFREQ^S$
ElseIf ($BRDDIAM^S$ < 12.7) Then
    $BRDFREQ^S$ = 0
Else
    $BRDFREQ^S$ = $BRDBA^S$ ÷ (3.141592654 * ($BRDDIAM^S$ ÷ 200) ^ 2)
End If
If ($BRDFREQ^S$ > $MERFREQ^S$) Then
    $BRDFREQ^S$ = $MERFREQ^S$
End If

Step 8 : Calculate merchantable height

$MERHGT^S$ measured in meters
$RATIO_8$ = $MERBA^S$ / BA^S
If ($RATIO_8$ = 1 or aht^S > 24.4) Then
    $MERHGT^S$ = aht^S
ElseIf ($RATIO_8$ = 0) Then
    $MERHGT^S$ = 0
Else
    A = 15.277829
    B = 1.4316388
    C = 0.53584452
    D = 185.70383
    E = 0.00000046086744
    F = 134.91889
    G = 68.753896
    H = 31.713797
    J = 0.030025568
    K = aht^S * 3.2808
    $MERHGT^S = (-A + B \times K - C \times K \times RATIO_8 + D \times (RATIO_8)^4 + E \times (K \div RATIO_8)^3 - F \times (RATIO_8)^5 - G \times (RATIO_8)^2 + H \times (RATIO_8)^{0.5} + J \times K \div RATIO_8) \div 3.2808$
End If
If ($MERHGT^S \neq 0$) Then

If ($MERHGT^S < aht^S$) Then
$MERHGT^S = aht^S$
End If
End If

Step 9 : Calculate board height

$BRDHGT^S$ measured in meters
If ($MERBA^S > 0$) Then
$RATIO_9 = BRDBA^S$ / $MERBA^S$
End If
If ($RATIO_9 \leq 0$) Then
$BRDHGT^S = 0$
ElseIf ($RATIO_9 > 1$) Then
$BRDHGT^S = MERHGT^S$
ElseIf ($MERHGT^S = 0$) Then
$BRDHGT^S = 0$
Else
A = 9.3709292
B = 0.046206221
C = 0.00074921477
D = 0.0000040028957
E = 2.8348874
F = 3.1858712
G = $MERHGT^S$ * 3.2808
$BRDHGT^S = (A + B \times G^2 - C \times G^3 + D \times G^4 - E \times RATIO_9 - F \times (RATIO_9)^5) \div 3.2808$
End If
If ($BRDHGT^S < MERHGT^S$ or $MERHGT^S > 24.4$) Then
$BRDHGT^S = MERHGT^S$
End If

Step 10 : Calculate merchantable volume and board volume

MV^S and SV^S measured in m^3 per hectare
A = 1.226
B = 315.832
If ($MERHGT^S > 0$ and $MERDIAM^S > 0$) Then
$C = ((3 \div (MERDIAM^S \div 2.54))^2) \times (1 + (0.5 \div (MERHGT^S \times 3.2808)))$
$MV^S = 0.069972228 \times (MERFREQ^S \times ((MERDIAM^S \div 2.54)^2 \div (A + (B \div (MERHGT^S \times 3.2808)))) \times (0.9604 - 0.166 \times C - 0.7868 \times C^2))$
If ($MV^S < 0$) Then
$MV^S = 0$
End If
Else
$MV^S = 0$
End If

If ($BRDHGT^S$ > 0 And $BRDDIAM^S$ > 0) Then

E = ((4 ÷ ($BRDDIAM^S$ ÷ 2.54))^2) × (1 + (0.5 ÷ ($BRDHGT^S$ × 3.2808)))

SV^S = 0.01396074 × ($BRDFREQ^S$ × (($BRDDIAM^S$ ÷ 2.54)^2 ÷ (A + (B ÷ ($BRDHGT^S$ × 3.2808)))) × (5.316 − 1.5928 × E − 4.3747 × E^2))

If (SV^S < 0) Then

SV^S = 0

End If

Else

SV^S = 0

End If

Procedure 2

Step 1: Calculate merchantable basal area as a fraction of stand total basal area

d^H measured in centimeters
$MERBA^H$ measured in m^2
BA^H measured in m^2
If (d^H < 4.1) Then

$MERBA^H$ = 0

Else

$MTBARAT^H = \left(1 - e^{-0.374584768 * d^H}\right)^{10.855823566}$

$MERBA^H$ = $MTBARAT^H$ × BA^H

End If

Step 2 : Calculate ratio of $MERBA^H$ to BA^H

$RATIO^H$ = $MERBA^H$ ÷ BA^H

Step 3 : Calculate merchantable diameter

$MERDIAM^H$ measured in centimeters
If (d^H < 4.1) Then

$MERDIAM^H$ = 0

ElseIf (d^H > 25) Then

$MERDIAM^H$ = d^H

Else

$MERDIAM^H$ = 0.76293 × d^H + 5.9041

End If

Step 4 : Calculate merchantable frequency

$MERFREQ^H$ measured in number of trees per hectare
If ($MERDIAM^H$ ≠ 0) Then

$MERFREQ^H$ = $MERBA^H$ ÷ (3.141592654 × ($MERDIAM^H$ ÷ 200)^2)

Else

$MERFREQ^H = 0$

End If

Step 5 : Calculate board basal area

$BRDBA^H$ measured in m^2

If ($MERDIAM^H < 11$) Then

$BRDBA^H = 0$

Else

$BMBARAT^H = \left(1 - e^{-0.3994689 * MERDIAM^H}\right)^{143.75231029}$

$BRDBA^H = MERBA^H \times BMBARAT^H$

End If

Step 6 : Calculate board diameter

$BRDDIAM^H$ measured in centimetres

If ($MERDIAM^H < 11$) Then

$BRDDIAM^H = 0$

ElseIf ($MERDIAM^H > 31$) Then

$BRDDIAM^H = MERDIAM^H$

Else

$BRDDIAM^H = 0.832332 \times MERDIAM^H + 5.214265$

Endif

Step 7 : Calculate board frequency

$BRDFREQ^H$ measured in number of trees per hectare

If ($MERDIAM^H < 11$) Then

$BRDFREQ^H = 0$

Else

$BRDFREQ^H = BRDBA^H \div (3.141592654 \times (BRDDIAM^H \div 200)^2)$

End If

Step 8 : Calculate merchantable height

$MERHGT^H$ measured in meters

If ($d^H < 4.1$) Then

$MERHGT^H = 0$

Else

$MERHGT^H = aht^H \times \left(1 + e^{-0.197080469 \times d^H}\right)^{0.489554121}$

End If

Step 9 : Calculate board height

$BRDHGT^H$ measured in meters

If $d^H < 11$ Then

    $BRDHGT^H = 0$

Else

    $BRDHGT^H = MERHGT^H \times \left(1 + \mathrm{e}^{-0.14338793 \times d^H}\right)^{0.377415145}$

End If

Step 10 : Calculate merchantable volume and board volume

MV^H and SV^H measured in m^3 per hectare

A = 1.046

B = 383.972

B2 = 0.145

R1 = 0.9057

R2 = -0.0708

R3 = -0.8375

R4 = 0.0043891

R5 = 0.04365

R6 = 0.3048

If ($MERHGT^H > 0$ and $MERDIAM^H > 0$) Then

    $TV2 = (R4 \times (MERDIAM^H)^2 \times (1 - R5 \times B2)^2) \div (A + R6 \times B \div MERHGT^H)$

    S = 0.15

    T= 7.0

    $X3 = (T^2 \div ((MERDIAM^H)^2 \times ((1 - R5 \times B2)^2))) \times (1 + S \div MERHGT^H)$

    $MVRAT^H = R1 + R2 \times X3 + R3 \times X3^2$

    $MV1 = TV2 \times MVRAT^H$

    $MV^H = MV1 \times MERFREQ^H$

    If ($MV^H < 0$) Then

        $MV^H = 0$

    End If

Else

    $MV^H = 0$

End If

If ($BRDHGT^H > 0$ And $BRDDIAM^H > 0$) Then

    $TV3 = (R4 \times (BRDDIAM^H)^2 \times (1 - R5 \times B2)^2) \div (A + R6 \times B \div BRDHGT^H)$

    S = 0.15

    t = 10.0

    $X4 = (T^2 \div ((BRDDIAM^H)^2 \times ((1 - R5 \times B2)^2))) \times (1 + S \div BRDHGT^H)$

    $BVRAT^H = R1 + R2 \times X4 + R3 \times X4^2$

    $BV1 = TV3 \times BVRAT^H$

    $SV^H = BV1 \times BRDFREQ^H$

    If ($SV^H < 0$) Then

        $SV^H = 0$

    End If

Else

$SV^{H} = 0$

End If

If ($SV^{H} > MV^{H}$) Then

$SV^{H} = MV^{H}$

End If

Procedure 3

Step 1: Calculate merchantable basal area as a fraction of stand total basal area

d^{S} measured in centimeters
$MERBA^{S}$ measured in m^2
BA^{S} measured in m^2
If ($d^{S} < 7.11$) Then

$RATIO_1 = 0$

Else

$RATIO_1 = 1 - \mathrm{e}^{-0.949286627 \times \left(\left(d^{S} \div 2.54\right) - 2.4\right)}$

If ($RATIO_1 < 0$) Then

$RATIO_1 = 0$

End If

End If

$MERBA^{S} = RATIO_1 \times BA^{S}$

If ($MERBA^{S} < 0.1$) Then

$MERBA^{S} = 0$

End If

Step 2 : Calculate merchantable frequency

$MERFREQ^{S}$ and $TFREQ^{S}$ measured in number of trees per hectare
If ($d^{S} < 7.11$) Then

$RATIO_2 = 0$

Else

$RATIO_2 = 1 - \mathrm{e}^{-0.505737453 \times \left(\left(d^{S} \div 2.54\right) - 2.4\right)^{1.272646184}}$

End If

$MERFREQ^{S} = RATIO_2 \times TFREQ^{S}$

If ($MERBA^{S} < 0.1$) Then

$MERFREQ^{S} = 0$

End If

Step 3 : Calculate merchantable diameter

$MERDIAM^{S}$ measured in centimeters

If ($MERFREQ^S < 1$) Then

$MERDIAM^S = 0$

Else

$MERDIAM^S = 200 \times (MERBA^S \div (MERFREQ^S \times 3.141592654))^{0.5}$

End If

Step 4 : Calculate board basal area

$BRDBA^S$ measured in m^2

If ($MERDIAM^S < 11.4$) Then

$BRDBA^S = 0$

Else

$RATIO_4 = 1 - e^{-0.63777207 \times \left((d^S \div 2.54) - 4.368490128\right)}$

If ($RATIO_4 < 0$) Then

$BRDBA^S = 0$

Else

$BRDBA^S = RATIO_4 \times MERBA^S$

End If

End If

Step 5 : Calculate board frequency

$BRDFREQ^S$ measured in number of trees per hectare

If ($MERDIAM^S < 11.4$) Then

$RATIO_5 = 0$

Else

$RATIO_5 = 1 - e^{-0.395102665 \times \left((d^S \div 2.54) - 4.425711477\right)}$

End If

If ($RATIO_5 < 0$) Then

$RATIO_5 = 0$

End If

$BRDFREQ^S = RATIO_5 \times MERFREQ^S$

Step 6 : Calculate board diameter

$BRDDIAM^S$ measured in centimetres

If $BRDFREQ^S < 1$ Then

$BRDDIAM^S = 0$

Else

$BRDDIAM^S = 200 \times (BRDBA^S \div (BRDFREQ^S \times 3.141592654))^{0.5}$

End If

Step 7 : Calculate stand average height

aht^S and dht^S measured in meters

A = − 1.466091
B = 0.861959
C = 0.00108537
aht^S = A + B × (dht^S × 3.2808) + C × (dht^S × 3.2808)^2) ÷ 3.2808

Step 8 : Calculate merchantable height

$MERHGT^S$ measured in meters
$RATIO_8 = MERBA^S / BA^S$
If ($RATIO_8$ = 1 or aht^S > 24.4) Then
    $MERHGT^S = aht^S$
ElseIf ($RATIO_8$ = 0) Then
    $MERHGT^S$ = 0
Else
    A = 15.277829
    B = 1.4316388
    C = 0.53584452
    D = 185.70383
    E = 0.00000046086744
    F = 134.91889
    G = 68.753896
    H = 31.713797
    J = 0.030025568
    K = aht^S × 3.2808
    $MERHGT^S$ = (−A + B × K − C × K × $RATIO_8$ + D × $(RATIO_8)^4$ + E × (K ÷ $RATIO_8)^3$ − F × $(RATIO_8)^5$ − G × $(RATIO_8)^2$ + H × $(RATIO_8)^{0.5}$ + J × K ÷ $RATIO_8$) ÷ 3.2808
End If
If ($MERHGT^S \neq 0$) Then
    If ($MERHGT^S < aht^S$) Then
        $MERHGT^S = aht^S$
    End If
End If

Step 9 : Calculate board height

$BRDHGT^S$ measured in meters
If ($MERBA^S$ > 0) Then
    $RATIO_9 = BRDBA^S / MERBA^S$
End If
If ($RATIO_9 \leq 0$) Then
    $BRDHGT^S$ = 0
ElseIf ($RATIO_9$ > 1) Then
    $BRDHGT^S = MERHGT^S$
ElseIf ($MERHGT^S$ = 0) Then

$BRDHGT^S = 0$
Else
$A = 9.3709292$
$B = 0.046206221$
$C = 0.00074921477$
$D = 0.0000040028957$
$E = 2.8348874$
$F = 3.1858712$
$G = MERHGT^S \times 3.2808$
$BRDHGT^S = (A + B \times G^2 - C \times G^3 + D \times G^4 - E \times RATIO_9 - F \times (RATIO_9)^5) \div 3.2808$
End If
If ($BRDHGT^S < MERHGT^S$ or $MERHGT^S > 24.4$) Then
$BRDHGT^S = MERHGT^S$
End If

Step 10 : Calculate merchantable volume and board volume

MV^S and SV^S measured in m^3 per hectare
$A = 1.226$
$B = 315.832$
If ($MERHGT^S > 0$ and $MERDIAM^S > 0$) Then
$C = ((3 \div (MERDIAM^S \div 2.54))^2) \times (1 + (0.5 \div (MERHGT^S \times 3.2808)))$
$MV^S = 0.069972228 \times (MERFREQ^S \times ((MERDIAM^S \div 2.54)^2 \div (A + (B \div (MERHGT^S \times 3.2808)))) \times (0.9604 - 0.166 \times C - 0.7868 \times C^2))$
If ($MV^S < 0$) Then
$MV^S = 0$
End If
Else
$MV^S = 0$
End If
If ($BRDHGT^S > 0$ And $BRDDIAM^S > 0$) Then
$E = ((4 \div (BRDDIAM^S \div 2.54))^2) \times (1 + (0.5 \div (BRDHGT^S \times 3.2808)))$
$SV^S = 0.01396074 \times (BRDFREQ^S \times ((BRDDIAM^S \div 2.54)^2 \div (A + (B \div (BRDHGT^S \times 3.2808)))) \times (5.316 - 1.5928 \times E - 4.3747 \times E^2))$
If ($SV^S < 0$) Then
$SV^S = 0$
End If
Else
$SV^S = 0$
End If

Procedure 4

Step 1: Calculate merchantable basal area as a fraction of stand total basal area

d^H measured in centimeters
$MERBA^H$ measured in m^2
BA^H measured in m^2
If (d^H < 7.11) Then
    $RATIO_1$ = 0
Else
    $RATIO_1 = 1 - e^{-0.949286627 \times \left((d^H \div 2.54) - 2.4\right)}$
    If ($RATIO_1$ < 0) Then
        $RATIO_1$ = 0
    End If
End If
$MERBA^H = RATIO_1 \times BA^H$
If ($MERBA^H$ < 0.1) Then
    $MERBA^H$ = 0
End If

Step 2 : Calculate merchantable frequency

$MERFREQ^H$ and $TFREQ^H$ measured in number of trees per hectare
If (d^H < 7.11) Then
    $RATIO_2$ = 0
Else
    $RATIO_2 = 1 - e^{-0.505737453 \times \left((d^H \div 2.54) - 2.4\right)^{1.272646184}}$
End If
$MERFREQ^H = RATIO_2 \times TFREQ^H$
If ($MERBA^H$ < 0.1) Then
    $MERFREQ^H$ = 0
End If

Step 3 : Calculate merchantable diameter

$MERDIAM^H$ measured in centimeters
If ($MERFREQ^H$ < 1) Then
    $MERDIAM^H$ = 0
Else
    $MERDIAM^H = 200 \times (MERBA^H \div (MERFREQ^H \times 3.141592654))^{0.5}$
End If

Step 4 : Calculate board basal area

$BRDBA^H$ measured in m^2
If ($MERDIAM^H$ < 11.4) Then
    $BRDBA^H$ = 0
Else

$RATIO_4 = 1 - e^{-0.63777207\times\left((d^H \div 2.54)-4.368490128\right)}$
If ($RATIO_4 < 0$) Then
$BRDBA^H = 0$
Else
$BRDBA^H = RATIO_4 \times MERBA^H$
End If
End If

Step 5 : Calculate board frequency

$BRDFREQ^H$ measured in number of trees per hectare
If ($MERDIAM^H < 11.4$) Then
$RATIO_5 = 0$
Else
$RATIO_5 = 1 - e^{-0.395102665\times\left((d^H \div 2.54)-4.425711477\right)}$
End If
If ($RATIO_5 < 0$) Then
$RATIO_5 = 0$
End If
$BRDFREQ^H = RATIO_5 \times MERFREQ^H$

Step 6 : Calculate board diameter

$BRDDIAM^H$ measured in centimetres
If $BRDFREQ^H < 1$ Then
$BRDDIAM^H = 0$
Else
$BRDDIAM^H = 200 \times (BRDBA^H \div (BRDFREQ^H \times 3.141592654))^{0.5}$
End If

Step 7 : Calculate stand average height

aht^H and dht^H measured in meters
$aht^H = 0.962 \times dht^H - 0.3979$

Step 8 : Calculate merchantable height

$MERHGT^H$ measured in meters
$RATIO_8 = MERBA^H / BA^H$
If ($RATIO_8 = 1$ or $aht^H > 24.4$) Then
$MERHGT^H = aht^H$
ElseIf ($RATIO_8 = 0$) Then
$MERHGT^H = 0$
Else
A = 15.277829

B = 1.4316388
C = 0.53584452
D = 185.70383
E = 0.00000046086744
F = 134.91889
G = 68.753896
H = 31.713797
J = 0.030025568
K = aht^H * 3.2808
$MERHGT^H = (-A + B \times K - C \times K \times RATIO_8 + D \times (RATIO_8)^4 + E \times (K \div RATIO_8)^3 - F \times (RATIO_8)^5 - G \times (RATIO_8)^2 + H \times (RATIO_8)^{0.5} + J \times K \div RATIO_8) \div 3.2808$
End If
If ($MERHGT^H \neq 0$) Then
If ($MERHGT^H < aht^H$) Then
$MERHGT^H = aht^H$
End If
End If

Step 9 : Calculate board height

$BRDHGT^H$ measured in meters
If ($MERBA^H > 0$) Then
$RATIO_9 = BRDBA^H / MERBA^H$
End If
If ($RATIO_9 \leq 0$) Then
$BRDHGT^H = 0$
ElseIf ($RATIO_9 > 1$) Then
$BRDHGT^H = MERHGT^H$
ElseIf ($MERHGT^H = 0$) Then
$BRDHGT^H = 0$
Else
A = 9.3709292
B = 0.046206221
C = 0.00074921477
D = 0.0000040028957
E = 2.8348874
F = 3.1858712
G = $MERHGT^H$ * 3.2808
$BRDHGT^H = (A + B \times G^2 - C \times G^3 + D \times G^4 - E \times RATIO_9 - F \times (RATIO_9)^5) \div 3.2808$
End If
If ($BRDHGT^H < MERHGT^H$ or $MERHGT^H > 24.4$) Then
$BRDHGT^H = MERHGT^H$
End If

Step 10 : Calculate merchantable volume and board volume

MV^H and SV^H measured in m^3 per hectare
A = 1.046
B = 383.972
B2 = 0.145
R1 = 0.9057
R2 = -0.0708
R3 = -0.8375
R4 = 0.0043891
R5 = 0.04365
R6 = 0.3048
If ($MERHGT^H$ > 0 and $MERDIAM^H$ > 0) Then
    $TV2 = (R4 \times (MERDIAM^H)^2 \times (1 - R5 \times B2)^2) \div (A + R6 \times B \div MERHGT^H)$
    S = 0.15
    T= 7.0
    $X3 = (T^2 \div ((MERDIAM^H)^2 \times ((1 - R5 \times B2)^2))) \times (1 + S \div MERHGT^H)$
    $MVRAT^H = R1 + R2 \times X3 + R3 \times X3^2$
    $MV1 = TV2 \times MVRAT^H$
    $MV^H = MV1 \times MERFREQ^H$
    If (MV^H < 0) Then
        $MV^H = 0$
    End If
Else
    $MV^H = 0$
End If
If ($BRDHGT^H$ > 0 And $BRDDIAM^H$ > 0) Then
    $TV3 = (R4 \times (BRDDIAM^H)^2 \times (1 - R5 \times B2)^2) \div (A + R6 \times B \div BRDHGT^H)$
    S = 0.15
    t = 10.0
    $X4 = (T^2 \div ((BRDDIAM^H)^2 \times ((1 - R5 \times B2)^2))) \times (1 + S \div BRDHGT^H)$
    $BVRAT^H = R1 + R2 \times X4 + R3 \times X4^2$
    $BV1 = TV3 \times BVRAT^H$
    $SV^H = BV1 \times BRDFREQ^H$
    If (SV^H < 0) Then
        $SV^H = 0$
    End If
Else
    $SV^H = 0$
End If

If ($SV^H > MV^H$) Then
    $SV^H = MV^H$
End If

Appendix B: List of Controls for the DP Model

This section lists controls $u_k \epsilon U_k(x_k)$ with the subset S_n in which they are applicable and the subset S_m to which x_{k+1} belongs after having taken decision u_k. After the table, a short section explains how $BARem$ and $BARem_{Split}$ are used to calculate the actual amount of basal area removed for each species type.

Decision (u_k)	S_n in which it applies	Resulting S_m for x_{k+1} after u_k is applied to x_k
1 – Do nothing and let grow	1	1
2 – Do nothing and let grow	2,3,4,5	Same S_n
3 – Regeneration harvest or site prep, survey and let regenerate naturally	1,2,3,4,5	1
4 - Regeneration harvest or site prep and fill plant without early competition control	1,2,3,4,5	1
5 - Regeneration harvest or site prep and plant 1000 trees/ha with 100% softwood	1,2,3,4,5	3
6 - Regeneration harvest or site prep and plant 1750 trees/ha with 100% softwood	1,2,3,4,5	3
7 - Regeneration harvest or site prep and plant 2500 trees/ha with 100% softwood	1,2,3,4,5	3
8 - Regeneration harvest or site prep and plant 3250 trees/ha with 100% softwood	1,2,3,4,5	3
9 - Regeneration harvest or site prep and plant 4000 trees/ha with 100% softwood	1,2,3,4,5	3
10 - Pre-Commercial thinning - keep mixed wood	1	2
11 - Pre-Commercial thinning - eliminate softwood and space hardwood if needed	1	2
12 - Pre-Commercial thinning - eliminate hardwood and space softwood if needed	1	2
13 - Commercial thinning (CT) - $BARem$ = 20%, $BARem_{Split}$ = 25% SW from below	1,2,3,4,5	4 if S_n = 3 or 4, 5 otherwise
14 - Commercial thinning (CT) - $BARem$ = 20%, $BARem_{Split}$ = 50% SW from below	1,2,3,4,5	4 if S_n = 3 or 4, 5 otherwise
15 - Commercial thinning (CT) - $BARem$ = 20%, $BARem_{Split}$ = 75% SW from below	1,2,3,4,5	4 if S_n = 3 or 4, 5 otherwise
16 - Commercial thinning (CT) - $BARem$ = 30%, $BARem_{Split}$ = 25% SW from below	1,2,3,4,5	4 if S_n = 3 or 4, 5 otherwise
17 - Commercial thinning (CT) - $BARem$ = 30%, $BARem_{Split}$ = 50% SW from below	1,2,3,4,5	4 if S_n = 3 or 4, 5 otherwise
18 - Commercial thinning (CT) - $BARem$ = 30%, $BARem_{Split}$ = 75% SW from below	1,2,3,4,5	4 if S_n = 3 or 4, 5 otherwise
19 - Commercial thinning (CT) - $BARem$ = 40%, $BARem_{Split}$ = 25% SW from below	1,2,3,4,5	4 if S_n = 3 or 4, 5 otherwise
20 - Commercial thinning (CT) - $BARem$ = 40%, $BARem_{Split}$ = 50% SW from below	1,2,3,4,5	4 if S_n = 3 or 4, 5 otherwise

Decision (u_k)	S_n in which it applies	Resulting S_m for x_{k+1} after u_k is applied to x_k
21 - Commercial thinning (CT) - $BARem$ = 40%, $BARem_{Split}$ = 75% SW from below	1,2,3,4,5	4 if S_n = 3 or 4, 5 otherwise
22 - Commercial thinning (CT) - $BARem$ = 20%, $BARem_{Split}$ = 25% SW across the diameter distribution	1,2,3,4,5	4 if S_n = 3 or 4, 5 otherwise
23 - Commercial thinning (CT) - $BARem$ = 20%, $BARem_{Split}$ = 50% SW across the diameter distribution	1,2,3,4,5	4 if S_n = 3 or 4, 5 otherwise
24 - Commercial thinning (CT) - $BARem$ = 20%, $BARem_{Split}$ = 75% SW across the diameter distribution	1,2,3,4,5	4 if S_n = 3 or 4, 5 otherwise
25 - Commercial thinning (CT) - $BARem$ = 30%, $BARem_{Split}$ = 25% SW across the diameter distribution	1,2,3,4,5	4 if S_n = 3 or 4, 5 otherwise
26 - Commercial thinning (CT) - $BARem$ = 30%, $BARem_{Split}$ = 50% SW across the diameter distribution	1,2,3,4,5	4 if S_n = 3 or 4, 5 otherwise
27 - Commercial thinning (CT) - $BARem$ = 30%, $BARem_{Split}$ = 75% SW across the diameter distribution	1,2,3,4,5	4 if S_n = 3 or 4, 5 otherwise
28 - Commercial thinning (CT) - $BARem$ = 40%, $BARem_{Split}$ = 25% SW across the diameter distribution	1,2,3,4,5	4 if S_n = 3 or 4, 5 otherwise
29 - Commercial thinning (CT) - $BARem$ = 40%, $BARem_{Split}$ = 50% SW across the diameter distribution	1,2,3,4,5	4 if S_n = 3 or 4, 5 otherwise
30 - Commercial thinning (CT) - $BARem$ = 40%, $BARem_{Split}$ = 75% SW across the diameter distribution	1,2,3,4,5	4 if S_n = 3 or 4, 5 otherwise
31 - Commercial thinning (CT) - $BARem$ = 20%, $BARem_{Split}$ = 25% SW from above	1,2,3,4,5	4 if S_n = 3 or 4, 5 otherwise
32 - Commercial thinning (CT) - $BARem$ = 20%, $BARem_{Split}$ = 50% SW from above	1,2,3,4,5	4 if S_n = 3 or 4, 5 otherwise
33 - Commercial thinning (CT) - $BARem$ = 20%, $BARem_{Split}$ = 75% SW from above	1,2,3,4,5	4 if S_n = 3 or 4, 5 otherwise
34 - Commercial thinning (CT) - $BARem$ = 30%, $BARem_{Split}$ = 25% SW from above	1,2,3,4,5	4 if S_n = 3 or 4, 5 otherwise
35 - Commercial thinning (CT) - $BARem$ = 30%, $BARem_{Split}$ = 50% SW from above	1,2,3,4,5	4 if S_n = 3 or 4, 5 otherwise
36 - Commercial thinning (CT) - $BARem$ = 30%, $BARem_{Split}$ = 75% SW from above	1,2,3,4,5	4 if S_n = 3 or 4, 5 otherwise
37 - Commercial thinning (CT) - $BARem$ = 40%, $BARem_{Split}$ = 25% SW from above	1,2,3,4,5	4 if S_n = 3 or 4, 5 otherwise
38 - Commercial thinning (CT) - $BARem$ = 40%, $BARem_{Split}$ = 50% SW from above	1,2,3,4,5	4 if S_n = 3 or 4, 5 otherwise
39 - Commercial thinning (CT) - $BARem$ = 40%, $BARem_{Split}$ = 75% SW from above	1,2,3,4,5	4 if S_n = 3 or 4, 5 otherwise
40 - Commercial thinning (CT) - $BARem$ = 20%, $BARem_{Split}$ = 100% SW from below	1,2,3,4,5	4 if S_n = 3 or 4, 5 otherwise
41 - Commercial thinning (CT) - $BARem$ = 30%, $BARem_{Split}$ = 100% SW from below	1,2,3,4,5	4 if S_n = 3 or 4, 5 otherwise
42 - Commercial thinning (CT) - $BARem$ = 40%, $BARem_{Split}$ = 100% SW from below	1,2,3,4,5	4 if S_n = 3 or 4, 5 otherwise
43 - Commercial thinning (CT) - $BARem$ = 20%, $BARem_{Split}$ = 100% SW across the diameter distribution	1,2,3,4,5	4 if S_n = 3 or 4, 5 otherwise

Decision (u_k)	S_n in which it applies	Resulting S_m for x_{k+1} after u_k is applied to x_k
44 - Commercial thinning (CT) - $BARem$ = 30%, $BARem_{Split}$ = 100% SW across the diameter distribution	1,2,3,4,5	4 if S_n = 3 or 4, 5 otherwise
45 - Commercial thinning (CT) - $BARem$ = 40%, $BARem_{Split}$ = 100% SW across the diameter distribution	1,2,3,4,5	4 if S_n = 3 or 4, 5 otherwise
46 - Commercial thinning (CT) - $BARem$ = 20%, $BARem_{Split}$ = 100% SW from above	1,2,3,4,5	4 if S_n = 3 or 4, 5 otherwise
47 - Commercial thinning (CT) - $BARem$ = 30%, $BARem_{Split}$ = 100% SW from above	1,2,3,4,5	4 if S_n = 3 or 4, 5 otherwise
48 - Commercial thinning (CT) - $BARem$ = 40%, $BARem_{Split}$ = 100% SW from above	1,2,3,4,5	4 if S_n = 3 or 4, 5 otherwise

The table above shows three values for $BARem$ and four values for $BARem_{Split}$ which yields 12 possible combinations. With some stands, it isn't possible to apply the combined $BARem$ and $BARem_{Split}$ as specified. Therefore, the following rules apply for commercial thinning:

$$BA^S\ removed = min\left[BA \times BARem \times BARem_{Split}; BA^S\right]$$

$$BA^H\ removed = min\left[BA \times BARem \times \left(1 - BARem_{Split}\right); BA^H\right]$$

$$Actual\ BARem = \frac{BA^H\ removed + BA^S\ removed}{BA}$$

$$Actual\ BARem_{Split} = \frac{BA^S\ removed}{BA^S\ removed + BA^H\ removed}$$

where BA = total stand basal area (m^2), $BA^\theta\ removed$ = actual amount of basal area removed for each species type (m^2) and $Actual\ BARem$ = actual percentage of the stand basal area removed. Therefore, if there isn't enough SW or HW to remove the amount prescribed by the CT action being applied, those amounts are adjusted accordingly. For example, a stand has $50m^2$ of total basal area which is split 80%SW / 20%HW. Therefore, the stand contains $40m^2$ of SW and $10m^2$ of HW. If action 37 were to be applied to this stand, it would require removal of 40% of the total basal area or $20m^2$ which should be split 25% SW and 75% HW or $5m^2$ SW and $15m^2$ HW. However, the stand only contains $10m^2$ HW. According to the equations given above, action 37, for the stand shown as an example here, means:

$$BA^S\ removed = min[50m^2 \times 40\% \times 25\%; 40\text{m}^2] = 5m^2$$

$$BA^H\ removed = min[0m^2 \times 40\% \times (1 - 25\%); 10m^2] = \ 10m^2$$

$$Actual\ BARem = \frac{5m^2 + 10m^2}{40m^2} = 37.5\%$$

$$Actual\ BARem_{Split} = \frac{5m^2}{5m^2 + 10m^2} = 33.3\%$$

Appendix C: Example List of Evaluation States

This section gives an example list of evaluation states for each subset in S^{Eval}.

Subset S_1^{Eval}

Age (years)	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95
Stocking %	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100

Age (years)	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95
Stocking %	75	75	75	75	75	75	75	75	75	75	75	75	75	75	75	75	75	75	75

Subset S_2^{Eval}

Units of measure: age = years, d^S = d^H = cm, cc = pct^S = percentage

Age	d^S	d^H	cc	pct^S
10	0.0	2.6	9	0
10	3.2	2.6	9	33
10	3.2	2.6	9	66
10	3.2	0.0	9	100
10	0.0	2.6	16	0
10	3.2	2.6	16	33
10	3.2	2.6	16	66
10	3.2	0.0	16	100
10	0.0	2.6	22	0
10	3.2	2.6	22	33
10	3.2	2.6	22	66
10	3.2	0.0	22	100
20	0.0	5.5	20	0
20	0.0	6.3	20	0
20	0.0	7.0	20	0
20	6.5	5.5	20	33
20	6.5	6.3	20	33
20	6.5	7.0	20	33
20	8.5	5.5	20	33
20	8.5	6.3	20	33
20	8.5	7.0	20	33
20	10.5	5.5	20	33
20	10.5	6.3	20	33
20	10.5	7.0	20	33
20	6.5	5.5	20	66
20	6.5	6.3	20	66
20	6.5	7.0	20	66

Age	d^S	d^H	cc %	pct^S
30	11.1	9.7	58	66
30	13.1	9.7	58	66
30	15.1	9.7	58	66
30	11.1	0.0	58	100
30	13.1	0.0	58	100
30	15.1	0.0	58	100
30	0.0	9.7	83	0
30	11.1	9.7	83	33
30	13.1	9.7	83	33
30	15.1	9.7	83	33
30	11.1	9.7	83	66
30	13.1	9.7	83	66
30	15.1	9.7	83	66
30	11.1	0.0	83	100
30	13.1	0.0	83	100
30	15.1	0.0	83	100
40	0.0	12.1	40	0
40	15.5	12.1	40	33
40	16.9	12.1	40	33
40	18.2	12.1	40	33
40	15.5	12.1	40	66
40	16.9	12.1	40	66
40	18.2	12.1	40	66
40	15.5	0.0	40	100
40	16.9	0.0	40	100
40	18.2	0.0	40	100
40	0.0	12.1	70	0

Age	d^S	d^H	cc %	pct^S
60	22.5	15.3	100	33
60	23.7	15.3	100	33
60	21.3	15.3	100	66
60	22.5	15.3	100	66
60	23.7	15.3	100	66
60	21.3	0.0	100	100
60	22.5	0.0	100	100
60	23.7	0.0	100	100
70	0.0	16.8	40	0
70	24.0	16.8	40	33
70	25.1	16.8	40	33
70	26.3	16.8	40	33
70	24.0	16.8	40	66
70	25.1	16.8	40	66
70	26.3	16.8	40	66
70	24.0	0.0	40	100
70	25.1	0.0	40	100
70	26.3	0.0	40	100
70	0.0	16.8	70	0
70	24.0	16.8	70	33
70	25.1	16.8	70	33
70	26.3	16.8	70	33
70	24.0	16.8	70	66
70	25.1	16.8	70	66
70	26.3	16.8	70	66
70	24.0	0.0	70	100
70	25.1	0.0	70	100

S_2^{Eval} continued…

Age	d^S	d^S	cc	pct^S
20	8.5	5.5	20	66
20	8.5	6.3	20	66
20	8.5	7.0	20	66
20	10.5	5.5	20	66
20	10.5	6.3	20	66
20	10.5	7.0	20	66
20	6.5	0.0	20	100
20	8.5	0.0	20	100
20	10.5	0.0	20	100
20	0.0	5.5	35	0
20	0.0	6.3	35	0
20	0.0	7.0	35	0
20	6.5	5.5	35	33
20	6.5	6.3	35	33
20	6.5	7.0	35	33
20	8.5	5.5	35	33
20	8.5	6.3	35	33
20	8.5	7.0	35	33
20	10.5	5.5	35	33
20	10.5	6.3	35	33
20	10.5	7.0	35	33
20	6.5	5.5	35	66
20	6.5	6.3	35	66
20	6.5	7.0	35	66
20	8.5	5.5	35	66
20	8.5	6.3	35	66
20	8.5	7.0	35	66
20	10.5	5.5	35	66
20	10.5	6.3	35	66
20	10.5	7.0	35	66
20	6.5	0.0	35	100
20	8.5	0.0	35	100
20	10.5	0.0	35	100
20	0.0	5.5	50	0
20	0.0	6.3	50	0
20	0.0	7.0	50	0
20	6.5	5.5	50	33
20	6.5	6.3	50	33
20	6.5	7.0	50	33
20	8.5	5.5	50	33
20	8.5	6.3	50	33
20	8.5	7.0	50	33
20	10.5	5.5	50	33
20	10.5	6.3	50	33
20	10.5	7.0	50	33

Age	d^S	d^S	cc %	pct^S
40	15.5	12.1	70	33
40	16.9	12.1	70	33
40	18.2	12.1	70	33
40	15.5	12.1	70	66
40	16.9	12.1	70	66
40	18.2	12.1	70	66
40	15.5	0.0	70	100
40	16.9	0.0	70	100
40	18.2	0.0	70	100
40	0.0	12.1	100	0
40	15.5	12.1	100	33
40	16.9	12.1	100	33
40	18.2	12.1	100	33
40	15.5	12.1	100	66
40	16.9	12.1	100	66
40	18.2	12.1	100	66
40	15.5	0.0	100	100
40	16.9	0.0	100	100
40	18.2	0.0	100	100
50	0.0	13.9	40	0
50	18.5	13.9	40	33
50	19.8	13.9	40	33
50	21.0	13.9	40	33
50	18.5	13.9	40	66
50	19.8	13.9	40	66
50	21.0	13.9	40	66
50	18.5	0.0	40	100
50	19.8	0.0	40	100
50	21.0	0.0	40	100
50	0.0	13.9	70	0
50	18.5	13.9	70	33
50	19.8	13.9	70	33
50	21.0	13.9	70	33
50	18.5	13.9	70	66
50	19.8	13.9	70	66
50	21.0	13.9	70	66
50	18.5	0.0	70	100
50	19.8	0.0	70	100
50	21.0	0.0	70	100
50	0.0	13.9	100	0
50	18.5	13.9	100	33
50	19.8	13.9	100	33
50	21.0	13.9	100	33
50	18.5	13.9	100	66
50	19.8	13.9	100	66

Age	d^S	d^S	cc %	pct^S
70	26.3	0.0	70	100
70	0.0	16.8	100	0
70	24.0	16.8	100	33
70	25.1	16.8	100	33
70	26.3	16.8	100	33
70	24.0	16.8	100	66
70	25.1	16.8	100	66
70	26.3	16.8	100	66
70	24.0	0.0	100	100
70	25.1	0.0	100	100
70	26.3	0.0	100	100
80	0.0	18.2	40	0
80	26.6	18.2	40	33
80	27.4	18.2	40	33
80	28.2	18.2	40	33
80	26.6	18.2	40	66
80	27.4	18.2	40	66
80	28.2	18.2	40	66
80	26.6	0.0	40	100
80	27.4	0.0	40	100
80	28.2	0.0	40	100
80	0.0	18.2	70	0
80	26.6	18.2	70	33
80	27.4	18.2	70	33
80	28.2	18.2	70	33
80	26.6	18.2	70	66
80	27.4	18.2	70	66
80	28.2	18.2	70	66
80	26.6	0.0	70	100
80	27.4	0.0	70	100
80	28.2	0.0	70	100
80	0.0	18.2	100	0
80	26.6	18.2	100	33
80	27.4	18.2	100	33
80	28.2	18.2	100	33
80	26.6	18.2	100	66
80	27.4	18.2	100	66
80	28.2	18.2	100	66
80	26.6	0.0	100	100
80	27.4	0.0	100	100
80	28.2	0.0	100	100
90	0.0	19.7	40	0
90	28.5	19.7	40	33
90	29.3	19.7	40	33
90	30.1	19.7	40	33

S_2^{Eval} continued…

Age	d^S	d^S	cc	pct^S
20	6.5	5.5	50	66
20	6.5	6.3	50	66
20	6.5	7.0	50	66
20	8.5	5.5	50	66
20	8.5	6.3	50	66
20	8.5	7.0	50	66
20	10.5	5.5	50	66
20	10.5	6.3	50	66
20	10.5	7.0	50	66
20	6.5	0.0	50	100
20	8.5	0.0	50	100
20	10.5	0.0	50	100
30	0.0	9.7	33	0
30	11.1	9.7	33	33
30	13.1	9.7	33	33
30	15.1	9.7	33	33
30	11.1	9.7	33	66
30	13.1	9.7	33	66
30	15.1	9.7	33	66
30	11.1	0.0	33	100
30	13.1	0.0	33	100
30	15.1	0.0	33	100
30	0.0	9.7	58	0
30	11.1	9.7	58	33
30	13.1	9.7	58	33
30	15.1	9.7	58	33

Age	d^S	d^S	cc %	pct^S
50	21.0	13.9	100	66
50	18.5	0.0	100	100
50	19.8	0.0	100	100
50	21.0	0.0	100	100
60	0.0	15.3	40	0
60	21.3	15.3	40	33
60	22.5	15.3	40	33
60	23.7	15.3	40	33
60	21.3	15.3	40	66
60	22.5	15.3	40	66
60	23.7	15.3	40	66
60	21.3	0.0	40	100
60	22.5	0.0	40	100
60	23.7	0.0	40	100
60	0.0	15.3	70	0
60	21.3	15.3	70	33
60	22.5	15.3	70	33
60	23.7	15.3	70	33
60	21.3	15.3	70	66
60	22.5	15.3	70	66
60	23.7	15.3	70	66
60	21.3	0.0	70	100
60	22.5	0.0	70	100
60	23.7	0.0	70	100
60	0.0	15.3	100	0
60	21.3	15.3	100	33

Age	d^S	d^S	cc %	pct^S
90	28.5	19.7	40	66
90	29.3	19.7	40	66
90	30.1	19.7	40	66
90	28.5	0.0	40	100
90	29.3	0.0	40	100
90	30.1	0.0	40	100
90	0.0	19.7	70	0
90	28.5	19.7	70	33
90	29.3	19.7	70	33
90	30.1	19.7	70	33
90	28.5	19.7	70	66
90	29.3	19.7	70	66
90	30.1	19.7	70	66
90	28.5	0.0	70	100
90	29.3	0.0	70	100
90	30.1	0.0	70	100
90	0.0	19.7	100	0
90	28.5	19.7	100	33
90	29.3	19.7	100	33
90	30.1	19.7	100	33
90	28.5	19.7	100	66
90	29.3	19.7	100	66
90	30.1	19.7	100	66
90	28.5	0.0	100	100
90	29.3	0.0	100	100
90	30.1	0.0	100	100

Subset S_3^{Eval}

Units of measure: age = years, d^S = cm, initial planting density = trees/hectare

Age	d^S	Initial planting density
5	0.0	1000
10	5.9	1000
15	8.8	1000
20	11.5	1000
25	13.9	1000
30	15.9	1000
35	17.7	1000
40	19.3	1000
45	20.7	1000
50	21.9	1000
55	23.0	1000
60	24.0	1000

Age	d^S	Initial planting density
70	26.2	1750
75	26.8	1750
80	27.4	1750
85	28.0	1750
90	28.7	1750
95	29.3	1750
5	0.0	2500
10	4.3	2500
15	8.0	2500
20	10.8	2500
25	13.1	2500
30	15.0	2500

Age	d^S	Initial planting density
40	16.6	3250
45	17.8	3250
50	18.8	3250
55	19.7	3250
60	20.5	3250
65	21.2	3250
70	21.8	3250
75	22.4	3250
80	22.8	3250
85	23.2	3250
90	23.6	3250
95	23.9	3250

S_3^{Eval} continued…

Age	d^S	Initial planting density
65	24.8	1000
70	25.6	1000
75	26.4	1000
80	27.0	1000
85	27.7	1000
90	28.3	1000
95	28.9	1000
5	0.0	1750
10	4.8	1750
15	9.0	1750
20	12.2	1750
25	14.8	1750
30	16.9	1750
35	18.7	1750
40	20.3	1750
45	21.6	1750
50	22.7	1750
55	23.8	1750
60	24.7	1750
65	25.5	1750

Age	d^S	Initial planting density
35	16.6	2500
40	18.0	2500
45	19.2	2500
50	20.3	2500
55	21.2	2500
60	22.1	2500
65	22.8	2500
70	23.5	2500
75	24.0	2500
80	24.5	2500
85	25.0	2500
90	25.3	2500
95	25.7	2500
5	0.0	3250
10	3.9	3250
15	7.3	3250
20	10.0	3250
25	12.1	3250
30	13.8	3250
35	15.3	3250

Age	d^S	Initial planting density
5	0.0	4000
10	3.7	4000
15	6.9	4000
20	9.3	4000
25	11.3	4000
30	13.0	4000
35	14.4	4000
40	15.6	4000
45	16.8	4000
50	17.7	4000
55	18.6	4000
60	19.4	4000
65	20.1	4000
70	20.7	4000
75	21.2	4000
80	21.7	4000
85	22.0	4000
90	22.4	4000
95	22.7	4000

Subset S_4^{Eval}

Units of measure: age is in years, d^S is in centimeters and cc is a percentage

Age	d^S	cc
10	3.7	40
10	5.7	40
10	7.7	40
10	3.7	55
10	5.7	55
10	7.7	55
10	3.7	70
10	5.7	70
10	7.7	70
10	3.7	85
10	5.7	85
10	7.7	85
10	3.7	100
10	5.7	100
10	7.7	100
20	9.3	40
20	11.4	40
20	13.5	40

Age	d^S	cc
40	22.6	55
40	15.8	70
40	19.2	70
40	22.6	70
40	15.8	85
40	19.2	85
40	22.6	85
40	15.8	100
40	19.2	100
40	22.6	100
50	18.6	40
50	22.3	40
50	26.0	40
50	18.6	55
50	22.3	55
50	26.0	55
50	18.6	70
50	22.3	70

Age	d^S	cc
70	27.7	85
70	31.3	85
70	24.1	100
70	27.7	100
70	31.3	100
80	26.7	40
80	30.0	40
80	33.3	40
80	26.7	55
80	30.0	55
80	33.3	55
80	26.7	70
80	30.0	70
80	33.3	70
80	26.7	85
80	30.0	85
80	33.3	85
80	26.7	100

S_4^{Eval} continued…

Age	d^S	cc
20	9.3	55
20	11.4	55
20	13.5	55
20	9.3	70
20	11.4	70
20	13.5	70
20	9.3	85
20	11.4	85
20	13.5	85
20	9.3	100
20	11.4	100
20	13.5	100
30	13.0	40
30	15.7	40
30	18.5	40
30	13.0	55
30	15.7	55
30	18.5	55
30	13.0	70
30	15.7	70
30	18.5	70
30	13.0	85
30	15.7	85
30	18.5	85
30	13.0	100
30	15.7	100
30	18.5	100
40	15.8	40
40	19.2	40
40	22.6	40
40	15.8	55
40	19.2	55

Age	d^S	cc
50	26.0	70
50	18.6	85
50	22.3	85
50	26.0	85
50	18.6	100
50	22.3	100
50	26.0	100
60	21.4	40
60	25.1	40
60	28.9	40
60	21.4	55
60	25.1	55
60	28.9	55
60	21.4	70
60	25.1	70
60	28.9	70
60	21.4	85
60	25.1	85
60	28.9	85
60	21.4	100
60	25.1	100
60	28.9	100
70	24.1	40
70	27.7	40
70	31.3	40
70	24.1	55
70	27.7	55
70	31.3	55
70	24.1	70
70	27.7	70
70	31.3	70
70	24.1	85

Age	d^S	cc
80	30.0	100
80	33.3	100
90	28.0	40
90	31.5	40
90	35.1	40
90	28.0	55
90	31.5	55
90	35.1	55
90	28.0	70
90	31.5	70
90	35.1	70
90	28.0	85
90	31.5	85
90	35.1	85
90	28.0	100
90	31.5	100
90	35.1	100
100	29.2	40
100	32.9	40
100	36.6	40
100	29.2	55
100	32.9	55
100	36.6	55
100	29.2	70
100	32.9	70
100	36.6	70
100	29.2	85
100	32.9	85
100	36.6	85
100	29.2	100
100	32.9	100
100	36.6	100

Subset S_5^{Eval}

Units of measure: age = years, d^S = d^H = cm, cc = pct^S = percentage

Age	d^S	d^S	cc	pct^S
30	0.0	8.4	40	0
30	0.0	10.0	40	0
30	0.0	11.6	40	0
40	0.0	11.0	40	0
40	0.0	13.0	40	0
40	0.0	15.0	40	0
50	0.0	13.4	40	0

Age	d^S	d^S	cc	pct^S
60	23.7	20.1	55	66
60	28.5	15.5	55	66
60	28.5	17.8	55	66
60	28.5	20.1	55	66
70	21.5	17.3	55	66
70	21.5	19.7	55	66
70	21.5	22.1	55	66

Age	d^S	d^S	cc	pct^S
70	26.2	19.7	85	33
70	26.2	22.1	85	33
70	31.0	17.3	85	33
70	31.0	19.7	85	33
70	31.0	22.1	85	33
80	23.7	18.8	85	33
80	23.7	21.2	85	33

S_5^{Eval} continued…

Age	d^S	d^S	cc	pct^S
50	0.0	15.6	40	0
50	0.0	17.8	40	0
60	0.0	15.5	40	0
60	0.0	17.8	40	0
60	0.0	20.1	40	0
70	0.0	17.3	40	0
70	0.0	19.7	40	0
70	0.0	22.1	40	0
80	0.0	18.8	40	0
80	0.0	21.2	40	0
80	0.0	23.7	40	0
90	0.0	20.1	40	0
90	0.0	22.6	40	0
90	0.0	25.2	40	0
100	0.0	21.1	40	0
100	0.0	23.8	40	0
100	0.0	26.5	40	0
30	9.8	8.4	40	33
30	9.8	10.0	40	33
30	9.8	11.6	40	33
30	13.8	8.4	40	33
30	13.8	10.0	40	33
30	13.8	11.6	40	33
30	17.8	8.4	40	33
30	17.8	10.0	40	33
30	17.8	11.6	40	33
40	13.0	11.0	40	33
40	13.0	13.0	40	33
40	13.0	15.0	40	33
40	17.6	11.0	40	33
40	17.6	13.0	40	33
40	17.6	15.0	40	33
40	22.1	11.0	40	33
40	22.1	13.0	40	33
40	22.1	15.0	40	33
50	16.1	13.4	40	33
50	16.1	15.6	40	33
50	16.1	17.8	40	33
50	20.8	13.4	40	33
50	20.8	15.6	40	33
50	20.8	17.8	40	33
50	25.6	13.4	40	33
50	25.6	15.6	40	33
50	25.6	17.8	40	33
60	18.9	15.5	40	33

Age	d^S	d^S	cc	pct^S
70	26.2	17.3	55	66
70	26.2	19.7	55	66
70	26.2	22.1	55	66
70	31.0	17.3	55	66
70	31.0	19.7	55	66
70	31.0	22.1	55	66
80	23.7	18.8	55	66
80	23.7	21.2	55	66
80	23.7	23.7	55	66
80	28.4	18.8	55	66
80	28.4	21.2	55	66
80	28.4	23.7	55	66
80	33.0	18.8	55	66
80	33.0	21.2	55	66
80	33.0	23.7	55	66
90	25.7	20.1	55	66
90	25.7	22.6	55	66
90	25.7	25.2	55	66
90	30.3	20.1	55	66
90	30.3	22.6	55	66
90	30.3	25.2	55	66
90	34.9	20.1	55	66
90	34.9	22.6	55	66
90	34.9	25.2	55	66
100	27.4	21.1	55	66
100	27.4	23.8	55	66
100	27.4	26.5	55	66
100	32.0	21.1	55	66
100	32.0	23.8	55	66
100	32.0	26.5	55	66
100	36.7	21.1	55	66
100	36.7	23.8	55	66
100	36.7	26.5	55	66
30	9.8	0.0	55	100
30	13.8	0.0	55	100
30	17.8	0.0	55	100
40	13.0	0.0	55	100
40	17.6	0.0	55	100
40	22.1	0.0	55	100
50	16.1	0.0	55	100
50	20.8	0.0	55	100
50	25.6	0.0	55	100
60	18.9	0.0	55	100
60	23.7	0.0	55	100
60	28.5	0.0	55	100

Age	d^S	d^S	cc	pct^S
80	23.7	23.7	85	33
80	28.4	18.8	85	33
80	28.4	21.2	85	33
80	28.4	23.7	85	33
80	33.0	18.8	85	33
80	33.0	21.2	85	33
80	33.0	23.7	85	33
90	25.7	20.1	85	33
90	25.7	22.6	85	33
90	25.7	25.2	85	33
90	30.3	20.1	85	33
90	30.3	22.6	85	33
90	30.3	25.2	85	33
90	34.9	20.1	85	33
90	34.9	22.6	85	33
90	34.9	25.2	85	33
100	27.4	21.1	85	33
100	27.4	23.8	85	33
100	27.4	26.5	85	33
100	32.0	21.1	85	33
100	32.0	23.8	85	33
100	32.0	26.5	85	33
100	36.7	21.1	85	33
100	36.7	23.8	85	33
100	36.7	26.5	85	33
30	9.8	8.4	85	66
30	9.8	10.0	85	66
30	9.8	11.6	85	66
30	13.8	8.4	85	66
30	13.8	10.0	85	66
30	13.8	11.6	85	66
30	17.8	8.4	85	66
30	17.8	10.0	85	66
30	17.8	11.6	85	66
40	13.0	11.0	85	66
40	13.0	13.0	85	66
40	13.0	15.0	85	66
40	17.6	11.0	85	66
40	17.6	13.0	85	66
40	17.6	15.0	85	66
40	22.1	11.0	85	66
40	22.1	13.0	85	66
40	22.1	15.0	85	66
50	16.1	13.4	85	66
50	16.1	15.6	85	66

S_5^{Eval} continued…

Age	d^S	d^S	cc	pct^S
60	18.9	17.8	40	33
60	18.9	20.1	40	33
60	23.7	15.5	40	33
60	23.7	17.8	40	33
60	23.7	20.1	40	33
60	28.5	15.5	40	33
60	28.5	17.8	40	33
60	28.5	20.1	40	33
70	21.5	17.3	40	33
70	21.5	19.7	40	33
70	21.5	22.1	40	33
70	26.2	17.3	40	33
70	26.2	19.7	40	33
70	26.2	22.1	40	33
70	31.0	17.3	40	33
70	31.0	19.7	40	33
70	31.0	22.1	40	33
80	23.7	18.8	40	33
80	23.7	21.2	40	33
80	23.7	23.7	40	33
80	28.4	18.8	40	33
80	28.4	21.2	40	33
80	28.4	23.7	40	33
80	33.0	18.8	40	33
80	33.0	21.2	40	33
80	33.0	23.7	40	33
90	25.7	20.1	40	33
90	25.7	22.6	40	33
90	25.7	25.2	40	33
90	30.3	20.1	40	33
90	30.3	22.6	40	33
90	30.3	25.2	40	33
90	34.9	20.1	40	33
90	34.9	22.6	40	33
90	34.9	25.2	40	33
100	27.4	21.1	40	33
100	27.4	23.8	40	33
100	27.4	26.5	40	33
100	32.0	21.1	40	33
100	32.0	23.8	40	33
100	32.0	26.5	40	33
100	36.7	21.1	40	33
100	36.7	23.8	40	33
100	36.7	26.5	40	33
30	9.8	8.4	40	66

Age	d^S	d^S	cc	pct^S
70	21.5	0.0	55	100
70	26.2	0.0	55	100
70	31.0	0.0	55	100
80	23.7	0.0	55	100
80	28.4	0.0	55	100
80	33.0	0.0	55	100
90	25.7	0.0	55	100
90	30.3	0.0	55	100
90	34.9	0.0	55	100
100	27.4	0.0	55	100
100	32.0	0.0	55	100
100	36.7	0.0	55	100
30	0.0	8.4	70	0
30	0.0	10.0	70	0
30	0.0	11.6	70	0
40	0.0	11.0	70	0
40	0.0	13.0	70	0
40	0.0	15.0	70	0
50	0.0	13.4	70	0
50	0.0	15.6	70	0
50	0.0	17.8	70	0
60	0.0	15.5	70	0
60	0.0	17.8	70	0
60	0.0	20.1	70	0
70	0.0	17.3	70	0
70	0.0	19.7	70	0
70	0.0	22.1	70	0
80	0.0	18.8	70	0
80	0.0	21.2	70	0
80	0.0	23.7	70	0
90	0.0	20.1	70	0
90	0.0	22.6	70	0
90	0.0	25.2	70	0
100	0.0	21.1	70	0
100	0.0	23.8	70	0
100	0.0	26.5	70	0
30	9.8	8.4	70	33
30	9.8	10.0	70	33
30	9.8	11.6	70	33
30	13.8	8.4	70	33
30	13.8	10.0	70	33
30	13.8	11.6	70	33
30	17.8	8.4	70	33
30	17.8	10.0	70	33
30	17.8	11.6	70	33

Age	d^S	d^S	cc	pct^S
50	16.1	17.8	85	66
50	20.8	13.4	85	66
50	20.8	15.6	85	66
50	20.8	17.8	85	66
50	25.6	13.4	85	66
50	25.6	15.6	85	66
50	25.6	17.8	85	66
60	18.9	15.5	85	66
60	18.9	17.8	85	66
60	18.9	20.1	85	66
60	23.7	15.5	85	66
60	23.7	17.8	85	66
60	23.7	20.1	85	66
60	28.5	15.5	85	66
60	28.5	17.8	85	66
60	28.5	20.1	85	66
70	21.5	17.3	85	66
70	21.5	19.7	85	66
70	21.5	22.1	85	66
70	26.2	17.3	85	66
70	26.2	19.7	85	66
70	26.2	22.1	85	66
70	31.0	17.3	85	66
70	31.0	19.7	85	66
70	31.0	22.1	85	66
80	23.7	18.8	85	66
80	23.7	21.2	85	66
80	23.7	23.7	85	66
80	28.4	18.8	85	66
80	28.4	21.2	85	66
80	28.4	23.7	85	66
80	33.0	18.8	85	66
80	33.0	21.2	85	66
80	33.0	23.7	85	66
90	25.7	20.1	85	66
90	25.7	22.6	85	66
90	25.7	25.2	85	66
90	30.3	20.1	85	66
90	30.3	22.6	85	66
90	30.3	25.2	85	66
90	34.9	20.1	85	66
90	34.9	22.6	85	66
90	34.9	25.2	85	66
100	27.4	21.1	85	66
100	27.4	23.8	85	66

S_5^{Eval} continued…

Age	d^S	d^S	cc	pct^S
30	9.8	10.0	40	66
30	9.8	11.6	40	66
30	13.8	8.4	40	66
30	13.8	10.0	40	66
30	13.8	11.6	40	66
30	17.8	8.4	40	66
30	17.8	10.0	40	66
30	17.8	11.6	40	66
40	13.0	11.0	40	66
40	13.0	13.0	40	66
40	13.0	15.0	40	66
40	17.6	11.0	40	66
40	17.6	13.0	40	66
40	17.6	15.0	40	66
40	22.1	11.0	40	66
40	22.1	13.0	40	66
40	22.1	15.0	40	66
50	16.1	13.4	40	66
50	16.1	15.6	40	66
50	16.1	17.8	40	66
50	20.8	13.4	40	66
50	20.8	15.6	40	66
50	20.8	17.8	40	66
50	25.6	13.4	40	66
50	25.6	15.6	40	66
50	25.6	17.8	40	66
60	18.9	15.5	40	66
60	18.9	17.8	40	66
60	18.9	20.1	40	66
60	23.7	15.5	40	66
60	23.7	17.8	40	66
60	23.7	20.1	40	66
60	28.5	15.5	40	66
60	28.5	17.8	40	66
60	28.5	20.1	40	66
70	21.5	17.3	40	66
70	21.5	19.7	40	66
70	21.5	22.1	40	66
70	26.2	17.3	40	66
70	26.2	19.7	40	66
70	26.2	22.1	40	66
70	31.0	17.3	40	66
70	31.0	19.7	40	66
70	31.0	22.1	40	66
80	23.7	18.8	40	66

Age	d^S	d^S	cc	pct^S
40	13.0	11.0	70	33
40	13.0	13.0	70	33
40	13.0	15.0	70	33
40	17.6	11.0	70	33
40	17.6	13.0	70	33
40	17.6	15.0	70	33
40	22.1	11.0	70	33
40	22.1	13.0	70	33
40	22.1	15.0	70	33
50	16.1	13.4	70	33
50	16.1	15.6	70	33
50	16.1	17.8	70	33
50	20.8	13.4	70	33
50	20.8	15.6	70	33
50	20.8	17.8	70	33
50	25.6	13.4	70	33
50	25.6	15.6	70	33
50	25.6	17.8	70	33
60	18.9	15.5	70	33
60	18.9	17.8	70	33
60	18.9	20.1	70	33
60	23.7	15.5	70	33
60	23.7	17.8	70	33
60	23.7	20.1	70	33
60	28.5	15.5	70	33
60	28.5	17.8	70	33
60	28.5	20.1	70	33
70	21.5	17.3	70	33
70	21.5	19.7	70	33
70	21.5	22.1	70	33
70	26.2	17.3	70	33
70	26.2	19.7	70	33
70	26.2	22.1	70	33
70	31.0	17.3	70	33
70	31.0	19.7	70	33
70	31.0	22.1	70	33
80	23.7	18.8	70	33
80	23.7	21.2	70	33
80	23.7	23.7	70	33
80	28.4	18.8	70	33
80	28.4	21.2	70	33
80	28.4	23.7	70	33
80	33.0	18.8	70	33
80	33.0	21.2	70	33
80	33.0	23.7	70	33

Age	d^S	d^S	cc	pct^S
100	27.4	26.5	85	66
100	32.0	21.1	85	66
100	32.0	23.8	85	66
100	32.0	26.5	85	66
100	36.7	21.1	85	66
100	36.7	23.8	85	66
100	36.7	26.5	85	66
30	9.8	0.0	85	100
30	13.8	0.0	85	100
30	17.8	0.0	85	100
40	13.0	0.0	85	100
40	17.6	0.0	85	100
40	22.1	0.0	85	100
50	16.1	0.0	85	100
50	20.8	0.0	85	100
50	25.6	0.0	85	100
60	18.9	0.0	85	100
60	23.7	0.0	85	100
60	28.5	0.0	85	100
70	21.5	0.0	85	100
70	26.2	0.0	85	100
70	31.0	0.0	85	100
80	23.7	0.0	85	100
80	28.4	0.0	85	100
80	33.0	0.0	85	100
90	25.7	0.0	85	100
90	30.3	0.0	85	100
90	34.9	0.0	85	100
100	27.4	0.0	85	100
100	32.0	0.0	85	100
100	36.7	0.0	85	100
30	0.0	8.4	100	0
30	0.0	10.0	100	0
30	0.0	11.6	100	0
40	0.0	11.0	100	0
40	0.0	13.0	100	0
40	0.0	15.0	100	0
50	0.0	13.4	100	0
50	0.0	15.6	100	0
50	0.0	17.8	100	0
60	0.0	15.5	100	0
60	0.0	17.8	100	0
60	0.0	20.1	100	0
70	0.0	17.3	100	0
70	0.0	19.7	100	0

S_5^{Eval} continued…

Age	d^S	d^S	cc	pct^S
80	23.7	21.2	40	66
80	23.7	23.7	40	66
80	28.4	18.8	40	66
80	28.4	21.2	40	66
80	28.4	23.7	40	66
80	33.0	18.8	40	66
80	33.0	21.2	40	66
80	33.0	23.7	40	66
90	25.7	20.1	40	66
90	25.7	22.6	40	66
90	25.7	25.2	40	66
90	30.3	20.1	40	66
90	30.3	22.6	40	66
90	30.3	25.2	40	66
90	34.9	20.1	40	66
90	34.9	22.6	40	66
90	34.9	25.2	40	66
100	27.4	21.1	40	66
100	27.4	23.8	40	66
100	27.4	26.5	40	66
100	32.0	21.1	40	66
100	32.0	23.8	40	66
100	32.0	26.5	40	66
100	36.7	21.1	40	66
100	36.7	23.8	40	66
100	36.7	26.5	40	66
30	9.8	0.0	40	100
30	13.8	0.0	40	100
30	17.8	0.0	40	100
40	13.0	0.0	40	100
40	17.6	0.0	40	100
40	22.1	0.0	40	100
50	16.1	0.0	40	100
50	20.8	0.0	40	100
50	25.6	0.0	40	100
60	18.9	0.0	40	100
60	23.7	0.0	40	100
60	28.5	0.0	40	100
70	21.5	0.0	40	100
70	26.2	0.0	40	100
70	31.0	0.0	40	100
80	23.7	0.0	40	100
80	28.4	0.0	40	100
80	33.0	0.0	40	100
90	25.7	0.0	40	100

Age	d^S	d^S	cc	pct^S
90	25.7	20.1	70	33
90	25.7	22.6	70	33
90	25.7	25.2	70	33
90	30.3	20.1	70	33
90	30.3	22.6	70	33
90	30.3	25.2	70	33
90	34.9	20.1	70	33
90	34.9	22.6	70	33
90	34.9	25.2	70	33
100	27.4	21.1	70	33
100	27.4	23.8	70	33
100	27.4	26.5	70	33
100	32.0	21.1	70	33
100	32.0	23.8	70	33
100	32.0	26.5	70	33
100	36.7	21.1	70	33
100	36.7	23.8	70	33
100	36.7	26.5	70	33
30	9.8	8.4	70	66
30	9.8	10.0	70	66
30	9.8	11.6	70	66
30	13.8	8.4	70	66
30	13.8	10.0	70	66
30	13.8	11.6	70	66
30	17.8	8.4	70	66
30	17.8	10.0	70	66
30	17.8	11.6	70	66
40	13.0	11.0	70	66
40	13.0	13.0	70	66
40	13.0	15.0	70	66
40	17.6	11.0	70	66
40	17.6	13.0	70	66
40	17.6	15.0	70	66
40	22.1	11.0	70	66
40	22.1	13.0	70	66
40	22.1	15.0	70	66
50	16.1	13.4	70	66
50	16.1	15.6	70	66
50	16.1	17.8	70	66
50	20.8	13.4	70	66
50	20.8	15.6	70	66
50	20.8	17.8	70	66
50	25.6	13.4	70	66
50	25.6	15.6	70	66
50	25.6	17.8	70	66

Age	d^S	d^S	cc	pct^S
70	0.0	22.1	100	0
80	0.0	18.8	100	0
80	0.0	21.2	100	0
80	0.0	23.7	100	0
90	0.0	20.1	100	0
90	0.0	22.6	100	0
90	0.0	25.2	100	0
100	0.0	21.1	100	0
100	0.0	23.8	100	0
100	0.0	26.5	100	0
30	9.8	8.4	100	33
30	9.8	10.0	100	33
30	9.8	11.6	100	33
30	13.8	8.4	100	33
30	13.8	10.0	100	33
30	13.8	11.6	100	33
30	17.8	8.4	100	33
30	17.8	10.0	100	33
30	17.8	11.6	100	33
40	13.0	11.0	100	33
40	13.0	13.0	100	33
40	13.0	15.0	100	33
40	17.6	11.0	100	33
40	17.6	13.0	100	33
40	17.6	15.0	100	33
40	22.1	11.0	100	33
40	22.1	13.0	100	33
40	22.1	15.0	100	33
50	16.1	13.4	100	33
50	16.1	15.6	100	33
50	16.1	17.8	100	33
50	20.8	13.4	100	33
50	20.8	15.6	100	33
50	20.8	17.8	100	33
50	25.6	13.4	100	33
50	25.6	15.6	100	33
50	25.6	17.8	100	33
60	18.9	15.5	100	33
60	18.9	17.8	100	33
60	18.9	20.1	100	33
60	23.7	15.5	100	33
60	23.7	17.8	100	33
60	23.7	20.1	100	33
60	28.5	15.5	100	33
60	28.5	17.8	100	33

S_5^{Eval} continued…

Age	d^S	d^S	cc	pct^S
90	30.3	0.0	40	100
90	34.9	0.0	40	100
100	27.4	0.0	40	100
100	32.0	0.0	40	100
100	36.7	0.0	40	100
30	0.0	8.4	55	0
30	0.0	10.0	55	0
30	0.0	11.6	55	0
40	0.0	11.0	55	0
40	0.0	13.0	55	0
40	0.0	15.0	55	0
50	0.0	13.4	55	0
50	0.0	15.6	55	0
50	0.0	17.8	55	0
60	0.0	15.5	55	0
60	0.0	17.8	55	0
60	0.0	20.1	55	0
70	0.0	17.3	55	0
70	0.0	19.7	55	0
70	0.0	22.1	55	0
80	0.0	18.8	55	0
80	0.0	21.2	55	0
80	0.0	23.7	55	0
90	0.0	20.1	55	0
90	0.0	22.6	55	0
90	0.0	25.2	55	0
100	0.0	21.1	55	0
100	0.0	23.8	55	0
100	0.0	26.5	55	0
30	9.8	8.4	55	33
30	9.8	10.0	55	33
30	9.8	11.6	55	33
30	13.8	8.4	55	33
30	13.8	10.0	55	33
30	13.8	11.6	55	33
30	17.8	8.4	55	33
30	17.8	10.0	55	33
30	17.8	11.6	55	33
40	13.0	11.0	55	33
40	13.0	13.0	55	33
40	13.0	15.0	55	33
40	17.6	11.0	55	33
40	17.6	13.0	55	33
40	17.6	15.0	55	33
40	22.1	11.0	55	33

Age	d^S	d^S	cc	pct^S
60	18.9	15.5	70	66
60	18.9	17.8	70	66
60	18.9	20.1	70	66
60	23.7	15.5	70	66
60	23.7	17.8	70	66
60	23.7	20.1	70	66
60	28.5	15.5	70	66
60	28.5	17.8	70	66
60	28.5	20.1	70	66
70	21.5	17.3	70	66
70	21.5	19.7	70	66
70	21.5	22.1	70	66
70	26.2	17.3	70	66
70	26.2	19.7	70	66
70	26.2	22.1	70	66
70	31.0	17.3	70	66
70	31.0	19.7	70	66
70	31.0	22.1	70	66
80	23.7	18.8	70	66
80	23.7	21.2	70	66
80	23.7	23.7	70	66
80	28.4	18.8	70	66
80	28.4	21.2	70	66
80	28.4	23.7	70	66
80	33.0	18.8	70	66
80	33.0	21.2	70	66
80	33.0	23.7	70	66
90	25.7	20.1	70	66
90	25.7	22.6	70	66
90	25.7	25.2	70	66
90	30.3	20.1	70	66
90	30.3	22.6	70	66
90	30.3	25.2	70	66
90	34.9	20.1	70	66
90	34.9	22.6	70	66
90	34.9	25.2	70	66
100	27.4	21.1	70	66
100	27.4	23.8	70	66
100	27.4	26.5	70	66
100	32.0	21.1	70	66
100	32.0	23.8	70	66
100	32.0	26.5	70	66
100	36.7	21.1	70	66
100	36.7	23.8	70	66
100	36.7	26.5	70	66

Age	d^S	d^S	cc	pct^S
60	28.5	20.1	100	33
70	21.5	17.3	100	33
70	21.5	19.7	100	33
70	21.5	22.1	100	33
70	26.2	17.3	100	33
70	26.2	19.7	100	33
70	26.2	22.1	100	33
70	31.0	17.3	100	33
70	31.0	19.7	100	33
70	31.0	22.1	100	33
80	23.7	18.8	100	33
80	23.7	21.2	100	33
80	23.7	23.7	100	33
80	28.4	18.8	100	33
80	28.4	21.2	100	33
80	28.4	23.7	100	33
80	33.0	18.8	100	33
80	33.0	21.2	100	33
80	33.0	23.7	100	33
90	25.7	20.1	100	33
90	25.7	22.6	100	33
90	25.7	25.2	100	33
90	30.3	20.1	100	33
90	30.3	22.6	100	33
90	30.3	25.2	100	33
90	34.9	20.1	100	33
90	34.9	22.6	100	33
90	34.9	25.2	100	33
100	27.4	21.1	100	33
100	27.4	23.8	100	33
100	27.4	26.5	100	33
100	32.0	21.1	100	33
100	32.0	23.8	100	33
100	32.0	26.5	100	33
100	36.7	21.1	100	33
100	36.7	23.8	100	33
100	36.7	26.5	100	33
30	9.8	8.4	100	66
30	9.8	10.0	100	66
30	9.8	11.6	100	66
30	13.8	8.4	100	66
30	13.8	10.0	100	66
30	13.8	11.6	100	66
30	17.8	8.4	100	66
30	17.8	10.0	100	66

S_5^{Eval} continued…

Age	d^S	d^S	cc	pct^S
40	22.1	13.0	55	33
40	22.1	15.0	55	33
50	16.1	13.4	55	33
50	16.1	15.6	55	33
50	16.1	17.8	55	33
50	20.8	13.4	55	33
50	20.8	15.6	55	33
50	20.8	17.8	55	33
50	25.6	13.4	55	33
50	25.6	15.6	55	33
50	25.6	17.8	55	33
60	18.9	15.5	55	33
60	18.9	17.8	55	33
60	18.9	20.1	55	33
60	23.7	15.5	55	33
60	23.7	17.8	55	33
60	23.7	20.1	55	33
60	28.5	15.5	55	33
60	28.5	17.8	55	33
60	28.5	20.1	55	33
70	21.5	17.3	55	33
70	21.5	19.7	55	33
70	21.5	22.1	55	33
70	26.2	17.3	55	33
70	26.2	19.7	55	33
70	26.2	22.1	55	33
70	31.0	17.3	55	33
70	31.0	19.7	55	33
70	31.0	22.1	55	33
80	23.7	18.8	55	33
80	23.7	21.2	55	33
80	23.7	23.7	55	33
80	28.4	18.8	55	33
80	28.4	21.2	55	33
80	28.4	23.7	55	33
80	33.0	18.8	55	33
80	33.0	21.2	55	33
80	33.0	23.7	55	33
90	25.7	20.1	55	33
90	25.7	22.6	55	33
90	25.7	25.2	55	33
90	30.3	20.1	55	33
90	30.3	22.6	55	33
90	30.3	25.2	55	33
90	34.9	20.1	55	33

Age	d^S	d^S	cc	pct^S
30	9.8	0.0	70	100
30	13.8	0.0	70	100
30	17.8	0.0	70	100
40	13.0	0.0	70	100
40	17.6	0.0	70	100
40	22.1	0.0	70	100
50	16.1	0.0	70	100
50	20.8	0.0	70	100
50	25.6	0.0	70	100
60	18.9	0.0	70	100
60	23.7	0.0	70	100
60	28.5	0.0	70	100
70	21.5	0.0	70	100
70	26.2	0.0	70	100
70	31.0	0.0	70	100
80	23.7	0.0	70	100
80	28.4	0.0	70	100
80	33.0	0.0	70	100
90	25.7	0.0	70	100
90	30.3	0.0	70	100
90	34.9	0.0	70	100
100	27.4	0.0	70	100
100	32.0	0.0	70	100
100	36.7	0.0	70	100
30	0.0	8.4	85	0
30	0.0	10.0	85	0
30	0.0	11.6	85	0
40	0.0	11.0	85	0
40	0.0	13.0	85	0
40	0.0	15.0	85	0
50	0.0	13.4	85	0
50	0.0	15.6	85	0
50	0.0	17.8	85	0
60	0.0	15.5	85	0
60	0.0	17.8	85	0
60	0.0	20.1	85	0
70	0.0	17.3	85	0
70	0.0	19.7	85	0
70	0.0	22.1	85	0
80	0.0	18.8	85	0
80	0.0	21.2	85	0
80	0.0	23.7	85	0
90	0.0	20.1	85	0
90	0.0	22.6	85	0
90	0.0	25.2	85	0

Age	d^S	d^S	cc	pct^S
30	17.8	11.6	100	66
40	13.0	11.0	100	66
40	13.0	13.0	100	66
40	13.0	15.0	100	66
40	17.6	11.0	100	66
40	17.6	13.0	100	66
40	17.6	15.0	100	66
40	22.1	11.0	100	66
40	22.1	13.0	100	66
40	22.1	15.0	100	66
50	16.1	13.4	100	66
50	16.1	15.6	100	66
50	16.1	17.8	100	66
50	20.8	13.4	100	66
50	20.8	15.6	100	66
50	20.8	17.8	100	66
50	25.6	13.4	100	66
50	25.6	15.6	100	66
50	25.6	17.8	100	66
60	18.9	15.5	100	66
60	18.9	17.8	100	66
60	18.9	20.1	100	66
60	23.7	15.5	100	66
60	23.7	17.8	100	66
60	23.7	20.1	100	66
60	28.5	15.5	100	66
60	28.5	17.8	100	66
60	28.5	20.1	100	66
70	21.5	17.3	100	66
70	21.5	19.7	100	66
70	21.5	22.1	100	66
70	26.2	17.3	100	66
70	26.2	19.7	100	66
70	26.2	22.1	100	66
70	31.0	17.3	100	66
70	31.0	19.7	100	66
70	31.0	22.1	100	66
80	23.7	18.8	100	66
80	23.7	21.2	100	66
80	23.7	23.7	100	66
80	28.4	18.8	100	66
80	28.4	21.2	100	66
80	28.4	23.7	100	66
80	33.0	18.8	100	66
80	33.0	21.2	100	66

S_5^{Eval} continued…

Age	d^S	d^S	cc	pct^S
90	34.9	22.6	55	33
90	34.9	25.2	55	33
100	27.4	21.1	55	33
100	27.4	23.8	55	33
100	27.4	26.5	55	33
100	32.0	21.1	55	33
100	32.0	23.8	55	33
100	32.0	26.5	55	33
100	36.7	21.1	55	33
100	36.7	23.8	55	33
100	36.7	26.5	55	33
30	9.8	8.4	55	66
30	9.8	10.0	55	66
30	9.8	11.6	55	66
30	13.8	8.4	55	66
30	13.8	10.0	55	66
30	13.8	11.6	55	66
30	17.8	8.4	55	66
30	17.8	10.0	55	66
30	17.8	11.6	55	66
40	13.0	11.0	55	66
40	13.0	13.0	55	66
40	13.0	15.0	55	66
40	17.6	11.0	55	66
40	17.6	13.0	55	66
40	17.6	15.0	55	66
40	22.1	11.0	55	66
40	22.1	13.0	55	66
40	22.1	15.0	55	66
50	16.1	13.4	55	66
50	16.1	15.6	55	66
50	16.1	17.8	55	66
50	20.8	13.4	55	66
50	20.8	15.6	55	66
50	20.8	17.8	55	66
50	25.6	13.4	55	66
50	25.6	15.6	55	66
50	25.6	17.8	55	66
60	18.9	15.5	55	66
60	18.9	17.8	55	66
60	18.9	20.1	55	66
60	23.7	15.5	55	66
60	23.7	17.8	55	66

Age	d^S	d^S	cc	pct^S
100	0.0	21.1	85	0
100	0.0	23.8	85	0
100	0.0	26.5	85	0
30	9.8	8.4	85	33
30	9.8	10.0	85	33
30	9.8	11.6	85	33
30	13.8	8.4	85	33
30	13.8	10.0	85	33
30	13.8	11.6	85	33
30	17.8	8.4	85	33
30	17.8	10.0	85	33
30	17.8	11.6	85	33
40	13.0	11.0	85	33
40	13.0	13.0	85	33
40	13.0	15.0	85	33
40	17.6	11.0	85	33
40	17.6	13.0	85	33
40	17.6	15.0	85	33
40	22.1	11.0	85	33
40	22.1	13.0	85	33
40	22.1	15.0	85	33
50	16.1	13.4	85	33
50	16.1	15.6	85	33
50	16.1	17.8	85	33
50	20.8	13.4	85	33
50	20.8	15.6	85	33
50	20.8	17.8	85	33
50	25.6	13.4	85	33
50	25.6	15.6	85	33
50	25.6	17.8	85	33
60	18.9	15.5	85	33
60	18.9	17.8	85	33
60	18.9	20.1	85	33
60	23.7	15.5	85	33
60	23.7	17.8	85	33
60	23.7	20.1	85	33
60	28.5	15.5	85	33
60	28.5	17.8	85	33
60	28.5	20.1	85	33
70	21.5	17.3	85	33
70	21.5	19.7	85	33
70	21.5	22.1	85	33
70	26.2	17.3	85	33

Age	d^S	d^S	cc	pct^S
80	33.0	23.7	100	66
90	25.7	20.1	100	66
90	25.7	22.6	100	66
90	25.7	25.2	100	66
90	30.3	20.1	100	66
90	30.3	22.6	100	66
90	30.3	25.2	100	66
90	34.9	20.1	100	66
90	34.9	22.6	100	66
90	34.9	25.2	100	66
100	27.4	21.1	100	66
100	27.4	23.8	100	66
100	27.4	26.5	100	66
100	32.0	21.1	100	66
100	32.0	23.8	100	66
100	32.0	26.5	100	66
100	36.7	21.1	100	66
100	36.7	23.8	100	66
100	36.7	26.5	100	66
30	9.8	0.0	100	100
30	13.8	0.0	100	100
30	17.8	0.0	100	100
40	13.0	0.0	100	100
40	17.6	0.0	100	100
40	22.1	0.0	100	100
50	16.1	0.0	100	100
50	20.8	0.0	100	100
50	25.6	0.0	100	100
60	18.9	0.0	100	100
60	23.7	0.0	100	100
60	28.5	0.0	100	100
70	21.5	0.0	100	100
70	26.2	0.0	100	100
70	31.0	0.0	100	100
80	23.7	0.0	100	100
80	28.4	0.0	100	100
80	33.0	0.0	100	100
90	25.7	0.0	100	100
90	30.3	0.0	100	100
90	34.9	0.0	100	100
100	27.4	0.0	100	100
100	32.0	0.0	100	100
100	36.7	0.0	100	100

Appendix D: Commercial Thinning Equations

This appendix shows the equations and supporting data for commercial thinning as it was implemented in this thesis. See section 3.2 for equations and graph for thinning from below for softwood. Equations (D-1) and (D-2) are the same when thinning from above and from below for softwood or hardwood. Equation (D-3) is used for thinning from below where $\tilde{d}^{\theta}$ goes up subsequent to thinning and equation (D-4) is used for thinning from above where $\tilde{d}^{\theta}$ goes down.

14Rem = ($BARem^{\theta}$ - 0.05) / 0.35 * (14_high - 14_low) + 14_low (D-1)
35Rem = ($BARem^{\theta}$ - 0.05) / 0.35 * (35_high - 35_low) + 35_low (D-2)

Thinning from below: $\tilde{d}^{\theta}$ = d^{θ}+ (d^{θ} - 14) / 21 * (35Rem - 14Rem) + 14Rem (D-3)

Thinning from above: $\tilde{d}^{\theta}$ = d^{θ}- ((d^{θ} - 14) / 21 * (35Rem - 14Rem) + 14Rem) (D-4)

where $BARem^{\theta}$ is the actual basal area percentage to be removed for species θ
14_low – Average diameter change for $BARem^{\theta}$ = 5% and d^{θ} = 14 cm
14_high – Average diameter change for $BARem^{\theta}$ = 40% and d^{θ} = 14 cm
35_low – Average diameter change for $BARem^{\theta}$ = 5% and d^{θ} = 35 cm
35_high – Average diameter change for $BARem^{\theta}$ = 40% and d^{θ} = 35 cm
14Rem – Average diameter change for a given $BARem^{\theta}$ with d^{θ} = 14 cm
35Rem – Average diameter change for a given $BARem^{\theta}$ with d^{θ} = 35 cm
$\tilde{d}^{\theta}$ is the average diameter of the stand immediately following the CT

Softwood thinning from above

For each combination of species type and thinning type, equations (D-1) and (D-2) are used to calculate the diameter growth for a stand with 14 cm and 35 cm diameters for the given $BARem^{\theta}$ percentage. The values in equations (D-1a) and (D-2a) are for softwood thinning from above.

14Rem = ($BARem^{\theta}$ - 0.05) / 0.35 * (1.5345 – 0.1947) + 0.1947 (D-1a)
35Rem = ($BARem^{\theta}$ - 0.05) / 0.35 * (2.9183 – 0.3254) + 0.3254 (D-2a)

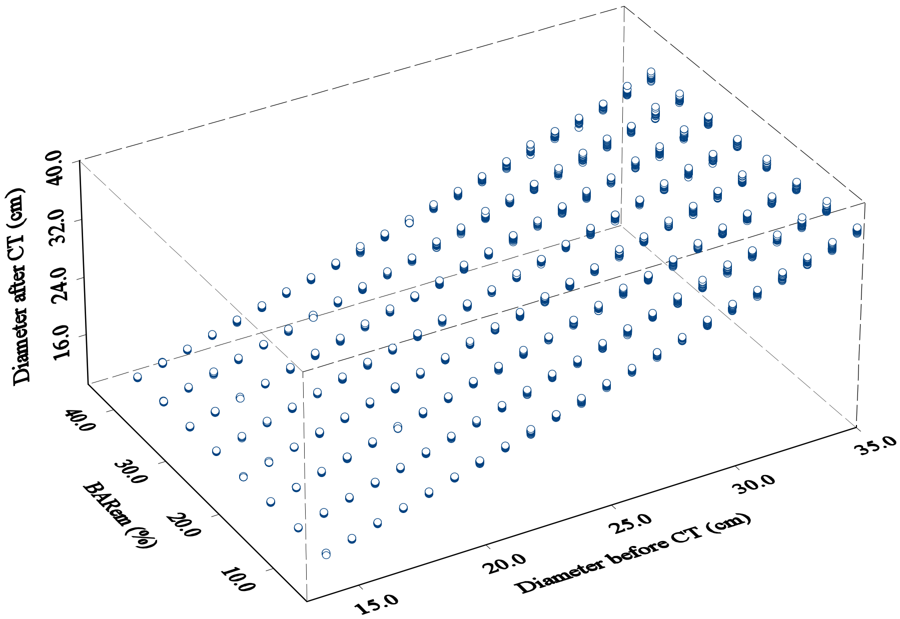

Figure D-1 - Plot of 5280 diameter change simulations for CT from above for SW

Hardwood thinning from below

$$14Rem = (BARem^{\theta} - 0.05) / 0.35 * (2.3199 - 0.6675) + 0.6675 \quad \text{(D-1b)}$$
$$35Rem = (BARem^{\theta} - 0.05) / 0.35 * (8.0899 - 1.0476) + 1.0476 \quad \text{(D-2b)}$$

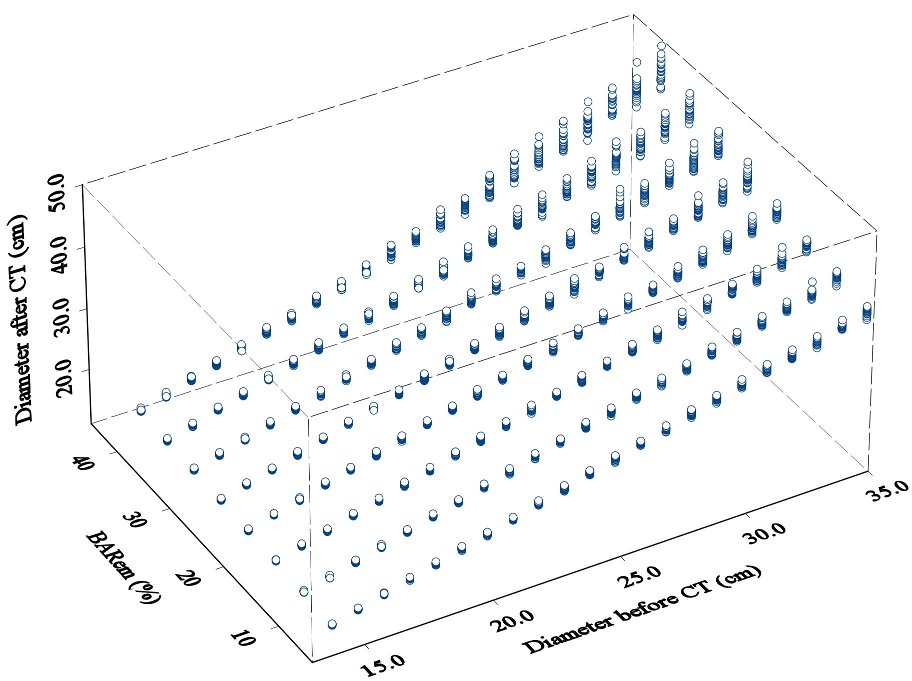

Figure D-2 - Plot of 5280 diameter change simulations for CT from below for HW

Hardwood thinning from above

$$14Rem = (BARem^{\theta} - 0.05) / 0.35 * (1.5243 - 0.215) + 0.215 \quad \text{(D-1c)}$$

$$35Rem = (BARem^{\theta} - 0.05) / 0.35 * (3.0908 - 0.3517) + 0.3517 \quad \text{(D-2c)}$$

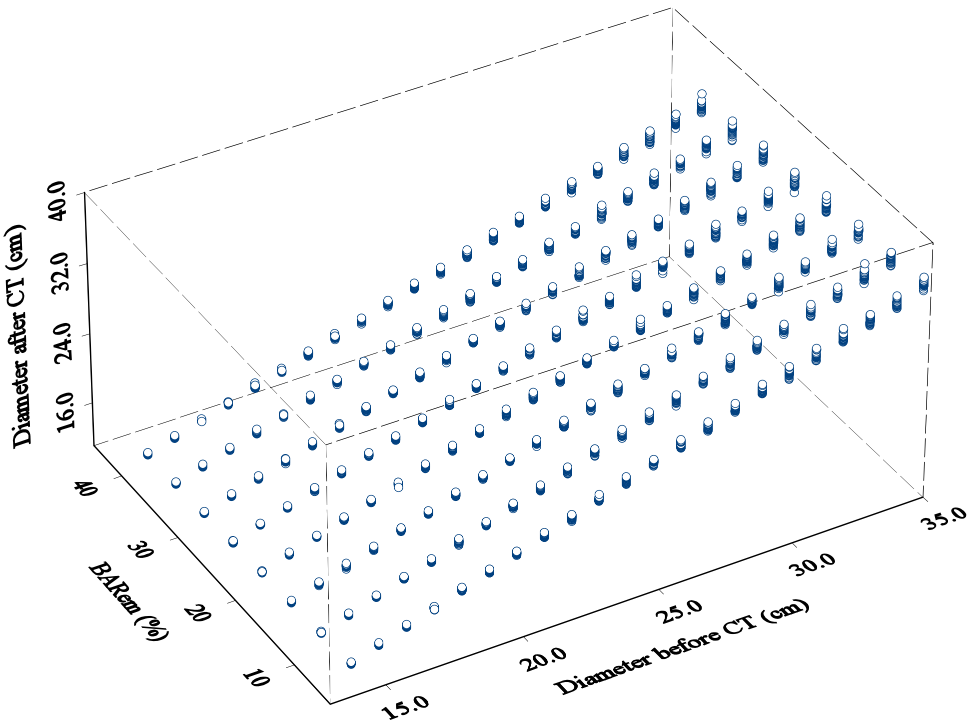

Figure D-3 - Plot of 5280 diameter change simulations for CT from above for HW

Appendix E: Visual Basic Code for the DP Model

The DP model used in this thesis was built using the Visual Basic coding language and is included in this thesis in its entirety. It is available as a download in text file format from Dalhousie University or from the author upon request. The code uses a form on which the user can enter scenario specific parameter values before launching the DP optimization. A screen shot of the form is shown in section 4.6. Please contact the author if this form is needed to run the code.